International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 7, 2025
https://www.theaspd.com/ijes.php

Observer Based Adaptive Event Triggered Control Scheme For A Class Nonlinear Systems With Uncertain Dynamics

Ajay Kulkarni [1*] **Nitesh Kumar Soni** [2] **and Vinit Gupta** [3]

[1,2,3]Medi-Caps University Indore India
* Corresponding author's Email: ajay.kulkarni@medicaps.ac.in

Abstract—*This work presents an adaptive event triggered control scheme for a class of uncertain nonlinear system with unmeasurable states. Controlling strategy presented in this work mainly includes the designing of a nonlinear state estimator to estimate the system states followed by the designing of an event triggered control mechanism for a networked control system. The proposed scheme ensures the uniform ultimate boundedness (UUB) of system variables along with prescribed performance of the closed loop system and at the same time ensuring the optimum sharing of the network resources. An output recurrent wavelet neural network (ORWNN) is used to approximate the nonlinear uncertainties of the system. Numerical simulation is carried out to illustrate theoretical developments.*
Keywords—*Networked Control System (NCS), Event Triggered Control, Zeno Behavior Output Recurrent Wavelet Neural Network (ORWNN) State Nonlinear Observer*

1. INTRODUCTION

Networked Control System (NCS) can be viewed as a typical case of distributed system composed of control system components such as plant, actuator, controller, and sensors which are connected through a communication channel for closed loop configuration. In networked control system, various closed loop signals are transmitted from one component to another in the form of information packets through the communication channel [1]. One of the profound advantages of networked control systems is the establishment of a distributed closed loop which allows the tele control of systems like unmanned arial vehicles, mining robots and others over a communication channel. However, practical communication channel is associated with issues like limited bandwidth, delay, packet loss, channel noise or the complicacies generated by a particular communication protocol used, these network imperfections often degrade the system performance and sometimes even makes the system unstable. In networked control scheme, classical control strategy is augmented with mechanisms like event triggering, use of predictor to compensate the network imperfections. In networked control system, event triggered scheme has been to be a highly effective methodology for the appropriate utilization of the network resources. This scheme allows the availability of network resources for data transmission only on the violation of some predesigned event triggering mechanisms [1-7].

Various control methodologies existing in the literature require an accurate mathematical model of the real time system to ensure the effective controlling of that system. These control schemes require effective cancellation of the nonlinear system dynamics. However, development of analytical models of such nonlinear phenomenon is a non-trivial task as the basic laws of physics cannot explain these nonlinearities with a desired level of accuracy. Nonparametric experimental modelling has been proved to be an effective way to model out such complexities. This flexible approach allows the use of universal approximators like neural networks, wavelet networks to approximation of nonlinear trends presents in the system. Augmentation of controllers with these approximators and making them learn by suitably adjusting their weights with the help of update laws has relaxed the model dependency constraint to a large extent [10-19]. There exist several approximator architectures with different features and capabilities. Output recurrent architecture has been proved to be an effective architecture, in comparison to conventional feedforward architectures, it offers the advantages of improved temporal dynamism due to the inclusion of feedback mechanism. Wavelet functions based approximators have been proved to be highly efficacious approximators due to their systematic construction methodology and unique identification capabilities [13].

The existence of observer designing schemes has greatly streamlined the designing of state feedback controllers for the systems where system states are not available for construction of control term. Observer is a control component used to get an estimate of the internal states of system through its output. The vitality of this component can be analyzed by considering the situations of physical systems where system states are not accessible due to sensor faults, unavailability of sensors or unavailability of sensor installation locations. Various design approaches starting from observer design for linear control systems and extending up to observer design for nonlinear systems with uncertain dynamics have been cited in the literature [14-17].

This works presents an adaptive observer-based event triggered control scheme for a class of uncertain nonlinear systems. The main contribution of this paper are as follows

a. Firstly, an output recurrent wavelet neural network based adaptive Luenberger observer is developed for effective estimation of the system states using the system output.

b. Adaptive observer is located at the plant site and there is continuous access to system output. This allows the continuous update of observer dynamics and weights of the wavelet neural network. State estimates from the observer are subsequently used for designing event triggering controller.

c. Output of the wavelet estimator is also transmitted to the controller along with the system states. Information transmission is carried out at discrete triggering instants.

d. Triggering mechanisms are designed to ensure effective exploration of event triggering mechanism and system performance within the allowable limits.

e. Zeno free behavior of the proposed event triggered control scheme is ensured by establishing the fact that there exist a finite lower limit of inter-execution time.

The rest of the paper is constituted as follows: Section 2 presents the framework of the system preliminaries in terms of mathematical model of the nonlinear system, proposed strategy is applicable to nonlinear systems which can be represented in this form. This section also details the output recurrent wavelet network and its characteristics. Section 3 describes the designing concepts of adaptive observer. Section 4 develops the event triggered control mechanism. Starting from the designing of baseline controller and subsequent transformation of this control scheme into event triggered form is described in section 4. This section also addresses issues like avoidance of Zeno behaviour. Section 5 illustrates the validity of theoretical development with the help of a simulation study whereas section 6 concludes the paper.

2. PRELIMINARY

2.1. System Formulation

Consider the following class of strict feedback nonlinear uncertain systems [18]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(x) + u \\ y = [c_1 \quad \cdots \quad c_n][x_1 \quad \cdots \quad x_n]^T \end{cases} \tag{1}$$

where $x = [x_1 \quad \cdots \quad x_n]^T \in R^n$ represents system states, $u \in R$ and $y \in R$ are control input and system output respectively. Nonlinear term $f(x): R^n \rightarrow R$ represents the system uncertainty and $c_i; i = 1, \dots, n$ are the elements of matrix C defined below. All the system states are unmeasurable; however, the output is available for measurement.

System (1) can be expressed in following state-space form

$$\begin{cases} \dot{x} = Ax + B(f(x) + u) \\ y = Cx \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & \cdots & 0 & 0 & 0 \end{bmatrix}_{nXn} ; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{nX1} \quad C = [c_1 \quad \cdots \quad c_n]_{1Xn}$$

The objective of this paper is to design an adaptive event triggered control scheme for the uncertain nonlinear system of the form (1) with unknown system states. An adaptive observer is designed to estimate the unknown states. Control term so designed must steer the system states towards the desired trajectory so that the resulting error dynamics converges to a compact set containing origin.

Following assumptions are considered for the system (1) [18 - 20]

Assumption 1: Nonlinear function $f(x(t))$ satisfies the condition of Lipschitz continuity i.e. following inequality is satisfied on every compact set $\Omega_x \subset R^n$

$$\|f(x(t)) - f(x(t_k))\| \leq L\|x_i(t) - x_i(t_k)\| \quad (3)$$

where $L > 0$ is the Lipschitz constant.

Assumption 2: Desired trajectory $y_d(t) \in R$ and its derivatives up to n^{th} order $\{y_d(t), \dot{y}_d(t), \ddot{y}_d(t), \dots\}$ are bounded and known.

Also, $\left|\overset{i}{y}_d(t_1) - \overset{i}{y}_d(t_2)\right| \leq \sigma \in L_\infty, \forall t_1, t_2 \in R^+, i = 1, \dots, n$

Assumption 3: Triple (A, B, C) is an controllable -observable triple.

Assumption 4: There exists a positive definite symmetric (PDS) matrix P such that the following equality holds

$$B^T P = C \quad (4)$$

Assumption 5: Let there exist observer gain matrix m such that the matrix $(A - mC)$ is Hurwitz stable. Thus, there exists an arbitrary positive definite matrix Q such that positive definite matrix P is the unique solution of the equation

$$(A - mC)^T P + P(A - mC) = -Q \quad (5)$$

Assumption 6: Let there exist controller gain matrix K such that the matrix $(A - BK)$ is Hurwitz stable. Thus, there exists an arbitrary positive definite matrix Q_1 such that positive definite matrix P_1 is the unique solution of the equation

$$(A - BK)^T P_1 + P_1(A - BK) = -Q_1 \quad (6)$$

Remark 1 Design matrices P and P_1 should be selected such that Assumptions 3 and 4 are satisfied.

Lemma 1. For any matrices X and Y, following inequality holds

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y \quad (7)$$

where $\varepsilon > 0$

2.2. Output Recurrent Wavelet Neural Network (ORWNN)

Insertion of recurrency traits such as output recurrency in the classical feedforward structure of function approximators results in the inclusion of feedback link from output layer to input layer. This mechanism allows the feedback of network output to input side so as to serve as an additional input component. Recurrency offers several advantages over conventional feedforward architecture such as improved dynamic characterization, better contextual recognition, efficient pattern recognition. These features have enhanced the applicability of recurrent architecture [13].

Wavelet families have displayed a promising potential in the realm of function approximation due to their characteristics of orthonormality and multiresolution analysis. Orthonormality allows unique function representation whereas multiresolution analysis allows a systematic approach for constructing an approximation framework. Thus, wavelet network can be viewed as a parsimonious approximation structure in which each wavelet approximates a unique attribute of the function to be approximated.

Any square integrable nonlinear function $f(x) \in L^2(R)$ defined on any compact set of system states $\Omega_x \subset R^n$ can be approximated by using output recurrent wavelet neural network as

$$\hat{f}(x) = \sum_{k=1,2,\dots}^{K_{J_0}} \alpha_{J_0,k} \prod_{i=1}^{n} \varphi_{J_0,k}(x_i)\, \varphi_{J_0,k}\left(\hat{f}(x)\right) + \sum_{j=J_0}^{J} \sum_{k=1,2\dots}^{K_j} \beta_{j,k} \prod_{i=1}^{n} \psi_{j,k}(x_i)\, \psi_{j,k}\left(\hat{f}(x)\right)$$

$$\hat{f}(x) = \sum_{k=1,2,\dots}^{K_{J_0}} \alpha_{J_0,k}\, \varphi_{J_0,k}(x') + \sum_{j=J_0}^{J} \sum_{k=1,2\dots}^{K_j} \beta_{j,k} \psi_{j,k}(x') \quad (8)$$

In vector matrix form (7) can be represented as

$$\hat{f}(x) = \alpha^T \varphi(x') + \beta^T \psi(x') \quad (9)$$

where $x' = [x_1, \ x_2, \ \dots, \ x_n, \ \hat{f}(x)]^T$ is the input vector, $\varphi_{J_{0,k}}(x')$ and $\psi_{j,k}(x')$ are the vectors of scaling and wavelet functions with α and β as the weight vectors of these functions respectively. Functions may be taken from an appropriate wavelet family satisfying the norms of multiresolution analysis. Also, $j \in Z$ and $k \in Z$ are wavelet parameters namely dilates and translates. From analytical point of view, these translates and dilates should extend over the entire temporal and spectral range of the function. This approach often results in an uncountable number of wavelet functions in (4) which is not feasible for constructing a practical network. However, universal approximation theorem and the norm of multiresolution analysis provides a pragmatic way to construct a parsimonious wavelet network structure.

Lemma 2: There exist finite number of dilates j with some coarsest and finest values J_0 and J respectively along with finite translates k at each dilation level such that a network constructed with these values can approximate and function $f(x) \in L^2(R)$ defined on a compact set $\Omega_x \subset R^n$ with arbitrary precision. Thus, there exist some optimal value of weight vectors α^* and β^* such that

$$f(x) = \alpha^{*T}\varphi(x') + \beta^{*T}\psi(x') + \varepsilon. \quad (10)$$

where $0 < \epsilon < |\epsilon_m|$

With α^* and β^* as the estimates of α^* and β^*, wavelet network estimation of the function $f(x)$ can be expressed as

$$\hat{f}(x) = \hat{\alpha}^T\varphi(x') + \hat{\beta}^T\psi(x') \quad (11)$$

Optimal weight vectors are now selected as

$$[\alpha^*, \quad \beta^*]^T = \underset{[\hat{\alpha} \quad \hat{\beta}]^T \in \Omega}{\arg min} \left\{ \sup_{x \in \Omega_x} \left| f(x) - \hat{f}(x) \right| \right\} \quad (12)$$

where Ω and Ω_x are compact sets of weights and state variables respectively.

Thus, estimation error can be defined as

$$\tilde{f}(x) = f(x) - \hat{f}(x) = \tilde{\alpha}^T\varphi(x') + \tilde{\beta}^T\psi(x') + \epsilon \quad (13)$$

where $\tilde{\alpha}^T = \alpha^{*T} - \hat{\alpha}^T$ and $\tilde{\beta}^T = \beta^{*T} - \hat{\beta}^T$

Corollary: With properly designed update laws, weight parameters can be steered towards their optimum values and weight estimation error can be reduced to arbitrarily small value and function estimation error can be confined to a compact set such that $\left|\tilde{f}(x)\right| < f_m$ for $\Omega_x \subset R^n$.

3. OBSERVER DESIGN AND ANALYSIS

3.1. Observer Design

This section describes the designing aspects of a Luenberger adaptive nonlinear observer for the state estimation of the system (2) under the assumptions 1-6. For the system of form (2), required observer can be modelled as [15]

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + B\left(\hat{f}(\hat{x}) + u\right) + m(y - \hat{y}) \\ \qquad\qquad \hat{y} = C\hat{x} \end{cases} \quad (14)$$

where $\hat{x} = [\hat{x}_1, \quad \dots \quad, \hat{x}_n]^T \in R^n$ are the estimates of the system states, $\hat{y}$ is the estimate of system output,$m = [m_1 \quad m_2 \quad \dots \quad m_n]^T$ is the observer gain matrix and $\hat{f}(\hat{x})$ is the wavelet network estimate of the uncertain term $f(x)$. As the function arguments are not available, their estimates are used in wavelet approximator as the input terms. Equation (14) can be expressed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + B\left(\hat{f}(\hat{x}) + u\right) + mC(x - \hat{x}) \\ \qquad\qquad \hat{y} = C\hat{x} \end{cases} \quad (15)$$

Defining the observer estimation error as

$$\tilde{x} = x - \hat{x} = [x_1 - \hat{x}_1 \quad x_2 - \hat{x}_2 \quad \dots \quad x_n - \hat{x}_n]^T \quad (16)$$

Differentiation of the error terms (16) results in the following error dynamics

$$\dot{\tilde{x}} = (A - mC)\tilde{x} + B\left(f(x) - \hat{f}(\hat{x})\right) \quad (17)$$

For an efficient observer design, the error dynamics (17) must settle down rapidly to a residual set containing origin. Next subsection presents the convergence analysis of observer error dynamics.

3.2. Convergence Analysis

To analyze the convergence, consider the Lyapunov function of the form

$$V_1 = \frac{1}{2}\tilde{x}^T P\tilde{x} + \frac{1}{2}(\tilde{\alpha}^T\tilde{\alpha} + \tilde{\beta}^T\tilde{\beta}) \quad (18)$$

where P is a positive definite symmetric matrix of appropriate dimensions.

Differentiating (18) along the system trajectories and making appropriate substitutions using (15)

$$\dot{V}_1 = \frac{1}{2}\{\dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}}\} + (\tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}}) \quad (19)$$

$$\dot{V}_1 = \frac{1}{2}\left\{\left\{(A - mC)\tilde{x} + B\left(f(x) - \hat{f}(\hat{x})\right)\right\}^T P\tilde{x} + \tilde{x}^T P\left\{(A - mC)\tilde{x} + B\left(f(x) - \hat{f}(\hat{x})\right)\right\}\right\} + (\tilde{\alpha}^T\dot{\tilde{\alpha}} + \tilde{\beta}^T\dot{\tilde{\beta}})$$

International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 7, 2025
https://www.theaspd.com/ijes.php

$$\dot{V}_1=\frac{1}{2}\left\{\left\{(A-mC)\tilde{x}+B\left(f(x)-\hat{f}(\hat{x})\right)\right\}^T P\tilde{x}+\tilde{x}^T P\left\{(A-mC)\tilde{x}+B\left(f(x)-\hat{f}(\hat{x})\right)\right\}\right\}+(\tilde{\alpha}^T\dot{\tilde{\alpha}}+\tilde{\beta}^T\dot{\tilde{\beta}})$$

$$\dot{V}_1=\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\big(f(x)-f(\hat{x})\big)^T B^T P\tilde{x}+\tilde{x}^T PB\big(f(x)-f(\hat{x})\big)+\left(f(\hat{x})-\hat{f}(\hat{x})\right)^T B^T P\tilde{x}+\tilde{x}^T PB\left(f(\hat{x})-\hat{f}(\hat{x})\right)\Big\}+(\tilde{\alpha}^T\dot{\tilde{\alpha}}+\tilde{\beta}^T\dot{\tilde{\beta}}) \quad (20)$$

Here the term $\left(f(\hat{x})-\hat{f}(\hat{x})\right)$ (14) represents the wavelet estimation error for nonlinear term $f(\hat{x})$ so above equation becomes

$$\dot{V}_1=\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\big(f(x)-f(\hat{x})\big)^T B^T P\tilde{x}+\tilde{x}^T PB\big(f(x)-f(\hat{x})\big)+\big(\tilde{\alpha}^T\varphi(x')+\tilde{\beta}^T\psi(x')+\epsilon\big)^T B^T P\tilde{x}+\tilde{x}^T PB\big(\tilde{\alpha}^T\varphi(x')+\tilde{\beta}^T\psi(x')+\epsilon\big)\Big\}+(\tilde{\alpha}^T\dot{\tilde{\alpha}}+\tilde{\beta}^T\dot{\tilde{\beta}})$$

Applying the conditions stated in assumption 4,

$$\dot{V}_1=\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\big(f(x)-f(\hat{x})\big)^T B^T P\tilde{x}+\tilde{x}^T PB\big(f(x)-f(\hat{x})\big)+\left(\tilde{\alpha}^T\varphi(x')+\tilde{\beta}^T\psi(x')\right)^T\tilde{y}+\tilde{y}^T\left(\tilde{\alpha}^T\varphi(x')+\tilde{\beta}^T\psi(x')\right)+\epsilon^T B^T P\tilde{x}+\tilde{x}^T PB\epsilon\Big\}+(\tilde{\alpha}^T\dot{\tilde{\alpha}}+\tilde{\beta}^T\dot{\tilde{\beta}})$$

With the update laws of the form

$$\begin{cases}\dot{\tilde{\alpha}}=-\varphi(x')\tilde{y}^T\\ \dot{\tilde{\beta}}=-\psi(x')\tilde{y}^T\end{cases} \quad (21)$$

above equation becomes

$$\dot{V}_1=\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\big(f(x)-f(\hat{x})\big)^T B^T P\tilde{x}+\tilde{x}^T PB\big(f(x)-f(\hat{x})\big)+\epsilon^T B^T P\tilde{x}+\tilde{x}^T PB\epsilon\Big\}$$

Applying Leema 1 to above equation

$$\dot{V}_1=\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\varepsilon_1\big(f(x)-f(\hat{x})\big)^T B^T B\big(f(x)-f(\hat{x})\big)+\frac{1}{\varepsilon_1}\tilde{x}^T PP\tilde{x}\Big\}+\frac{1}{2}(\varepsilon_2\tilde{x}^T PBB^T P\tilde{x}+\frac{1}{\varepsilon_2}\epsilon^T\epsilon)$$

$$\dot{V}_1\le\frac{1}{2}\Big\{\tilde{x}^T\{(A-mC)^T P+P(A-mC)\}\tilde{x}+\varepsilon_1\|f(x)-f(\hat{x})\|^2+\frac{1}{\varepsilon_1}\tilde{x}^T P^2\tilde{x}\Big\}+\frac{1}{2}(\varepsilon_2\tilde{x}^T PBB^T P\tilde{x}+\frac{1}{\varepsilon_2}\epsilon^T\epsilon)$$

Applying assumption 1

$$\dot{V}_1\le\frac{1}{2}\Big\{-\tilde{x}^T Q\tilde{x}+\varepsilon_1 L_1^2\|x-\hat{x}\|^2+\frac{1}{\varepsilon_1}\tilde{x}^T P^2\Big\}+\frac{1}{2}(\varepsilon_2\tilde{x}^T PBB^T P\tilde{x}+\frac{1}{\varepsilon_2}\epsilon^T\epsilon)$$

$$\dot{V}\le\frac{1}{2}\Big\{-\tilde{x}^T Q\tilde{x}+\tilde{x}^T\varepsilon_1 L_1^2 I\tilde{x}+\frac{1}{\varepsilon_1}\tilde{x}^T P^2\tilde{x}\Big\}+\frac{1}{2}(\varepsilon_2\tilde{x}^T P^2\tilde{x}+\frac{1}{\varepsilon_2}\epsilon^T\epsilon)$$

$$\dot{V}_1\le\frac{1}{2}\tilde{x}^T\Big\{-Q+\varepsilon_1 L_1^2 I+\frac{1}{\varepsilon_1}P^2+\varepsilon_2 P^2\Big\}\tilde{x}+\frac{1}{2}\frac{1}{\varepsilon_2}\epsilon^T\epsilon$$

$$\dot{V}_1\le\frac{1}{2}\tilde{x}^T\Big\{-\lambda_{min}(Q)+\varepsilon_1 L_1^2+\left(\frac{1}{\varepsilon_1}+\varepsilon_2\right)\lambda_{max}^2(P)\Big\}\tilde{x}+\frac{1}{2}\frac{1}{\varepsilon_2}|\epsilon_m|^2 \quad (22)$$

where $\lambda_{min}(Q)$ and $\lambda_{max}(P)$ are minimum and maximum eigen values of the matrices Q and P respectively.

Thus $\dot{V}_1$ is negative outside a compact set defined as

$$\Omega_{\tilde{x}}=\left\{\tilde{x}\,\middle|\,\|\tilde{x}\|\le\frac{\frac{1}{\varepsilon_2}|\epsilon_m|^2}{\left\{\lambda_{min}(Q)-\varepsilon_1 L_1^2-\left(\frac{1}{\varepsilon_1}+\varepsilon_2\right)\lambda_{max}^2(P)\right\}}\right\} \quad (23)$$

By appropriately selecting the values of parameters, defined set can be made arbitrarily small.

Next section describes the designing of event triggered control policy using the observer state estimates and analysis of closed loop system performance.

4. EVENT TRIGGERING CONTROL POLICY

4.1. Baseline Controller Design

With the desired trajectory $y_d(t)$ satisfying assumption 2, tracking error vector for the system of form (1) can be defined as

International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 7, 2025
https://www.theaspd.com/ijes.php

$$e = [e_1 \quad e_2 \quad \dots \quad e_n]^T \tag{24}$$

where

$$\begin{cases} e_1 = x_1 - y_d \\ e_2 = x_2 - \dot{y}_d \\ \vdots \\ e_n = x_n - \overset{n-1}{y}_d \end{cases} \tag{25}$$

Following dynamics results from the differentiation of error variables

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \vdots \\ \dot{e}_n = f(x) + u - \overset{n}{y}_d \end{cases} \tag{26}$$

Tracking error dynamics can be expressed as

$$\dot{e} = Ae + B\left\{f(x) + u - \overset{n}{y}_d\right\} \tag{27}$$

Formulation of a state feedback control term for the stabilization of (3) requires the error terms e (24) and nonlinear system dynamics $f(x)$ to be known. However, as the system states are not available and $f(x)$ is uncertain system dynamics, control law is framed by considering the observer state estimates $\hat{x}$ (15) and wavelet approximation of the uncertain dynamics $\hat{f}(\hat{x})$ (11). With the estimated state variables, control law can be framed as

$$u = -K\hat{e} - \hat{f}(\hat{x}) + \overset{n}{y}_d \tag{28}$$

where

$\hat{e} = [\hat{e}_1, \quad \hat{e}_2, \quad \dots \quad , \hat{e}_n]^T$ is the observer tracking error vector, its elements are defined as

$\hat{e}_i = \hat{x} - \overset{i-1}{y}_d \; ; i = 1, \dots, n$

$K = [k_1 \quad k_2 \quad \dots \quad k_{n-1} \quad 1]$ is the gain vector with $k_i < 0, i = 1, \dots, n-1$.

Substitution of the control term (27) in (26) and subsequent rearrangement of the terms result in following equation

$$\dot{e} = Ae + B\{f(x) + Ke - Ke - K\hat{e} - \hat{f}(\hat{x})\} = (A - BK)e + B\{f(x) + Ke - K\hat{e} - \hat{f}(\hat{x})\} = (A - BK)e + B\{K\tilde{x} + f(x) + f(\hat{x}) - f(\hat{x}) - \hat{f}(\hat{x})\} = (A - BK)e + B\{K\tilde{x} + f(x) - f(\hat{x}) + f(\hat{x}) - \hat{f}(\hat{x})\} \tag{29}$$

Closed loop error dynamics so generated (29) should settle down to a residual set containing origin. To analyze the convergence of closed loop signals with observer (14) and controller scheme (28), consider the following Lyapunov function

$$V_2 = \frac{1}{2}e^T P_1 e \tag{30}$$

Differentiation of the above equation and substitution of (29) results in

$$\dot{V}_2 = \frac{1}{2}\{e^T(A - BK)^T P_1 e + e^T P_1 (A - BK)e\} + \frac{1}{2}\{\tilde{x}^T K^T B^T P_1 e + e^T P_1 BK\tilde{x}\} + \frac{1}{2}\{(f(x) - f(\hat{x}))^T B^T P_1 e + e^T P_1 B(f(x) - f(\hat{x}))\} + \frac{1}{2}\left\{(f(\hat{x}) - \hat{f}(\hat{x}))^T B^T P_1 e + e^T P_1 B(f(\hat{x}) - \hat{f}(\hat{x}))\right\} \tag{31}$$

Applying Leema 1 to above equation

$$\dot{V}_2 \le \frac{1}{2}e^T\{(A - BK)^T P_1 + P_1(A - BK)\}e + \frac{1}{2}\left\{\varepsilon_3 e^T P_1 P_1 e + \frac{1}{\varepsilon_3}\tilde{x}^T K^T B^T BK\tilde{x}\right\} + \frac{1}{2}\left\{\varepsilon_4 e^T P_1 P_1 e + \frac{1}{\varepsilon_4}(f(x) - f(\hat{x}))^T B^T B(f(x) - f(\hat{x}))\right\} + \frac{1}{2}\left\{\varepsilon_5 e^T P_1 P_1 e + \frac{1}{\varepsilon_5}(f(\hat{x}) - \hat{f}(\hat{x}))^T B^T B(f(\hat{x}) - \hat{f}(\hat{x}))\right\}$$

$$\dot{V}_2 \le \frac{1}{2}e^T\{(A - BK)^T P_1 + P_1(A - BK)\}e + \frac{1}{2}\{\varepsilon_3 e^T P_1 P_1 e + \varepsilon_4 e^T P_1 P_1 e + \varepsilon_5 e^T P_1 P_1 e\} + \frac{1}{2}\frac{1}{\varepsilon_3}\tilde{x}^T K^T B^T BK\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_4}\|(f(x) - f(\hat{x}))\|^2 + \frac{1}{2}\frac{1}{\varepsilon_5}\left\|(f(\hat{x}) - \hat{f}(\hat{x}))\right\|^2$$

$$\dot{V}_2 \le \frac{1}{2}e^T\{(A - BK)^T P_1 + P_1(A - BK)\}e + \frac{1}{2}e^T\{(\varepsilon_3 + \varepsilon_4 + \varepsilon_5)P_1^2\}e + \frac{1}{2}\frac{1}{\varepsilon_3}\langle\tilde{x}^T K^T B^T, BK\tilde{x}\rangle + \frac{1}{2}\frac{1}{\varepsilon_4}\|(f(x) - f(\hat{x}))\|^2 + \frac{1}{2}\frac{1}{\varepsilon_5}\left\|(f(\hat{x}) - \hat{f}(\hat{x}))\right\|^2 \tag{32}$$

In above equation, the term $\langle\tilde{x}^T K^T B^T, BK\tilde{x}\rangle$ is positive semidefinite i.e. $\langle\tilde{x}^T K^T B^T, BK\tilde{x}\rangle \ge 0$. Nonexistence of this term implies the existence of the following set

$$\Omega_1 = \{ \tilde{x}_i, i = 1,..,n | \langle \tilde{x}^T K^T B^T, BK\tilde{x} \rangle = 0\} \qquad (33)$$

Existence of this set does not impose any constraint on the controllability or observability norms secondly using Young's inequality this term can be expressed as

$$\langle \tilde{x}^T K^T B^T, BK\tilde{x} \rangle \le \{\sum_{i=1}^{n} k_i x_i\}^2 \le \{\sum_{i=1}^{n} k_i^2 \tilde{x}_i^2 + 2\sum_{i=1}^{n-1}\sum_{j+1}^{n} k_i k_j |\tilde{x}_i| |\tilde{x}_j|\} \le \sum_{i=1}^{n} k_i^2 \tilde{x}_i^2 + \sum_{i=1}^{n-1}\left\{k_i^2 \tilde{x}_i^2 \{\sum_{j=i+1}^{n} \mu_{ij}\} + \sum_{j=i+1}^{n} \frac{1}{\mu_{ij}} k_j^2 \tilde{x}_j^2 \right\}$$

where $\mu_{ij} > 0$

$$\langle \tilde{x}^T K^T B^T, BK\tilde{x} \rangle \le \sum_{i=1}^{n}\left\{\tilde{x}_i^T \left\{k_i^2 (1 + \sum_{j=i+1}^{n} \mu_{ij} + \sum_{\substack{j=1 \\ i>1}}^{i} \frac{1}{\mu_{ji}})\right\} \tilde{x}_i\right\} \qquad (34)$$

Due to inequality (34), corollary of lemma2 and assumption 2, equation (32) results in

$$\dot{V}_2 \le \frac{1}{2} e^T \{(A - BK)^T P_1 + P_1(A - BK)\}e + \frac{1}{2} e^T\{ (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)P_1^2\}e + \frac{1}{2}\frac{1}{\varepsilon_3}\sum_{i=1}^{n}\left\{\tilde{x}_i^T \left\{k_i^2 (1 + \sum_{j=i+1}^{n} \mu_{ij} + \sum_{\substack{j=1 \\ i>1 \\ i\neq j}}^{i} \frac{1}{\mu_{ji}})\right\} \tilde{x}_i\right\} + \frac{1}{2}\frac{1}{\varepsilon_4}\tilde{x}^T L_2^2 I \tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_5} f_m^2$$

Considering assumption 5,

$$\dot{V}_2 \le -\frac{1}{2} e^T Q_1 e + \frac{1}{2} e^T\{ (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)P_1^2\}e + \frac{1}{2}\frac{1}{\varepsilon_3}\sum_{i=1}^{n}\{\tilde{x}_i^T \rho \tilde{x}_i\} + \frac{1}{2}\frac{1}{\varepsilon_4}\tilde{x}^T L_2^2 I \tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_5} f_m^2$$

$$\dot{V}_2 \le \frac{1}{2} e^T\{-\lambda_{min}(Q_1) + (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)\, \lambda_{max}^2(P_1)\}e + + \frac{1}{2}\frac{1}{\varepsilon_5} f_m^2 + \tilde{x}^T (\frac{1}{2}\frac{1}{\varepsilon_4} L_2^2 I + \frac{1}{2}\frac{1}{\varepsilon_3}\rho I)\, \tilde{x} \quad (34)$$

where $\lambda_{min}(Q_1)$ and $\lambda_{max}(P_1)$ are minimum and maximum eigen values of the matrices Q_1 and P_1 respectively and

$$\rho = \left\{k_i^2 (1 + \sum_{j=i+1}^{n} \mu_{ij} + \sum_{\substack{j=1 \\ i>1 \\ i\neq j}}^{i} \frac{1}{\mu_{ji}})\right\} \qquad (36)$$

Appearance of the observer error term in (35) requires the redefining of observer and controller Lyapunov functions. This redefinition will allow the separate analysis of controller and observer signals and smooth establishment of stability norms. Considering the complete Lyapunov function as the combination of (18) and (30)

$$V = V_1 + V_2 = V_a + V_b \qquad (37)$$

with

$$\dot{V}_a = \frac{1}{2}\tilde{x}^T \left\{-\lambda_{min}(Q) + \varepsilon_1 L_1^2 + \left(\frac{1}{\varepsilon_1} + \varepsilon_2\right) \lambda_{max}^2(P) + \frac{1}{\varepsilon_4} L_2^2 + \frac{1}{\varepsilon_3}\rho\right\}\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_2}|\epsilon_m|^2 \quad (38)$$

and

$$\dot{V}_b = \frac{1}{2} e^T\{-\lambda_{min}(Q_1) + (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)\lambda_{max}^2(P_1)\}e + \frac{1}{2}\frac{1}{\varepsilon_5} f_m^2 \qquad (39)$$

Negativeness of (38) and (39) allows the establishment of separate converging residual sets for observer estimation error (17) and controller tracking error (27). Considering the derivative of (37)

$$\dot{V} = \frac{1}{2}\tilde{x}^T \left\{-\lambda_{min}(Q) + \varepsilon_1 L_1^2 + \left(\frac{1}{\varepsilon_1} + \varepsilon_2\right) \lambda_{max}^2(P) + \frac{1}{\varepsilon_4} L_2^2 + \frac{1}{\varepsilon_3}\rho\right\}\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_2}|\epsilon_m|^2 + \frac{1}{2} e^T\{-\lambda_{min}(Q_1) + (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)\lambda_{max}^2(P_1)\}e + \frac{1}{2}\frac{1}{\varepsilon_5} f_m^2 \qquad (40)$$

Above equation allows the redefining of (23) and defining of following convergence sets

$$\Omega_{\tilde{x}} = \left\{\tilde{x} |\ \|\tilde{x}\| \le \frac{\frac{1}{\varepsilon_2}|\epsilon_m|^2}{\left\{\lambda_{min}(Q) - \varepsilon_1 L_1^2 - \left(\frac{1}{\varepsilon_1} + \varepsilon_2\right) \lambda_{max}^2(P)\right\} - \frac{1}{2}\frac{1}{\varepsilon_4} L_2^2 - \frac{1}{2}\frac{1}{\varepsilon_3}\rho}\right\} \qquad (41)$$

and

$$\Omega_e = \left\{e |\ \|e\| \le \frac{\frac{1}{\varepsilon_5} f_m^2}{\{\lambda_{min}(Q_1) - (\varepsilon_3 + \varepsilon_4 + \varepsilon_5)\, \lambda_{max}^2(P_1)\}}\right\} \qquad (42)$$

With the definition of above sets, the terms $\dot{V}_a$ and $\dot{V}_b$ and hence $\dot{V}$ are negative when the error terms $\tilde{x}$ and e are outside the residual sets (41) and (42) respectively.

4.2. Event Triggered Control Design

International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 7, 2025
https://www.theaspd.com/ijes.php

This section describes the transformation of controller term (28) into event triggered form. Event triggered control scheme has been proved highly effective in the case of control on the network. Under the conditions of limited network resources and multiple control loops sharing a communication channel, this strategy often allows the optimal sharing of network resources.

In the event triggering scheme, control policy is updated at some nonuniformly spaced time instants known as triggering instants. These triggering instants are governed by some appropriately designed triggering mechanisms which ensure the system performance to be within some prescribed performance limits. Violation of these delimiting conditions reflects the considerable detuning of system variables and requires the updation of control term by invoking a triggering instant. This scheme also reduces the computational complexity of the controller as controller computations are performed at discrete instants only [1,7].

Event triggered form of the control term (28) can be defined as

$$v(t) = u(t_k) = -K\hat{e}(t_k) - \hat{f}\big(\hat{x}(t_k)\big) + \overset{n}{y}_d(t_k) \tag{43}$$

where the terms are the values of the variables in (28) at some instant $t = t_k$. Time instant, $t_k, k \in Z$ is the current triggering instant i.e. the instant at which control term is updated. Considering t_{k+1} as the next triggering instant, the control term $v(t)$ (43) is held constant for the duration $\Delta t = t_{k+1} - t_k$ known as inter execution time. To control the inter-execution time following triggering mechanisms are considered

$$\big(\hat{e}(t) - \hat{e}(t_k)\big)^T \big(\hat{e}(t) - \hat{e}(t_k)\big) \le m_1 \tag{44}$$

$$\Big(\big(\hat{f}(\hat{x}\,(t)) - \hat{f}(\hat{x}(t_k))\big)\Big)^T \Big(\big(\hat{f}(\hat{x}\,(t)) - \hat{f}(\hat{x}(t_k))\big)\Big) \le m_2 \tag{45}$$

where $m_i < 0, i = 1,2$are the threshold values reflecting the limit of permissible deviations for various control components, whenever the deviation exceeds the threshold value updation is carried out. Thus, the next update instant can be defined as

$$t_{k+1} = inf\Big\{inf\Big\{t \ge t_k; \big(\hat{e}(t) - \hat{e}(t_k)\big)^T \big(\hat{e}(t) - \hat{e}(t_k)\big) \ge m_1\Big\}, inf\Big\{t \ge t_k; \Big(\hat{f}(\hat{x}\,(t)) - \hat{f}(\hat{x}(t_k)\Big)^T \Big(\hat{f}(\hat{x}\,(t)) - \hat{f}(\hat{x}(t_k)\Big) \ge m_2\Big\}\Big\} \tag{46}$$

Next subsection details the convergence of closed loop with event triggered control.

4.3. Convergence Analysis with Event Triggered Control

In this section, convergence of the closed loop system under the action of event triggered control scheme is analyzed. It is carried out by examining the convergence of Lyapunov function (30) due to control term (43). The analysis is carried out at an instant $t \in [t_k, t_{k+1})$.

Under the effect of event triggered control scheme (43), the error dynamics (29) becomes

$$\dot{e} = (A - BK)e(t) + B\Big\{K\tilde{x}(t) + K\,\big(\hat{e}(t) - \hat{e}(t_k)\big) + f\big(x(t)\big) - f\big(\hat{x}\,(t)\big) + f\big(\hat{x}\,(t)\big) - \hat{f}\big(\hat{x}\,(t)\big) + \hat{f}\big(\hat{x}\,(t)\big) - \hat{f}\big(\hat{x}(t_k)\big) - \big(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\big)\Big\} \tag{47}$$

As compared to (29), (47) contains some additional dynamics invoked due to event triggering control term. These terms may result in degeneration of the system up to a certain extent. Proposed event triggering mechanisms not only minimize the degenerative effect but also reduces the number of triggering instants.

Reconsidering the equation (30)

$$V_2(\mathrm{t}) = \frac{1}{2}e^T(t)P_1e(t) \tag{48}$$

Differentiation of (48) and subsequent substitution of (47) leads to following analytical developments

$$\dot{V}_2 = \frac{1}{2}\{e^T(t)(A - BK)^T P_1 e(t) + e^T(t)P_1(A - BK)e(t)\} + \frac{1}{2}\{\tilde{x}^T(t)K^T B^T P_1 e(t) + e^T(t)P_1BK\tilde{x}(t)\} + \frac{1}{2}\Big\{\big(\hat{e}(t) - \hat{e}(t_k)\big)^T K^T B^T P_1 e(t) + e^T(t)P_1BK\,\big(\hat{e}(t) - \hat{e}(t_k)\big)\Big\} + \frac{1}{2}\Big\{\Big(f\big(x(t)\big) - f\big(\hat{x}\,(t)\big)\Big)^T B^T P_1 e(t) + e^T(t)P_1B\Big(f\big(x(t)\big) - f\big(\hat{x}\,(t)\big)\Big)\Big\} + \frac{1}{2}\Big\{\Big(f\big(\hat{x}\,(t)\big) - \hat{f}\big(\hat{x}\,(t)\big)\Big)^T B^T P_1 e(t) + e^T(t)P_1B\Big(f\big(\hat{x}\,(t)\big) - \hat{f}\big(\hat{x}\,(t)\big)\Big)\Big\} + \frac{1}{2}\Big\{\Big(\hat{f}(\hat{x}\,(t)) - \hat{f}(\hat{x}(t_k)\Big)^T B^T P_1 e(t) + e^T(t)P_1B\,\Big(\hat{f}(\hat{x}\,(t)) -$$

$\hat{f}(\hat{x}(t_k))\Big\} - \frac{1}{2}\Big\{\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)^T B^T P_1 e(t) + e^T(t) P_1 B\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)\Big\}$ (49)

$\dot{V}_2 \leq \frac{1}{2}e^T(t)\{(A-BK)^T P_1 + P_1(A-BK)\}e(t) + \frac{1}{2}\left\{\varepsilon_3 e^T(t)P_1P_1e(t) + \frac{1}{\varepsilon_3}\tilde{x}^T(t)K^TB^TBK\tilde{x}(t)\right\} + \frac{1}{2}\left\{\varepsilon_6 e^T(t)P_1P_1e(t) + \frac{1}{\varepsilon_6}\left(\hat{e}(t)-\hat{e}(t_k)\right)^T(t)K^TB^TBK\left(\hat{e}(t)-\hat{e}(t_k)\right)\right\} + \frac{1}{2}\Big\{\varepsilon_4 e^T P_1 P_1 e + \frac{1}{\varepsilon_4}\left(f(x(t)) - f(\hat{x}(t))\right)^T B^T B\left(f(\hat{x}(t)) - \hat{f}(\hat{x}(t))\right)\Big\} + \frac{1}{2}\Big\{\varepsilon_7 e^T(t)P_1P_1e(t) + \frac{1}{\varepsilon_7}\left(f(\hat{x}(t)) - \hat{f}(\hat{x}(t))\right)^T B^TB\left(f(\hat{x}(t)) - f(\hat{x}(t_k))\right)\Big\} + \frac{1}{2}\Big\{\varepsilon_8 e^T(t)P_1P_1e(t) + \frac{1}{\varepsilon_8}\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)^T B^TB\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)\Big\} - \frac{1}{2}\Big\{\varepsilon_9 e^T(t)P_1P_1e(t) + \frac{1}{\varepsilon_9}\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)^T B^TB\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)\Big\}$

$\dot{V}_2 \leq \frac{1}{2}e^T(t)\{(A-BK)^TP_1 + P_1(A-BK)\}e(t) + \frac{1}{2}e^T(t)\{(\varepsilon_3+\varepsilon_4+\varepsilon_6+\varepsilon_5+\varepsilon_8+\varepsilon_9)P_1^2\}e(t) + \frac{1}{2}\frac{1}{\varepsilon_3}\langle\tilde{x}^T(t)K^TB^T, BK\tilde{x}(t)\rangle + \frac{1}{2}\frac{1}{\varepsilon_6}\langle\left(\hat{e}(t)-\hat{e}(t_k)\right)^TK^TB^T, BK\left(\hat{e}(t)-\hat{e}(t_k)\right)\rangle + \frac{1}{2}\frac{1}{\varepsilon_4}\left\|\left(f(x(t)) - f(\hat{x}(t))\right)\right\|^2 + \frac{1}{2}\frac{1}{\varepsilon_5}\left\|\left(f(\hat{x}(t)) - \hat{f}(\hat{x}(t))\right)\right\|^2 + \frac{1}{2}\frac{1}{\varepsilon_8}\left\|\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)\right\|^2 + \frac{1}{2}\frac{1}{\varepsilon_9}\left\|\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)\right\|^2$

$\dot{V}_2 \leq \frac{1}{2}e^T\{(A-BK)^TP_1 + P_1(A-BK)\}e + \frac{1}{2}e^T(t)\{(\varepsilon_3+\varepsilon_4+\varepsilon_6+\varepsilon_5+\varepsilon_8+\varepsilon_9)P_1^2\}e(t) + \frac{1}{2}\frac{1}{\varepsilon_3}\sum_{i=1}^n\{\tilde{x}_i^T\rho\tilde{x}_i\} + \frac{1}{2}\frac{1}{\varepsilon_3}\sum_{i=1}^n\left\{\left(\hat{e}(t)-\hat{e}(t_k)\right)^T\rho\left(\hat{e}(t)-\hat{e}(t_k)\right)\right\} + \frac{1}{2}\frac{1}{\varepsilon_4}\tilde{x}^TL_2^2I\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_5}f_m^2 + \frac{1}{2}\frac{1}{\varepsilon_8}\left\|\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)\right\|^2 + \frac{1}{2}\frac{1}{\varepsilon_9}\left\|\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)\right\|^2$

$\dot{V}_2 \leq \frac{1}{2}e^T\{-\lambda_{min}(Q_1) + (\varepsilon_3+\varepsilon_4+\varepsilon_6+\varepsilon_5+\varepsilon_8+\varepsilon_9)\lambda_{max}^2(P_1)\}e + + \frac{1}{2}\frac{1}{\varepsilon_5}f_m^2 + \tilde{x}^T(\frac{1}{2}\frac{1}{\varepsilon_4}L_2^2I + \frac{1}{2}\frac{1}{\varepsilon_3}\rho I)\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_3}\left(\hat{e}(t)-\hat{e}(t_k)\right)^T\rho I\left(\hat{e}(t)-\hat{e}(t_k)\right) + \frac{1}{2}\frac{1}{\varepsilon_8}\left\|\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)\right\|^2 + \frac{1}{2}\frac{1}{\varepsilon_9}\left\|\left(\overset{n}{y}_d(t) - \overset{n}{y}_d(t_k)\right)\right\|^2$

$\dot{V}_2 \leq \frac{1}{2}e^T\{-\lambda_{min}(Q_1) + (\varepsilon_3+\varepsilon_4+\varepsilon_6+\varepsilon_5+\varepsilon_8+\varepsilon_9)\lambda_{max}^2(P_1)\}e + + \frac{1}{2}\frac{1}{\varepsilon_5}f_m^2 + \tilde{x}^T(\frac{1}{2}\frac{1}{\varepsilon_4}L_2^2I + \frac{1}{2}\frac{1}{\varepsilon_3}\rho I)\tilde{x} + \frac{1}{2}\frac{1}{\varepsilon_3}\rho {m_1}^2 + \frac{1}{2}\frac{1}{\varepsilon_8}{m_2}^2 + \frac{1}{2}\frac{1}{\varepsilon_9}\sigma^2$ (50)

Thus, $\dot{V}$ is negative outside the compact sets defined as

$$\Omega_{\tilde{x}} = \left\{\tilde{x}\,|\,\|\tilde{x}\| \leq \frac{\frac{1}{\varepsilon_2}|\epsilon_m|^2}{\left\{\lambda_{min}(Q) - \varepsilon_1L_1^2 - \left(\frac{1}{\varepsilon_1}+\varepsilon_2\right)\lambda_{max}^2(P)\right\} - \frac{1}{2}\frac{1}{\varepsilon_4}L_2^2 - \frac{1}{2}\frac{1}{\varepsilon_3}\rho}\right\} \quad (51)$$

and

$$\Omega_e = \left\{e\,|\,\|e\| \leq \frac{\frac{1}{\varepsilon_5}f_m^2 + \frac{1}{\varepsilon_3}\rho{m_1}^2 + \frac{1}{\varepsilon_8}{m_2}^2 + \frac{1}{\varepsilon_9}\sigma^2}{\{\lambda_{min}(Q_1) - (\varepsilon_3+\varepsilon_4+\varepsilon_6+\varepsilon_5+\varepsilon_8+\varepsilon_9)\lambda_{max}^2(P_1)\}}\right\} \quad (52)$$

Thus, for the networked closed loop system containing the dynamics (1) with controllable – observable triple $(A \quad B \quad C)$ and Lipschitz continuous uncertain nonlinearities, adaptive observer and event triggering control scheme (14) and (43) ensures the ultimate upper boundedness of the closed loop dynamics with error variables approaching to residual sets (51) and (52).

Also, the boundedness of closed loop signals implies that $\dot{V} \in L_\infty$ which means Lyapunov function V is bounded and does not contain any discontinuity, so

$\{V(t)|V(t) \in C; V(t) \leq V(0)\}\,; \forall t \in [0, \infty)$ (53)

This mathematical development can be viewed as an extension of the convergence analysis resulting in residual sets (41) and (42). The effect of event triggering can be observed by comparing the sets (41) and (51) and (42) and (52) respectively. Comparison reveals that observer dynamics is not affected by the event triggering and the convergence set is preserved. However, in the case of controller the effect of event triggering is reflected by the additional terms appearing in (52). These terms are mainly controlled by the threshold values of the triggering mechanisms (44) and (45). The effect of these terms can be viewed as the enlargement of the residual set thereby causing a deterioration in system performance, higher threshold values result in larger sets. However, as proved in subsequent section, higher threshold values may result in lesser switching instants and thus there is a tradeoff between system performance and triggering instants. By properly selecting the threshold limits an acceptable system performance with an appropriate number of triggering instants can be achieved.

Next section analyses the admissibility issues of the event triggering schemes under the constraints imposed by issues like Zeno Behavior.

4.4. Admissibility with Event Triggered Control

Zeno behavior refers to an infinite number of triggering instants within a finite time duration. Zeno behavior often imposes pragmatic constraints on the effective implementation of event control scheme and are required to be excluded. To avoid the piling of triggering instants it is required to ensure that there exists a finite positive lower bound for inter execution time [1].

Consider the triggering mechanisms described in (44) and (45)

$$\Delta\hat{e}(t) = \left(\hat{e}(t) - \hat{e}(t_k)\right)^T\left(\hat{e}(t) - \hat{e}(t_k)\right) \tag{54}$$

$$\Delta\hat{f}(t) = \left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)^T\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right) \tag{55}$$

Differentiation of (54) and (55) and application of Lemma 1 results in

$$\Delta\dot{\hat{e}}(t) = \varepsilon_{10}\left(\hat{e}(t) - \hat{e}(t_k)\right)^T\left(\hat{e}(t) - \hat{e}(t_k)\right) + \frac{1}{\varepsilon_{10}}\left(\dot{\hat{e}}(t)\right)^T\left(\dot{\hat{e}}(t)\right) \tag{56}$$

$$\Delta\dot{\hat{f}}(t) = \varepsilon_{11}\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)^T\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right) + \frac{1}{\varepsilon_{11}}\left(\dot{\hat{f}}(\hat{x}(t))\right)^T\left(\dot{\hat{f}}(\hat{x}(t))\right) \tag{55}$$

$$\Delta\dot{\hat{e}}(t) = \varepsilon_{10}\left\|\left(\hat{e}(t) - \hat{e}(t_k)\right)\right\|^2 + \frac{1}{\varepsilon_{10}}\left\|\dot{\hat{e}}(t)\right\|^2$$

$$\Delta\dot{\hat{f}}(t) = \varepsilon_{11}\left\|\left(\hat{f}(\hat{x}(t)) - \hat{f}(\hat{x}(t_k))\right)\right\|^2 + \frac{1}{\varepsilon_{11}}\left\|\dot{\hat{f}}(\hat{x}(t))\right\|^2 \tag{57}$$

As all the closed loop signals, update laws and wavelet basis are bounded and continuous it implies that

$$\dot{\hat{e}}(t) \in L_\infty \ , \ \dot{\hat{f}}\left(\hat{x}(t)\right) \in L_\infty \tag{58}$$

It means there exist positive constants ρ_1 and ρ_2 such that

$$\left\|\dot{\hat{e}}(t)\right\|^2 \leq \rho_1 \ ; \ \left\|\dot{\hat{f}}(\hat{x}(t))\right\|^2 \leq \rho_2 \tag{59}$$

with

$$\Delta\dot{\hat{e}}(t) \leq \varepsilon_{10}m_1 + \frac{1}{\varepsilon_{10}}\rho_1 \tag{60}$$

$$\Delta\dot{\hat{f}}(t) \leq \varepsilon_{11}m_2 + \frac{1}{\varepsilon_{11}}{\rho_2}^2 \tag{61}$$

Above equations implies that both the dynamics require finite time to make transition from zero to threshold value and thus ensures the presence of a finite time bound in both the cases. Integrating the above dynamics

$$\int_{t_k}^{t}\Delta\dot{\hat{e}}(t)\,dt \ \leq \int_{t_k}^{t}\left(\varepsilon_{10}m_1 + \frac{1}{\varepsilon_{10}}\rho_1\right)dt \tag{62}$$

$$\int_{t_k}^{t}\Delta\dot{\hat{f}}(t)\,dt \ \leq \int_{t_k}^{t}\left(\varepsilon_{11}m_2 + \frac{1}{\varepsilon_{11}}\rho_2\right)dt \tag{63}$$

Considering the limiting cases

$$m_1 = \left(\varepsilon_{10}m_1 + \frac{1}{\varepsilon_{10}}\rho_1\right)(t_1 - t_k) \tag{64}$$

$$m_2 = \left(\varepsilon_{11}m_2 + \frac{1}{\varepsilon_{11}}\rho_2\right)(t_2 - t_k) \tag{65}$$

As the constants m_1, $\left(\varepsilon_{10}m_1 + \frac{1}{\varepsilon_{10}}\rho_1\right)$, m_2, $\left(\varepsilon_{11}m_2 + \frac{1}{\varepsilon_{11}}\rho_2\right) > 0$, it indicates that

$t_1 > t_k$ and $t_2 > t_k$ with

$$t_{k+1} \geq inf(t_1, t_2) \tag{66}$$

Thus, Zeno behavior is successfully avoided.

Equation (64) and (65) clearly reflect the positive monotonic relation between the threshold value and next triggering instant and thus substantiate the conclusions drawn in previous subsection.

Remark 2. Equation (57) poses the requirement of continuously differentiable wavelet basis for the construction of wavelet network. Norms of Lipschitz continuity is used for the selection of wavelets and the wavelet basis functions with vanishing moments $p \geq 3$ are selected for the construction of wavelet network [36].

Next section illustrates the simulation study carried out to validate the effectiveness of the control scheme.

5. SIMULATION

Following system dynamics is considered to conduct the simulation

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + u \\ y = x_1 + x_2 \end{cases} \tag{67}$$

where $f(x) = 0.5\sin(x_2^2) + 0.75x_1^2(1 - x_2)$ is the uncertain nonlinear dynamics. System belongs to the class of strict feedback systems (1) with unknown system states and satisfies all the assumptions specified. Vector matrix notation for the system is

$$\begin{cases} \dot{x} = Ax + B(f + u) \\ y = Cx \end{cases} \tag{68}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Firstly, an observer is designed for the estimation of system states, observer dynamics (14) is implemented with following settings

Initial condition are $x(0) = [0.3 \quad 0]^T$

Observer gain is taken as $m = [12.54 \quad 2.56]^T$

Observer is augmented with an output recurrent wavelet network for the estimation of the system uncertainty. Wavelet network is constructed by using Daubechies wavelet (db4) with n = 2, coarsest and finest resolution levels are selected as 2 and 3 respectively. Number of translates at coarsest resolution level are taken as 4 and translates are doubled every time when resolution is incremented by1. Also, vanishing moments of Daubechies wavelet (db4) is $p = 4$ and it satisfies the required conditions of smoothness as demanded by (57). Weights of the wavelet network are adjusted using tunning laws (21) and initial setting is taken as zero.

Secondly, an event triggered state feedback law is formulated using (43) to ensure the effective tracking performance of the system (1) configured in closed loop. Control scheme utilizes the observer state estimates (15) and wavelet network output (11) for framing of control term. Data transmission from observer to controller is carried out at non-regularly spaced instants controlled by triggering mechanisms. Controller implementation is performed with following parameter settings

Initial condition: $x(0) = [0.3 \quad 0]^T$; gain settings: $K = [10.67 \quad 1]^T$; desired trajectory: $y_d = \sin(t)$; event triggering thresholds: $m_1 = 0.75$, $m_2 = 0.95$

Results of the simulations conducted are shown in figures 1,2,3 and 4. Figures clearly reveals the efficacy of the proposed event triggered observer – controller scheme. Figures 1 and 2 display the estimation ability of the observer formulated. As clear from the figures, observer effectively estimate the system states and the estimation error converges to small neighborhood of origin. Convergence time and rms values of the state estimation errors are

Table I

Estimation Error Attributes

Estimation Eror	Convergence time	RMS value
$x_1 - \hat{x}_1$	0.1 sec	0.0071
$x_2 - \hat{x}_2$	5.1 sec	$4.04\ X\ 10^{-4}$

The data also accounts for the accurate and rapid approximation capability of the wavelet network used for the estimation of the system uncertainty.

Figure 3 shows the tracking performance of the system with event triggered control. Even with event triggered control scheme the system state closely tracks the desired trajectory with tracking error bounded within the permissible limits of

$[-0.15 \quad 0.15]$ with rms value of the order of 0.0608. Figure 4 displays the event triggered control policy evolved with the control law (43). Control policy seems feasible and acceptable from the point of

view of event triggered norms and performance of the closed loop system. Control term is free from eno behaviour with minimum and maximum value of inter-execution time equal to 0.2 sec and 1.2 sec respectively. On an average there are 6 triggering instants over a span of
2 sec. These attributes seem feasible from pragmatic point of view and thus the proposed scheme demonstrates an effective performance in terms of control on the network.

6. CONCLUSION

An event triggered control algorithm is presented for a class of strict feedback nonlinear system with unmeasured states and uncertain nonlinear dynamics. An adaptive observer is framed for the estimation of the system states. Observer location has been chosen at the sensor site as it allows the continuous evolution of the observer estimates without using network resources. An output recurrent wavelet neural network is constructed by using orthonormal wavelets is used for estimation of nonlinearity. It is augmented with the conventional observer to ensure effective estimation of system states under the conditions of unknown dynamics. Event triggered scheme allows the effective sharing of network resources and at the same time reduces the information release instants of the sensor / observer and computational complexity of the controller. Analytical proofs have been developed to testify the uniform ultimate boundedness of closed loop signals and Zeno free behavior of the control term. Finally, a simulation study is conducted to validate the effectiveness of observer – controller scheme. Simulation results illustrate the Zeno free control policy with controller and observer error dynamics bounded within the permissible limits.

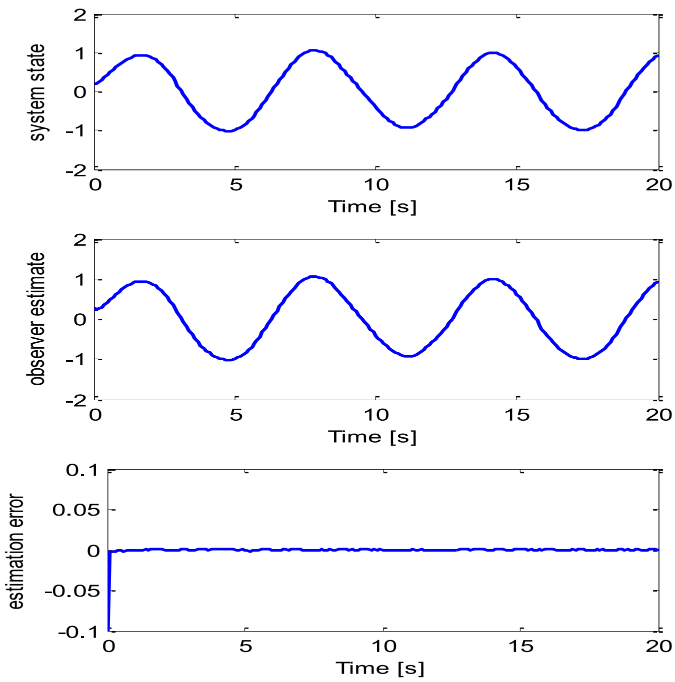

Fig. 1. System performance a. System state x_1 b. Observer estimation $\hat{x}_1$ c, Estimation error $x_1 - \hat{x}_1$

International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 7, 2025
https://www.theaspd.com/ijes.php

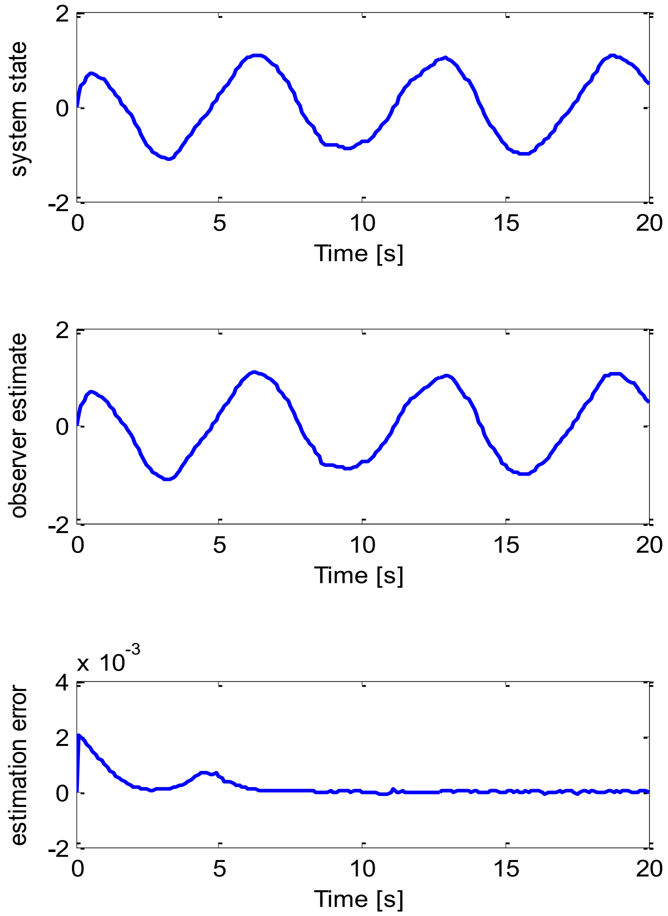

Fig. 2. a. System state x_2 b. Observer estimation $\hat{x}_2$ c. Estimation error $x_2 - \hat{x}_2$

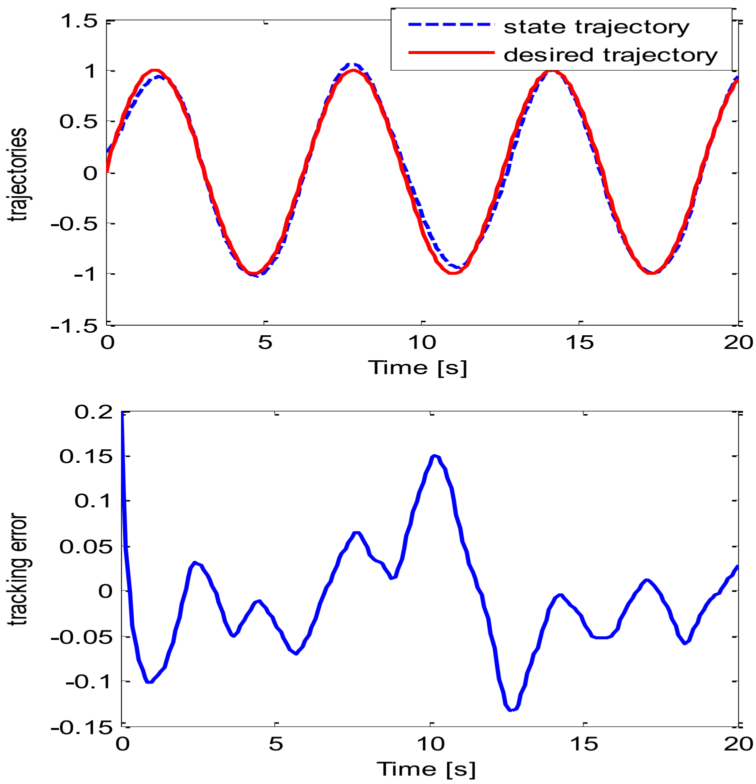

Fig. 3. a. Trajectory followed by state variable x_1 and desired trajectory y_d b. Tracking error $x_1 - y_d$

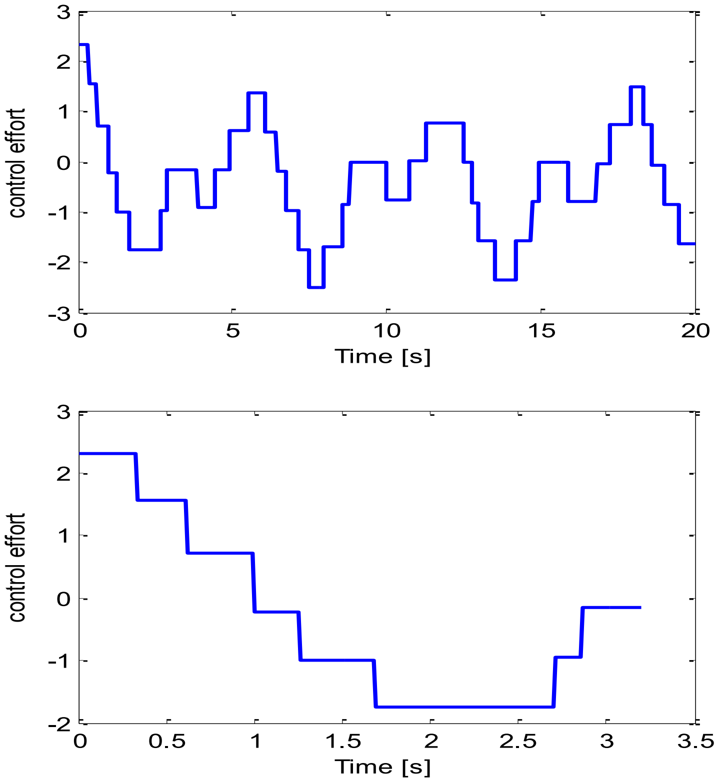

Fig. 4. a. Control Effort b. Zoom of control effort

REFERENCES

[1] P. Tabuada, "Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks," IEEE Transactions on Automatic Control, vol. 52, no. 9, pp. 1680-1685, 2007. doi: 10.1109/TAC.2007.904277

[2] J. Cai, R. Yu, Q. Yan, C. Mei and L. Shen, "Event-Triggered Control for Strict-Feedback Nonlinear Systems With External Disturbances," in IEEE Access, vol. 7, pp. 38390-38396, 2019, doi: 10.1109/ACCESS.2019.2906672

[3] J. Huang, W. Wang, C. Wen and G. Li, "Adaptive Event-Triggered Control of Nonlinear Systems with Controller and Parameter Estimator Triggering," in IEEE Transactions on Automatic Control, vol. 65, no. 1, pp. 318-324, Jan. 2020, doi: 10.1109/TAC.2019.2912517

[4] Y. Xie, Q. Ma and S. Xu, "Adaptive Event-Triggered Finite-Time Control for Uncertain Time Delay Nonlinear System," in IEEE Transactions on Cybernetics, vol. 53, no. 9, pp. 5928-5937, 2023. doi: 10.1109/TCYB.2022.3219098

[5] L. Chu and Y. Liu, “Adaptive event-triggered control for nonlinear systems with time-varying parameter uncertainties”, International Journal of Robust and Nonlinear Control, vol. 34, issue 3, pp. 2094-2108, 2024.

[6] L. Cao, Q. Zhou, G. Dong and H. Li, "Observer-Based Adaptive Event-Triggered Control for Nonstrict-Feedback Nonlinear Systems with Output Constraint and Actuator Failures," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 51, no. 3, pp. 1380-1391, 2021. doi: 10.1109/TSMC.2019.2895858

[7] N. Pang, X. Wang and Z. Wang, "Observer-Based Event-Triggered Adaptive Control for Nonlinear Multiagent Systems with Unknown States and Disturbances," in IEEE Transactions on Neural Networks and Learning Systems, vol. 34, no. 9, pp. 6663-6669, Sept. 2023, doi : 10.1109/TNNLS.2021.3133440

[8] Q. Zhang and A. Benveniste, “Wavelet networks,” IEEE Transactions on Neural Networks, Vol. 3, no. 6, pp.889-898, 1992. doi : 10.1109/72.165591

[9] J. Zhang, G. G. Walter, Y. Miao, and. W. Lee, “Wavelet neural networks for function learning,” IEEE Transactions on Signal Processing, Vol. 43, no. 6, pp.1485-1497, 1995.doi: 10.1109/78.388860

[10] B.Delyon, and A. Juditsky, and A. Benveniste, “Accuracy analysis for wavelet approximations,” IEEE Transactions on. Neural Networks, Vol. 6, no. 2, pp.332-348 1995. doi: 10.1109/72.363469

[11] S.A. Billings and H.L.Wei, “A New Class of Wavelet Networks for Nonlinear System Identification,” IEEE Transactions on Neural Networks, vol. 16, pp.862–874, 2005. doi: 10.1109/TNN.2005.849842

[12] C. F. Hsu, C. M. Lin, T. T. Lee, “Wavelet adaptive backstepping control for a class of nonlinear systems” IEEE Transactions on. Neural Networks, Vol. 17, no. 5, pp.1175-1183, 2006. doi: 10.1109/TNN.2006.878122

[13] L.T. Teng, F.J. Lin, H.C. Chiang and J.W. Lin, “Recurrent wavelet neural network controller with improved particle swarm optimisation for induction generator system”, IET Electric Power Applications , vol. 3 , issue 2, pp. 147 – 159, 2009. doi: https://doi.org/10.1049/iet-epa:20080038

[14] M. Ekramian, S. Hosseinnia, and F. Sheikholeslam, "General framework in designing Luenberger-like non-linear observer," IET Control Theory & Applications, vol. 7, no. 2, pp. 253–260, 2013. doi : https://doi.org/10.1049/iet-cta.2012.0284

[15] JZ. Ebrahimi, M. H. Asemani and A, A. Safavi, "Observer-based controller design for uncertain disturbed Takagi-Sugeno fuzzy systems: A fuzzy wavelet neural network approach" International Journal of Adaptive Control and Signal Processing, , vol. 35, issue 1, pp. 122-144, 2021. doi : https://doi.org/10.1002/acs.3195

[16] Q. T. Dam, R. E. H. Thabet, S. A. Ali and F. Guerin, "Observer Design for a Class of Uncertain Nonlinear Systems with Sampled-Delayed Output Using High-Gain Observer and Low-Pass Filter: Application for a Quadrotor UAV," in IEEE Transactions on Industrial Electronics, vol. 71, no. 1, pp. 933-942, 2024. doi : 10.1109/TIE.2023.3247786

[17] K. J. Astrom and B. Wittenmark, "Adaptive control". New York: Addison Wesley, 1995.

[18] H.K5, pp..lil, "Nonlinear systems". Upper Saddle River, NJ Prentice Hall, 2002.

[19] J.J.E. Slotine and W. Li, "Applied nonlinear control", Prentice Hall, 1991.

[20] S. Mallat, "A wavelet tour of signal processing", Academic Press, San Diego, 1999.