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Total Domination On Bipolar Anti-Fuzzy Graph

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Abstract
The bipolar anti-fuzzy graph (BAFG) was introduced in this paper; its size order and degree are defined along with appropriate examples, and its characteristics are covered. The domination number and total domination number of a bipolar anti-fuzzy graph are examined additionally; this concept has produced a few basic theorems.
Keywords: *BFG (BFG), BAFG (BAFG), dominating set, dominating set of a BAFG, and total domination number of BAFG.*

1. INTRODUCTION

The concept of fuzzy graph was proposed by Kaufmann, from the fuzzy relations introduced by Zadeh [11]. In 1975, Rosenfeld [10] introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. Muhammad Akram [4], Nagoor Gani and M. Basheer Ahmed [9], R. Muthuraj, A. Sasireka and A. Kanimozhi [6, 8-14] have all produced a great deal of creative work.

2. Preliminaries

In this section, the basic definitions needed to develop the subsequent sections definitions are discussed. Throughout this paper, the dominating set denoted as D, Throughout this paper,

1. The edge between the vertices u and v as uv.
2. G_{BA} be a BAFG, mean that G be a BFG with underlying graph $G^* = (V, E)$.

2.4 Definition [4]

A BFG, is denoted as a pair G=(A,B),
where
$A=(\mu_A^P, \mu_A^N)$ and $B=(\mu_B^P, \mu_B^N)$ are bipolar fuzzy sets and $\mu_A^P: V \to [0,1]$, $\mu_A^N: V \to [-1,0]$, and $\mu_B^P: V\times V \to [0,1]$, $\mu_B^N: V\times V \to [-1,0]$ are bipolar fuzzy mappings such that $\mu_B^P(uv) \le \min\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) \ge \max\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$. A is called the bipolar fuzzy vertex set of V and B the bipolar fuzzy edge set of E respectively. Note that B is a symmetric bipolar fuzzy relation on A. That is, G=(A,B) is a BFG of the underlying crisp graph G* = (V, E), where V is a vertex set and the edge set E $\subseteq$ V×V such that,

$$\mu_B^P(uv) \le \min\{\mu_A^P(u), \mu_A^P(v)\} \text{ and } \mu_B^N(uv) \ge \max\{\mu_A^N(u), \mu_A^N(v)\} \text{ for all } uv \in E.$$

2.5 Definition [5]

Let $\mathcal{G} = (\alpha, \beta)$ be a BFG.

Then the order of $\mathcal{G}$, the size of $\mathcal{G}$ and deg(u) is defined by $p = |V| = \sum_{u\in V} \frac{1+\mu_A^P(u)+\mu_A^N(u)}{2}$.,

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$q = |E| = \sum_{uv \in E} \frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2}$. and $\deg(u) = \sum_{v \in V} \frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2}$. respectively.

2.8 Definition [5]

Let $\mathcal{G} = (\alpha, \beta)$ be a BFG. The maximum degree $\Delta(\mathcal{G})$ and minimum degree $\delta(\mathcal{G})$ of a BFG, degree of an edge $(deg(uv))$ and the neighborhood degree $(deg_N(u))$ is

$\Delta(\mathcal{G}) = \max\{deg(u)/u \in V\}$, $\delta(\mathcal{G}) = \min\{deg(u)/u \in V\}$, $\deg(uv) = \sum_{uv \in E} \frac{1+\mu_B^P(uv)+\mu_B^N(uv)}{2}$. and

$$\deg_N(u) = \sum_{v \in N(u)} \frac{1+\mu_A^P(v)+\mu_A^N(v)}{2}.$$ respectively.

2.18 Definition [6]

A fuzzy graph $\mathcal{G} = (\sigma, \mu)$ is said to be an AFG with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all $u, v \epsilon V$, we have $\mu(u, v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $\mathcal{G}_A(\sigma, \mu)$.

2.19 Example

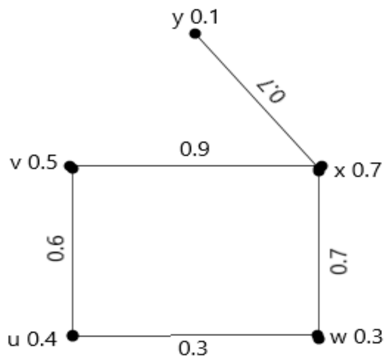

Fig. 2.19 (I)

2.20 Definition [6]

Every vertex in an AFG $\mathcal{G}_A$ has a unique fuzzy values, then $\mathcal{G}_A$ is said to be v —nodal anti fuzzy graph. (i.e) $\sigma(u) = c$ for all $u \in V(\mathcal{G}_A)$.

3. Total domination on BAFG (TBAFG)

3.1 Definition

A BAFG, is denoted as a pair $\mathcal{G}_{BA} = (\alpha, \beta)$,
where $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ and $\beta = (\mu_\beta^P, \mu_\beta^N)$ are BFS and
$\mu_\alpha^P: V \to [0,1]$, $\mu_\alpha^N: V \to [-1,0]$, and
$\mu_\beta^P: V \times V \to [0,1]$, $\mu_\beta^N: V \times V \to [-1,0]$ are BAF mappings such that
$\mu_\beta^P(uv) \geq \max\{\mu_\alpha^P(u), \mu_\alpha^P(v)\}$ and
$\mu_\beta^N(uv) \leq \min\{\mu_\alpha^N(u), \mu_\alpha^N(v)\}$ for all
α is called the BAF vertex set of V and β the BAF edge set of E respectively.

3.2 Example

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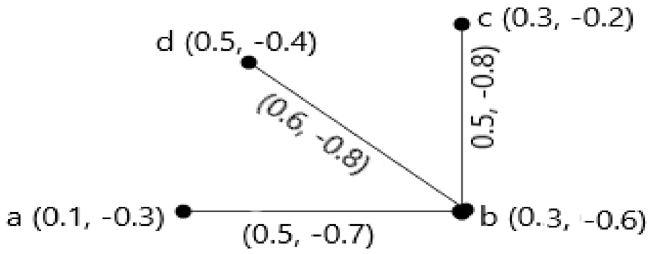

BAFG G_{BA}
Fig. 3.2 (I)

3.3 Definition
Let $\mathcal{G}_{BA}$ be a BAFG.
Then, the order of $\mathcal{G}_{BA}$ or the size of $\mathcal{G}_{BA}$ and $deg(u)$

$$p=|V|=\sum_{u\in V}\frac{1+\mu^{P}(u)+\mu^{N}_{BA}(u)}{2}.$$

3.4 Definition
Let $\mathcal{G}_{BA}$ be a BAFG. Then cardinality of E or the size of G_{BA} is defined as

$$q=|E|=\sum_{uv\in E}\frac{1+\mu^{P}_{Ba}(uv)+\mu^{N}_{BA}(uv)}{2}.$$

3.5 Definition
Let $\mathcal{G}_{BA}$ be a BAFG, then the degree of the vertex is denoted by $deg(u)$ and it is defined as

$$deg(u)=\sum_{v\in V}\frac{1+\mu^{P}_{BA}(uv)+\mu^{N}_{BA}(uv)}{2}.$$

3.6 Definition
Let $\mathcal{G}_{BA}$ be a BAFG. The maximum degree of a BFG is denoted by

$$\Delta(\mathcal{G}_{BA}) = max\{deg(u)/u\in V\}.$$

3.7 Definition
Let $\mathcal{G}_{BA}$ be a BAFG. The minimum degree of a BFG is denoted by

$$\delta(\mathcal{G}_{BA}) = min\{deg(u)/u\in V\}.$$

3.8 Definition
Let $\mathcal{G}_{BA}$ be a BAFG. The degree of an edge $uv\in E$ is denoted as $deg(uv)$ and it is defined as,

$$deg(uv)=\sum_{uv\in E}\frac{1+\mu^{P}_{BA}(uv)+\mu^{N}_{BA}(uv)}{2}.$$

3.9 Definition
Let $\mathcal{G}_{BA}$ be a BAFG. Then the neighbors (neighborhood) of u denoted by N(u) and is defined as $N(u)=\{v\in V/\ \mu^{P}_{B}(uv)=\min\{\mu^{P}_{A}(u),\mu^{P}_{A}(v)\}$ and $\mu^{N}_{B}(uv)=\max\{\mu^{N}_{A}(u),\mu^{N}_{A}(v)\}$ and $uv\in E\}$.
The closed neighbors of u ∈ V of G_{BA} is denoted by N[u] and is defined as N[u] = N(u) ∪ {u}.

3.10 Definition
Let $\mathcal{G}_{BA}$ be a BAFG.

Then $deg_{N}(u)=\sum_{v\in N(u)}\frac{1+\mu^{P}_{A}(v)+\mu^{N}_{A}(v)}{2}.$

3.11 Definition
Let $\mathcal{G}_{BA}$ be a BAFG.
Then $\Delta_N(G_{BA})$ = max {$deg_N(u)$ / u ∈V}.

3.12 Definition
Let $\mathcal{G}_{BA}$ be a BAFG.
Then, $\delta_N(G_{BA})$ = min {$deg_N(u)$ / u ∈V}.
3.13 Definition
Let G_{BA} be a BAFG. An edge of G_{BA} is said to be an effective edge if $\mu_B^P(uv) = \max\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \min\{\mu_A^N(u), \mu_A^N(v)\}$ for all $uv \in E$.
3.14 Definition
Let $\mathcal{G}_{BA}$ be a BAFG.
Then $\delta_E(G_{BA})$ = min {deg $_E$(u) / u ∈V}.
3.15 Definition
A BAFG $\mathcal{G}_{BA}$ is said to be a strong BFG.
If $\mu_B^P(uv) = \max\{\mu_A^P(u), \mu_A^P(v)\}$ and $\mu_B^N(uv) = \min\{\mu_A^N(u), \mu_A^N(v)\}$ and $uv \in E$.

3.16 Example

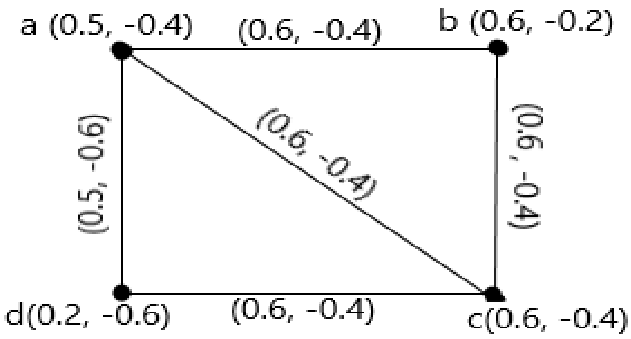

Fig. 3.16 (I)

Here all the edges are strong edges.

3.17 Definition
The strong neighborhood of an edge e_i in a BAFG refers to the effective edge that has the highest fuzzy value within the neighborhood of e_i in $\mathcal{G}_{BA} \Rightarrow N(e_i) = \{e_j \in E(G_{BA}) / e_j$

3.18 Definition
A set D⊆ V(G_{BA}) is said to be a dominating set of a BAFG if for every vertex V∈V(G_{BA}) \ D, there exist u in D such that V is a strong neighborhood of u with μ (u,v) = σ(u) ∨ σ(v). Otherwise, it dominates itself.
A subset D of V is said to be a dominating set in G_{BA} if for every v ∈V – D there exist u ∈ D such that u dominates v.
A dominating set D of V is said to be a minimal dominating set if no proper subset of D is a dominating set of G_{BA}.
The maximum fuzzy cardinality of a minimal dominating set in G is called the domination number of G_{BA} and is denoted by $\gamma_{BA}(G)$.

3.19 Definition
Let G_{BA} be a BFG. Then a dominating set D_s of V is said to be a total dominating set T of G_{BA} if the induced subgraph of T has no isolated vertex.
A total dominating set T of a BFG G_{BA} is called minimal total dominating set on BFG G, if no proper subset of T is a total dominating set of G_{BA}.
The minimum fuzzy cardinality among all minimal total dominating set on bipolar fuzzy G is called total dominating number of a BFG G and is denoted by $\gamma_{TBA}(G_{BA})$.

3.20 Example

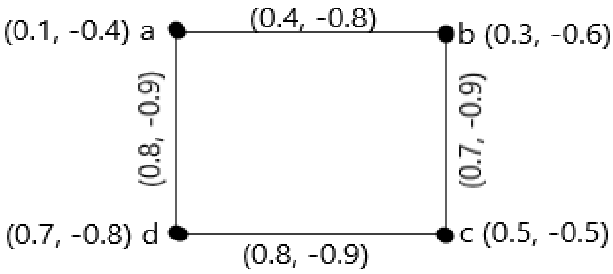

Fig.3.3

Here, the total dominating set D = {d, c}
The domination number $\gamma_{BA}(G) = 0.95$

3.21 Theorem
Let G_{BA} be a BAFG. A total dominating set (T) of G_{BA} is a minimal total dominating set (M(T)) iff if for each $u \in T$, one of the following two conditions holds.
1. $N(u) \cap T = \phi$.
2. There is a vertex $v \in V - T$ such that $N(v) \cap T = \{u\}$.

Proof
Let G_{BA} be a BAFG.
Let T be a MTDS of G_{BA} and $u \in T$.
Let $T_u = T - \{u\}$.
Then T_u is not a total dominating set as T is a minimal total dominating set.
Hence there exists $v \in V - T_u$ such that v is not dominated by any element of T_u.

Case i:
If v = u, then v = u is not dominated by any element of T_u and hence it is not dominated by any element of T and hence, $N(u) \cap T = \phi$.

Case ii: If $v \neq u$ then u dominates v as T is a minimal total dominating set of G_{BA} and hence,
$N(v) \cap T = \{u\}$.
Conversely,
let T be a total dominating set of G_{BA} and for each $u \in T$, one of the following two conditions holds.
i. $N(u) \cap T = \phi$.
ii. There is a vertex $v \in V - T$ such that $N(v) \cap T = \{u\}$.
Suppose if T is not a minimal total dominating set of G then $T_1 \subset T$ is a dominating set of G_{BA}.
Consider an element $u \in T$ and $u \notin T_1$.
Then $u \in V - T_1$ and there exists $w \in T_1$ such that w dominates u and so $w \in N(u)$.
Also $w \in T_1 \subset D$ and hence $N(u) \cap T \neq \phi$.
Given T is not a minimal total dominating set, then there is a vertex $v \in V - T$ such that either v is dominated by more than one vertex of T or there exist an element $u \in T$ such that u does not dominate any v for all $v \in V - T$.
Case i:
Let $u, w \in T$ dominates v and

$$u, w \in N(v).$$

Then $N(v) \cap T = \{u, w\} \neq \{u\}$.

Case ii:
Then for this $u \in T$,
$N(v) \cap T \neq \{u\}$ for all $v \in V - T$.

Then,
Conditions i and ii do not hold due to the assumption that T is not a minimal total dominating set of G. Therefore, T is a minimal total dominating set of G_{BA}.

3.22 Theorem
If G_{BA} is an BAFG without isolated vertices and T is a minimal total dominating set then $V\backslash T$ is a total dominating set in G_{BA}.
Proof
Let G_{BA} be a BAFG without isolated vertices. Let T be the minimal total dominating set of G_{BA}.
Let $u \in T$. Since G has no isolated vertices then $v \in N(u)$.
Case i:
If $v \in V\backslash T$, then, every element of T is dominated by some element of $V - T$.
Hence, $V\backslash T$ is a total dominating set of G_{BA}.
Case ii:
If $v \in T$ and T is a minimal total dominating set, then, there exists an element
$x \in V \setminus T$ such that $x \in N(u)$.
That is, for every element $u \in T$, there exists an element $x \in V \setminus T$ such that x dominates u. Hence $V \setminus T$ is a total dominating set of G_{BA}.

If $v \in T$ and T is a minimal total dominating set, then some element $x \in V \setminus T$ is adjacent to u. In other words, for every vertex $u \in T$, some vertex $x \in V \setminus T$ dominates u. Therefore, $V \setminus T$ forms a total dominating set of G_{BA}.

3.23 Theorem
Let G_{BA} be any completely bipartite BAFG with vertex sets M = $\{u_1, u_2, ...u_n\}$ and N = $\{v_1, v_2....v_n\}$, then $\gamma_T(G_{BA})$ = min $\{\sigma(u_i)\}$ + min $\{\sigma(v_i)\}$ for all $u_i \in M$ and $v_i \in N$.

Proof
Let G_{BA} be any completely bipartite BAFG. Then, $V = M \cup N$ and $M \cap N = \phi$. Let T = u_i, v_j, where $u_i \in M$ and $v_j \in N$ be a total dominating set of G_{BA} . By the definition of completely bipartite BAFG every vertex in M dominates every vertex in N and vice versa. The minimal total dominating set dominates exactly one vertex in M and N. Hence $\gamma_T(G_{BA})$ = min $\{\sigma(u_i)\}$ + min $\{\sigma(v_i)\}$ for all $u_i \in M$ and $v_j \in N$

3.24 Conclusion
We presented the idea of total domination in BAFGs along with its key definitions. The categorization of other domination types within BAFGs will be explored in the relevant papers.

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