

Inventory Optimization with Salvage Value: A Weibull-Based Deterioration with Linear Demand and Partial Backlogging

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Abstract

This paper Optimizes the inventory stock model that aligns with the real-world complexities by including salvage value into the analysis of deteriorating or decaying goods. Its extends the study done in the research work done by Sharma, G., & Vyas, B. (2018) [12]. On optimization of inventory cost using the EOQ model taking linear demand and the Weibull decay distribution of the items and the concept of partial backlogging. We have further added salvage value in this research work. Rate of demand is taken as to vary linearly over time. The deterioration rate of system's inventory follows a Weibull distribution, reflecting a more flexible and realistic inventory decay behaviour. Also, the model accounts for partial backlogging, considering the fact that not all unmet demand can be satisfied. Our main motive of this paper is to minimize overall inventory cost by obtaining the value of optimal quantity of order and cycle length under these taken dynamic conditions. All the analytical and graphical calculations are solved with the help of mathematical software MAPLE 2025, also a numerical demo example is solved to observe the overall effect of salvage value on cost elements of the inventory model and the inventory decision variables. Sensitivity analysis is also performed to explain how the key inventory parameters are affecting the optimal strategy, giving valuable results for inventory planning in perishable or decaying goods and time dependent inventory environments.

Keywords: EOQ model, Partial backlogging, Salvage value, Decision variables, Sensitivity analysis.

INTRODUCTION

Inventory simply means by the stock of goods, raw materials, or the finished products that a business keeps at any given point of time to manage the smooth functioning of overall process of the production of the inventory system and the industry sales management. This is a buffer stock for the purpose of production and the inventory stock consumption, making companies able to satisfy the customer demand without making of any kind of delay or damage. Early studies such as work done by Dave (1989) [1][2] further started the discussion on considerations of linearly increasing demand rate and the rate of shortages in the inventory model systems. As supply chains became more complex and comparatively more sophisticated, the researchers began working on additional realistic factors into the inventory models to make them align with the real-world inventory systems. For example, Ouyang and Cheng (2005) [3] and Hardik Soni et al. (2006) [4] considered the exponential demand decrement and credit policies, whereas the authors Alamri and Balkhi (2007) [5] considered learning and the forgetting effects. Teng et al. (2007) [6] and Chung and Wee (2008) [7] further improved this line of research as by combining the concept of partial backlogging and deteriorating items into pricing and production strategies. Roy (2008) [8], Tripathy and Mishra (2010) [9], and Begum et al. (2012) [10] worked upon the models with time dependent demand rate and holding costs, improving the realism of such formulations. The deterioration is usually taken to be followed as the Weibull distribution rate types because of to its flexibility in representation of different types of item decay behaviours. Very recently, Sharma and Suman (2019) [14] studied time-dependent deterioration rates with the variable holding cost which explains that the multiple cost dynamics in consideration can substantially affect the inventory model decision variables. Even after these developments, the concepts that remains less worked in the inventory development is the consideration of the salvage value which simply means the residual worth of items at the end of their complete inventory life cycle, which can significantly affect the cost optimization process. Whereas the recent studies have started to consider this factor [19][20], prior research has not combined it into models featuring Weibull deterioration and the linear demand rate considerations. This research paper is inspired from the base paper on the work of Sharma and Vyas (2018), which formulated an inventory EOQ model with considering linear demand, Weibull

deterioration, and partial backlogging. We extend their work by including the concept of salvage value, giving a more improved results for minimizing overall inventory expenses. This addition increases overall practical applicability of the given model to industries dealing with perishable or decaying goods where the unsold inventory items or goods may still hold some value. Next, we performed graphical and analytical computations using the mathematical software **MAPLE 2025** and solved a numerical **example** to verify and analysis overall model's performance. Also the sensitivity analysis shown to analyse how the inventory key parameters i.e. deterioration rate, demand rate, and the salvage value affect the optimal solutions. By combining theoretical model with the practical requirements, this study helps to the significant improvement of inventory optimization in settings where both the concepts of decay and salvage value are important considerations.

Notations

C_1 : It shows the cost for holding the inventory per unit and per unit time

C_2 : It shows the shortage cost per unit and per unit time

C_3 : It shows the cost of orders of inventory in each complete cycle

C_4 : Represents the Deterioration cost

C_5 : Opportunity cost because of the sales that are lost /per unit

δ : Salvage value constant

t_1 : Time when the shortages start

T : It shows the size of a full ordering cycle

D : Constant demand rate in $[t_1, T]$

W : It is the maximum level of the inventory level at time $t = 0$ for complete ordering cycle

S : It shows the peak value of the level of inventory shortage that is backlogged for each complete ordering cycle

Q : It shows the quantity of the orders for the given ordering cycle ($Q = W + S$)

$I(t)$: It shows the level of inventory at some time t

TIC : It is denoted for the Total inventory cost in the full cycle period $(0, T)$

Assumptions

1. This inventory stock model is made for only one product or item.

2. We have taken lead time as zero.

3. The rate of item or product demand is linear time dependent as:

$$R(t) = \begin{cases} (a + bt); & 0 \leq t < t_1 \\ D; & t_1 < t \leq T \end{cases}$$

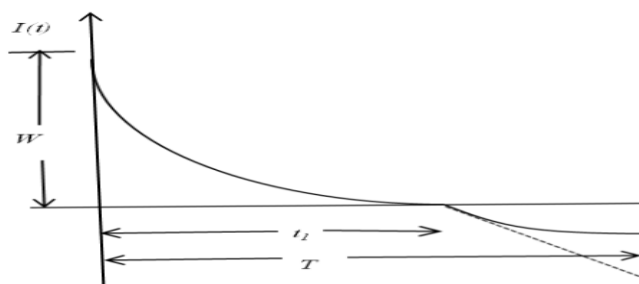
4. The rate of item decay of deterioration at any point of the time $t > 0$ follows the Weibull Distribution with two parameters. $\theta(t) = \alpha\beta t^{1-\beta}$; $0 < \alpha < 1$; $\beta > 0$

5. Backlogging rate is taken as: $e^{-u(T-t)}$

6. Salvage value constant is δ ; $0 < \delta < 1$

Mathematical formulation

Our main goal is to minimize or optimise the total expense per unit of time by optimizing key decision variables: the complete inventory cycle length T , the inventory depletion time t_1 , and overall inventory expense TIC and order quantity Q . The overall cost comprises cost ordering, cost of holding the goods, deterioration cost, shortage cost (with partial backlogging), and the salvage return. To analyse the nonlinear nature of the inventory expense function and to check if the convexity of curves is as expected. The Maple 2025 software was used for computation, solving the system of equations, and observing the model behaviour. The convexity ensures the existence of a global minimum.



During the positive inventory time period $[0, t_1]$ and the shortage inventory time period $[t_1, T]$, the inventory changes over time and is expressed by differential equations:

$$\frac{d}{dt} I_1(t) = -\alpha\beta t^{\beta-1} - bt - a \quad ; \quad 0 < t < t_1 \quad (1)$$

$$\frac{d}{dt} I_2(t) = -De^{-u(T-t)} \quad ; \quad t_1 < t < T \quad (2)$$

With the boundary conditions (obtained from the modelling) given as:

$$I_1(t_1) = 0$$

$$I_1(0) = W$$

$$I_2(t_1) = 0$$

$$I_2(T) = -S$$

Solving the above both differential Equations with the help of the Boundary Conditions we get:

$$I_1 = -\frac{bt^2}{2} - \alpha t^\beta - at + \frac{bt_1^2}{2} + \alpha t_1^\beta + at_1 \quad (3)$$

$$I_2 = \frac{e^{-uT}D(-e^{ut} + e^{ut_1})}{u} \quad (4)$$

At time $t = 0$, the inventory is at its maximum level, denoted by $I(0) = W$:

$$W = \frac{bt_1^2}{2} + \alpha t_1^\beta + at_1 \quad (5)$$

At time $t = T$, the inventory in the shortage period $[t_1, T]$ reaches its peak level, which is S :

$$S = -\frac{e^{-uT}D(-e^{uT} + e^{ut_1})}{u} \quad (6)$$

Now, the Order Quantity is: $Q = W + S$

$$Q = \frac{bt_1^2}{2} + \alpha t_1^\beta + at_1 - \frac{e^{-uT}D(-e^{uT} + e^{ut_1})}{u} \quad (7)$$

Inventory stock Holding Cost is: $HC = C_1 \int_0^{t_1} I_1(t) dt$

$$HC = \frac{C_1 t_1 (2b\beta t_1^2 + 6t_1^\beta \alpha \beta + 3a\beta t_1 + 2bt_1^2 + 3at_1)}{6(1+\beta)} \quad (8)$$

Shortage Cost is $SC = -C_2 \int_{t_1}^T I_2 dt$

$$SC = -\frac{C_2 e^{-uT}D(e^{ut_1}uT - e^{ut_1}ut_1 + e^{ut_1} - e^{uT})}{u^2} \quad (9)$$

Ordering Cost: $OC = C_3$ (10)

Deterioration Cost is: $DC = C_4 \{W - \int_0^{t_1} (a + bt) dt\}$

$$DC = C_4 \alpha t_1^\beta \quad (11)$$

Salvage Value: $SV = \delta \cdot DC$

$$SV = \delta C_4 \alpha t_1^\beta \quad (12)$$

Backlogging because of the sale that is lost is given by: $BC = C_5 \int_{t_1}^T (1 - \frac{1}{1+\delta(T-t)}) D dt$

$$BC = C_5 \left(-\frac{Du^2 * (T^3 - t_1^3)}{6} + \frac{D(Tu^2 - u)(T^2 - t_1^2)}{2} + D \left(uT - \frac{1}{2}u^2T^2 \right) (T - t_1) \right) \quad (13)$$

Total inventory Cost: $TIC = \frac{1}{T} \cdot (SC + HC + OC + DC + BC - SV)$

$$TIC = -\frac{C_2 e^{-uT}D(e^{ut_1}uT - e^{ut_1}ut_1 + e^{ut_1} - e^{uT})}{Tu^2} + \frac{C_1 t_1 (2b\beta t_1^2 + 6t_1^\beta \alpha \beta + 3a\beta t_1 + 2bt_1^2 + 3at_1)}{6T(1+\beta)} + \frac{1}{T} (C_3 + C_4 \alpha t_1^\beta) + C_5 \left(-\frac{Du^2(T^3 - t_1^3)}{6T} + \frac{D(Tu^2 - u)(T^2 - t_1^2)}{2T} + D \left(u - \frac{1}{2T}u^2T^2 \right) (T - t_1) \right) - \frac{\delta C_4 \alpha t_1^\beta}{T} \quad (14)$$

Our goal of this research paper is to get result of the optimal values of t_1 and T to minimize the overall inventory expenses. We have two decision variables t_1 and T . Hence the optimal or suitable values of the parameters are obtained by keeping both the partial derivative equal to zero. This is the essential condition for minimizing the overall inventory expense.

$$\frac{\partial TIC(t_1, T)}{\partial t_1} = 0, \quad \frac{\partial TIC(t_1, T)}{\partial T} = 0 \quad (15)$$

And, $\frac{\partial^2 TIC(t_1, T)}{\partial T^2} > 0$ (16)

$$\frac{\partial TIC(t_1, T)}{\partial T} = -\frac{C_2 e^{-uT}D(e^{ut_1}uT - e^{ut_1}ut_1 + e^{ut_1} - e^{uT})}{u^2T^2} + \frac{C_1 t_1 (2b\beta t_1^2 + (6t_1^\beta \alpha \beta) + 3a\beta t_1 + 2bt_1^2 + 3at_1)}{6(1+\beta)T^2} + \frac{1}{T^2} (C_3 + C_4 \alpha t_1^\beta) + \frac{C_5}{T^2} \left(-\frac{Du^2(T^3 - t_1^3)}{6} + \frac{D(Tu^2 - u)(T^2 - t_1^2)}{2} + D \left(uT - \frac{1}{2}u^2T^2 \right) (T - t_1) \right) - \frac{1}{T^2} \delta C_4 \alpha t_1^\beta +$$

$$\frac{C_2 e^{-uT} D(e^{ut_1} u T - e^{ut_1} u t_1 + e^{ut_1} - e^{uT})}{uT} - \frac{C_2 e^{-uT} D(e^{ut_1} u - u e^{uT})}{u^2 T} + \frac{C_5}{T} \left(-\frac{D u^2 (T^2)}{2} + \frac{D u^2 (T^2 - t_1^2)}{2} + D(T u^2 - u)T + D(-T * u^2 + u)(T - t_1) + \frac{D}{T} \left(uT - \frac{1}{2} u^2 T^2 \right) \right) \quad (17)$$

$$\frac{\partial TIC(t_1, T)}{\partial T} = -\frac{C_2 e^{-uT} D(e^{ut_1} u^2 T - e^{ut_1} u^2 t_1)}{u^2 T} + \frac{C_1 (2b\beta t_1^2 + 6t_1^\beta \alpha(\beta) + 3a\beta t_1 + 2b t_1^2 + 3a t_1)}{6(1+\beta)T} + \frac{C_1 t_1 \left(4b\beta t_1 + \frac{6t_1^\beta \beta^2 \alpha}{t_1} + 3a\beta + 4b t_1 + 3a \right)}{6(1+\beta)T} + \frac{C_4 \alpha t_1^\beta \beta}{t_1 T} + \frac{C_5}{T} \left(\frac{D u^2 t_1^2}{2} - D(T u^2 - u)t_1 - D \left(uT - \frac{1}{2} u^2 T^2 \right) \right) - \frac{\delta C_4 \alpha t_1^\beta \beta}{t_1 T} \quad (18)$$

Solving equations (17) and (18) using maple 2025 with the numerical example:

$$a = 2, b = 7, D = 10, u = 0.01, C_1 = 20, C_2 = 4, C_3 = 500, C_4 = 50, C_5 = 40, \alpha = 0.15, \beta = 2, \delta = 0.99$$

We get the results:

Optimal values of T and t₁ (Solving Using Maple 2025):

$$T = 5.297179588 \text{ Years}, t_1 = 0.9873969504 \text{ Years}$$

Optimal Order Quantity:

$$Q_1 = 47.71568573 \text{ Units per Cycle}$$

Optimal Value of Total Inventory cost:

$$OTIC = \text{Rs. } 181.9871150$$

Verification Of Convexity of Graphs Using Maple (2025):

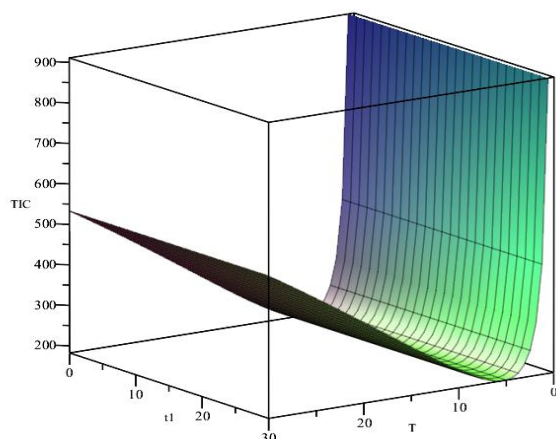
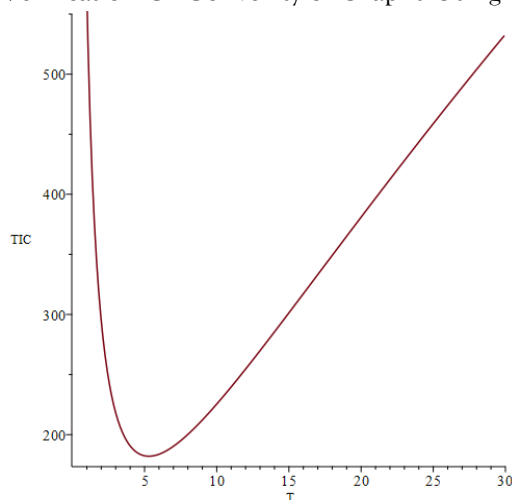


Figure 2: Graph of T v\vs Total inventory expense TIC

Figure 3: Graph of T and t₁ v\vs Total inventory

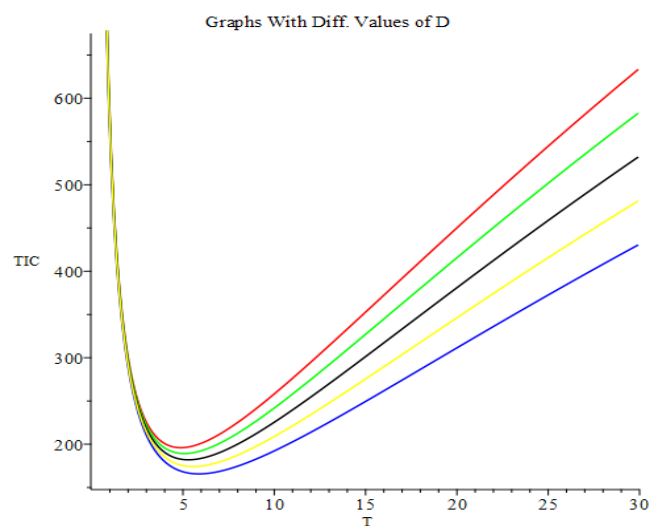
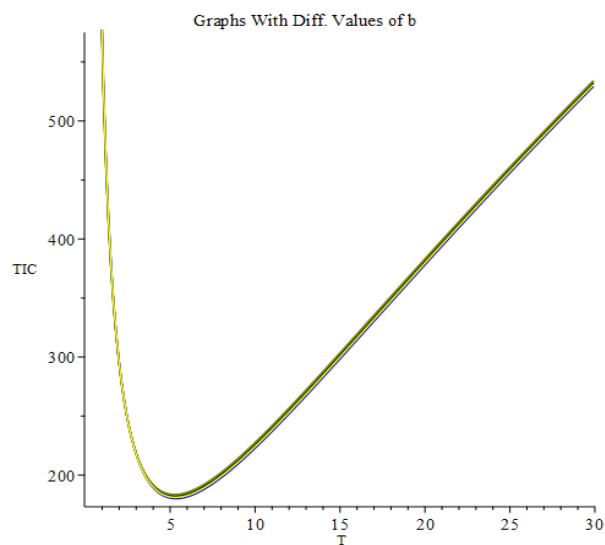
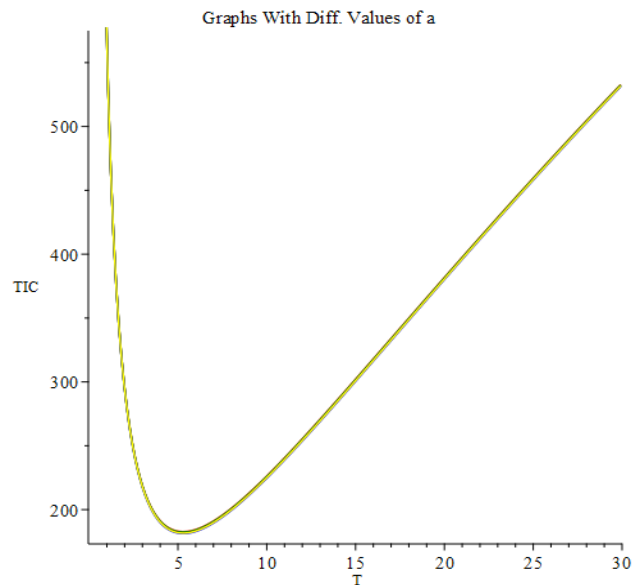
Sensitivity Analysis of Parameters

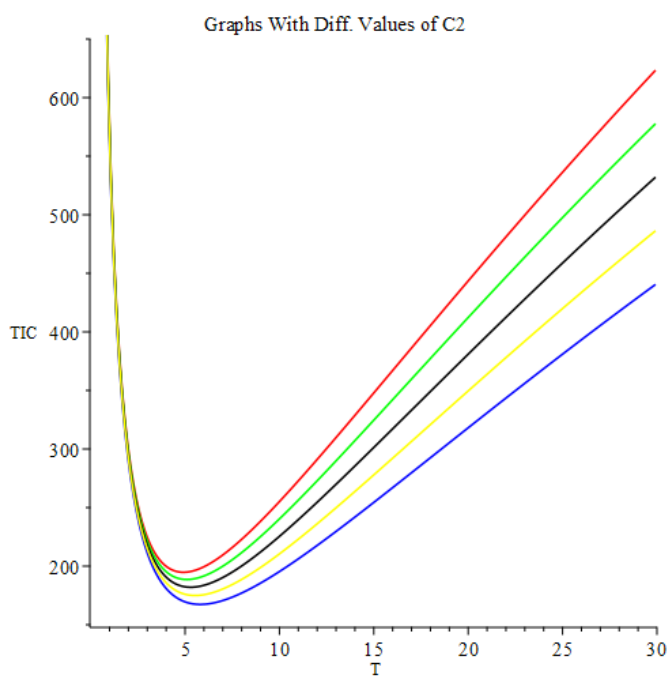
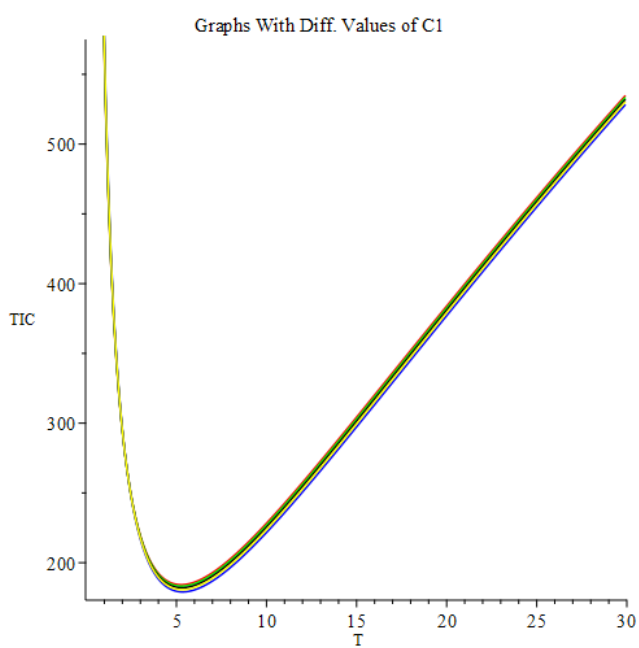
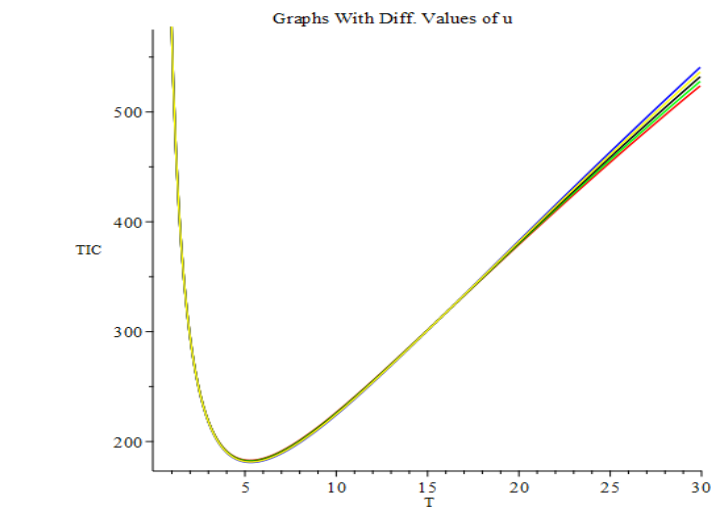
Sensitivity analysis of the key inventory model parameters $a, b, D, C_1, C_2, C_3, C_4, C_5, \alpha, \beta, \delta$ and u is given in the tabulated form as below:

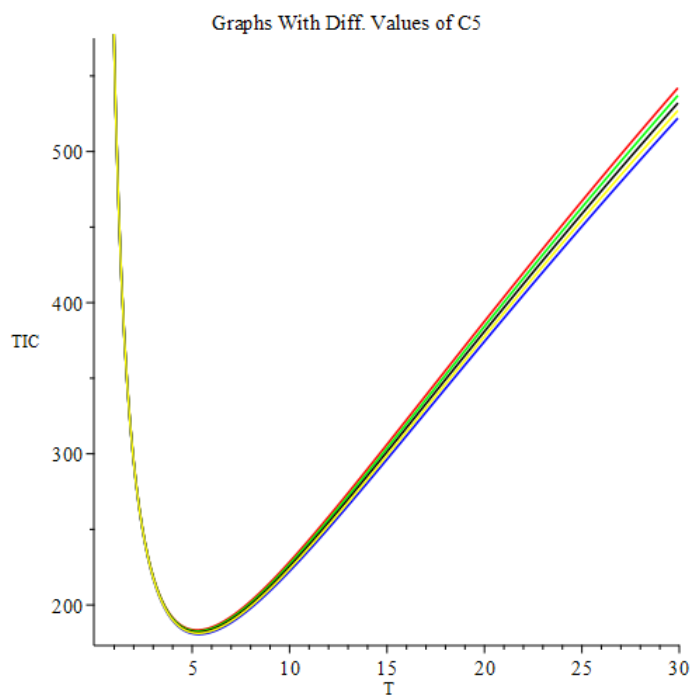
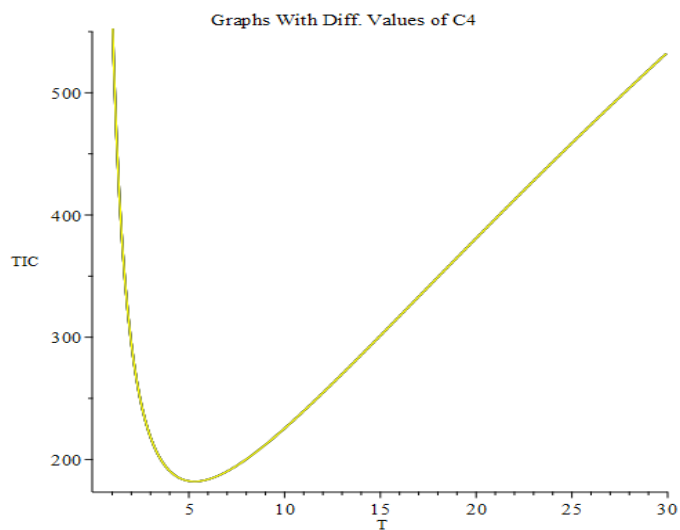
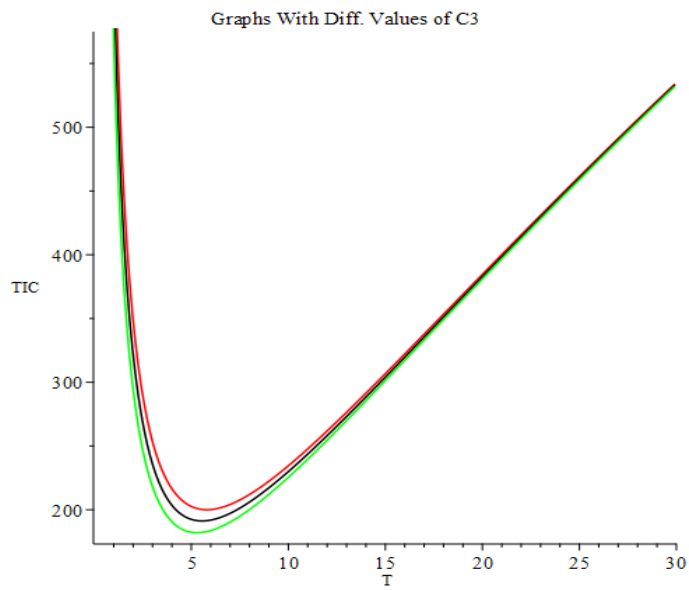
S. No.	Parameter	% Change	Values of			
			t_1	T	Q	TIC
1.	a	+20%	0.9658	5.2934	48.0758	182.7075
2.		+10%	0.9765	5.2953	47.8977	182.3512
3.		-20%	1.0094	5.3006	47.3389	181.2346
4.		-10%	0.9983	5.2989	47.5294	181.6149
5.	b	+20%	0.9180	5.2654	48.0452	183.5109
6.		+10%	0.9507	5.2803	47.8890	182.7894
7.		-20%	1.0768	5.3391	47.3022	180.0654
8.		-10%	1.0290	5.3165	47.5216	181.0867
9.	D	+20%	1.0291	4.8797	51.2525	195.9700
10.		+10%	1.0092	5.0734	49.5463	189.2237
11.		-20%	0.9367	5.8715	43.5965	165.7133
12.		-10%	0.9633	5.5592	45.7408	174.1831
13.	u	+20%	0.9903	5.2779	47.3521	182.9542
14.		+10%	0.9888	5.2875	47.5330	182.4719
15.		-20%	0.9844	5.3166	48.0863	181.0099
16.		-10%	0.9859	5.3068	47.9001	181.4997
17.	C_1	+20%	0.8970	5.2614	47.4361	184.1936
18.		+10%	0.9392	5.2780	47.5585	183.1597
19.		-20%	1.1082	5.3461	48.1925	179.0742
20.		-10%	1.0429	5.3195	47.9204	180.6413
21.	C_2	+20%	1.0256	4.9137	44.0259	194.7629
22.		+10%	1.0072	5.0929	45.7508	188.5792
23.		-20%	0.9418	5.8086	52.6238	167.3314
24.		-10%	0.9656	5.5328	49.9795	174.9286
25.	C_3	+20%	1.0409	5.7979	52.4931	200.0133
26.		+10%	1.0151	5.5530	50.1563	191.2035
27.		-20%	0.9250	4.7452	42.4552	162.0713
28.		-10%	0.9575	5.0286	45.1554	172.3026
29.	C_4	+20%	0.9873	5.2971	47.7155	181.9898
30.		+10%	0.9873	5.2971	47.7156	181.9884
31.		-20%	0.9874	5.2971	47.7157	181.9842
32.		-10%	0.9874	5.2971	47.7157	181.9856
33.	C_5	+20%	0.9915	5.2528	47.2897	183.3598
34.		+10%	0.9894	5.2748	47.5011	182.6783
35.		-20%	0.9831	5.3428	48.1544	180.5947
36.		-10%	0.9825	5.3198	47.9334	181.2934
37.	α	+20%	0.9839	5.2956	47.7311	182.062
38.		+10%	0.9856	5.2964	47.7234	182.0247
39.		-20%	0.9908	5.2987	47.7000	181.9112
40.		-10%	0.9891	5.2979	47.7078	181.9492
41.	β	+20%	0.9839	5.2942	47.6876	182.0060
42.		+10%	0.9856	5.2957	47.7017	181.9973
43.		-20%	0.9909	5.3000	47.7425	181.9605
44.		-10%	0.9891	5.2986	47.7293	181.9749
45.	δ	+20%	0.9955	5.2985	47.7256	181.7115
46.		+10%	0.9914	5.2978	47.7206	181.8498
47.		-20%	0.9793	5.2958	47.7058	182.2581
48.		-10%	0.9833	5.2965	47.7107	182.1231

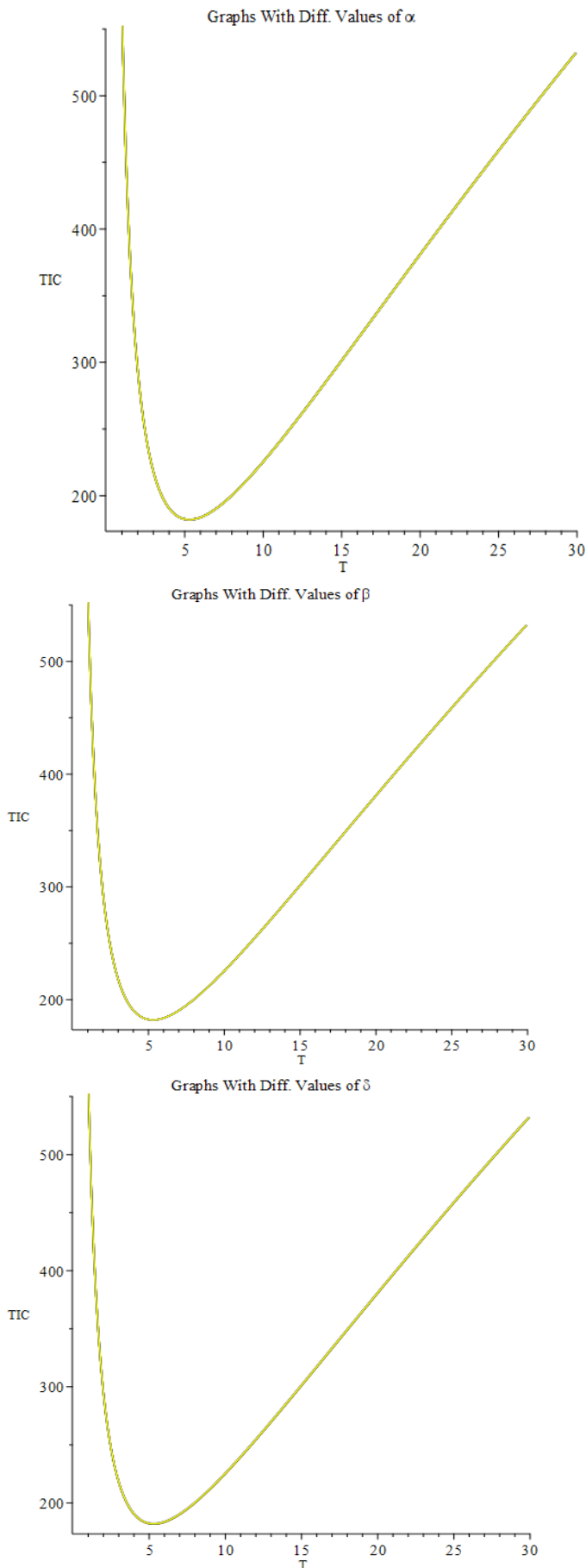
Effects of Different Parameters on the overall Inventory expense (TIC):

We have plotted graphs between T and the Total Inventory expense (TIC) by varying all key parameters with $\pm 10\%$ and $\pm 20\%$ changes. Red represents a 20% increase, green a 10% increase, black shows the original (no change), blue a 20% decrease, and yellow a 10% decrease. These graphs help analyse the level of sensitivity of TIC to parameter variations.









The results that are found from the sensitivity analysis are:

- Effect of demand rate parameters a and b : The demand rate parameters a and b significantly influence all variables, with increases leading to higher order quantities and cost. It leads to more inventory costs

for the real-life inventory systems. It also makes systems to order more quantity leading to more ordering costs.

- Effect of demand rate D : Demand rate D in time t_1 to T shows a strong effect, as D increases, order quantity and cost also increase considerably. It shows that higher demand rate will lead for more ordering cost and hence more total inventory costs.
- Effect of holding cost C_1 : Holding cost effects inversely to the complete cycle length T time and the order size Q . That means higher holding cost will result into less orders and hence more frequent orders. That simply means, it will make inventory models to order more frequently and that will cause higher ordering costs.
- Effect of shortage cost C_2 : Shortage have an inverse effect on the cycle length and the order size. That means if the shortage cost increases, the order size will decree leading to less time interval between two orders.
- Effect of ordering cost C_3 : highly affects TIC and Q , a higher ordering cost leads to increased order quantity and total cost. That means more ordering cost will make order quantity large and also it will increase overall inventory cost. It rises inflation in the market and leads to higher holding costs too.
- Effect of deterioration cost C_4 : It has minimal effect, reflecting model robustness to changes in deterioration cost. It slightly changes the value of cycle length T and Total inventory costs TIC , that means If items are decaying with slightly more rapidly, it will not occur any change or damage to the inventory system.
- Effect of opportunity cost C_5 : It affects Q and TIC moderately which means its effect should not be neglected and system should give importance to reduce it. Higher opportunity cost will result into higher total inventory cost TIC .
- Effect of deterioration rate constant u : It affects moderately to Q and TIC . Increase in u is increasing TIC and Q , decrease in u is decreasing TIC , T and Q . For a system, it is necessary to keep the deterioration rate constant as minimum as possible to optimize the inventory model smoothly.
- Effect of Weibull parameters α and β : These two parameters show low sensitivity, showing how stable the model is. Slight changes in these inventory parameter does not create any huge level of change in the inventory model which shows the higher stability of the system.
- Effect of salvage value constant δ : It shows a stabilizing effect on TIC and helps in reducing cost for higher salvage values. Hence, with the less salvage value, system will get less inventory costs and it will lead to overall profit to real world market systems.

CONCLUSION

All calculations and analytical computations were performed on the mathematical software Maple 2025, Giving the accurate determination of expressions for cost components and decision variables. The convexity of the graph of the total expense function was verified using graphical analysis on Maple which makes sure that unique optimal inventory system's solution exists for the inventory model. Among all parameters, demand (D) and ordering cost (C_3) shows the most substantial effect on the inventory decisions. The model shows robustness with respect to the deterioration parameters and salvage value, supporting its practical applicability in perishable goods inventory models. By the of the results of the sensitivity analysis, it can be easily concluded that the demand rate constant D and the ordering cost C_3 are the most sensitive inventory parameters to our inventory model system. Little changes in these parameters can cause a significant level of variations in the overall order quantity Q , the complete cycle time T , and the total inventory expense TIC . An increase in D during the shortage interval $[t_1, T]$ substantially brings increment in the order quantity and inventory costs, making demand forecasting a crucial factor in real-life inventory systems. Similarly, an increase in ordering cost C_3 pushes the order quantity upward and raises the overall cost, which can strain procurement budgets in real-world scenarios.

Future Scope

The given model can be further extended by considering stochastic or seasonal changes demand, variable lead times, and inflationary effects to align with more dynamic market conditions and make it more relatable for practical world systems. Future research can also work with applications of AI and machine learning for real-time monitoring of demand and deterioration can further enhance inventory decisions,

especially for perishable and time-sensitive goods. Further a software environment can be developed to make inventory solutions for industries handy and easy.

REFERENCES

- [1] Dave, U. (1989). On a heuristic inventory replenishment rule for items with a linearly increasing demand incorporating shortages. *Journal of the Operational Research Society*, 40, 827–830.
 - [2] Dave, U. (1989c). A comment on “An inventory replenishment policy for linearly increasing demand considering shortages – an optimal solution” by Murdeshwar, T.M. *Journal of the Operational Research Society*, 40, 412–416.
 - [3] Ouyang, W.U., and Cheng. (2005). An inventory model for deteriorating items with exponential declining demand and partial backlogging. *Yugoslav Journal of Operations Research*, 15(2), 277–288.
 - [4] Hardik Soni, Nita H. Shah, and Bhavin J. Shah. (2006). An EOQ model for two level credit policy order random input. *Revista Investigacion Operacional*, 27(3), 227–238.
 - [5] Alamri, A.A., and Balkhi, Z.T. (2007). The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. *International Journal of Production Economics*, 107, 125–138.
 - [6] Teng, J.T., Ouyang, L.Y., and Chang, C.T. (2007). A comparison between two pricing and lot-sizing models with partial backlogging and deteriorated items. *International Journal of Production Economics*, 105, 190–203.
 - [7] Chung, C.J., and Wee, H.M. (2008). An integrated production-inventory deteriorating model for pricing policy considering imperfect production, inspection planning and warranty-period- and stock-level-dependent demand. *International Journal of Systems Science*, 39(8), 823–837.
 - [8] Roy, A. (2008). An inventory model for deteriorating items with price dependent demand and time varying holding cost. *Advanced Modeling and Optimization*, 10, 25–37.
 - [9] Tripathy, C.K., and Mishra, U. (2010). An inventory model for Weibull deteriorating items with price dependent demand and time-varying holding cost. *Applied Mathematical Sciences*, 4(44), 2171–2179.
 - [10] Begum, R., Sahu, S.K., and Sahoo, R.R. (2012). An inventory model for deteriorating items with quadratic demand and partial backlogging. *British Journal of Applied Science & Technology*, 2(2), 112–131.
 - [11] Amutha, R., and Chandrasekaran, E. (2013). An EOQ model for deteriorating items with quadratic demand and time dependent holding cost. *International Journal of Emerging Science and Engineering*, 1(5).
 - [12] Sharma, G., & Vyas, B. (2018). An Approach To Positive Reflectance On Inventory Cost By Developing EOQ Model With Linear Demand By Weibull Distribution Deteriorating Items And Partial Backlogging. *ZENITH International Journal of Multidisciplinary Research*, 8(5), 125-133.
 - [13] Sharma, G., & Mahawar, A. (2019). Study of an economic order quantity model under linear demand, constant deterioration and shortages are permitted. *International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)*, 8(3), 1–8.
 - [14] Sharma, G., & Suman, S. (2019). An inventory model for time dependent deteriorate rate and variable holding cost. *International Journal of Mathematics and Computer Applications Research (IJMCAR)*, 9(1), 37–44.
 - [15] Li, K., Li, D., and Wu, D. (2022). Carbon transaction-based location-routing-inventory optimization for cold chain logistics. *Alexandria Engineering Journal*, 61(10), 7979–7986.
 - [16] Khare, G., & Sharma, G. (2022). Effects on Inventory Model in Time Deteriorate Rate with Variable Cost. *International Journal of Intelligent Systems and Applications in Engineering*, 10(3s), 45–50.
 - [17] Panigrahi, R.R., Jena, D., Sahoo, A., and Nayak, M.R. (2023). Inventory management practice and performances of manufacturing firms: An empirical study of RFID and VMI. In *Emerging Trends in Decision Sciences and Business Operations* (pp. 46–68). Routledge India.
 - [18] Prameswari, B.G., Rahman, A., Muharam, H., and Tjahjana, R.H. (2024). Product inventory optimization with EOQ approach in the context of circular economy. *Research Horizon*, 4(4), 389–398.
 - [19] Khare, G., & Sharma, G. (2024, March). Model for inventory having variable ordering and holding costs with salvage value. In *American Institute of Physics Conference Series* (Vol. 3025, No. 1, p. 030005).
 - [20] Khare, G., & Sharma, G. (2024). An Inventory Model with Fluctuate Ordering and Holding Cost with Salvage Value for Time Sensitive Demand and Partial Backlogging. *Commun. Appl. Nonlinear Anal.*, 31(1), 177–186.
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