

Study Of The Exotic Structure Of Light Nuclei ^8He And ^{12}Be Using The Hartree-Fock And Bear-Hodgson Potentials

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Abstract

The ground-state properties like the nuclear densities and the rms radii for exotic neutron-rich nuclei ^8He and ^{12}Be have been investigated using the Skyrme-Hartree-Fock and Bear-Hodgson potentials. The evaluated results are compared with available experimental data. It found that a common feature of the neutron and matter densities for above selected exotic nuclei is the long tail behavior. The two valence neutrons of ^8He and ^{12}Be are assumed to be in the ZBM model space (consists of $1p_{1/2}$, $1d_{5/2}$ and $2s_{1/2}$). It has been obvious that the structure of the outer two neutrons for these nuclei is mixed configuration with dominant $(1p_{1/2})^2$. The elastic charge form factors of above selected exotic nuclei are evaluated using the plane wave Born approximation and compared with those of their stable isotopes ^4He and ^9Be .

Keywords: Exotic light nuclei; Elastic form factors

1. INTRODUCTION

The study on the structure of short-lived nuclei far from β -stability has become a hot point in nuclear physics due to their exotic properties [1-4]. This led to the discovery of neutron halo in some exotic light neutron-rich nuclei as ^6He , ^{11}Li , ^{11}Be , ^{14}Be , ^{19}C , etc [5-7]. The nuclear halo is a quantum effect that arising from the very weak binding of the valence nucleons and their occupation on the orbits have $l = 0, 1$ (low angular momentum), which allow the wave function of these valence nucleons takes on an extended radial dimension [8]. The presence of a low-density tail at large radial distances is the main property of the nuclear-matter density distribution in halo nuclei [9].

The information about such nuclear structure can extract from the fragment momentum distribution of fragmentation reaction, total reaction cross section, Coulomb dissociation and quadrupole moment [10,11]. An additional information on the nuclear structure can be obtained by the proton elastic scattering at intermediate energies. It gives insights on both the nuclear-matter density distribution and the nuclear-matter radius [12]. This method is well established to study of the stable nuclei and it can be applied for the investigation of unstable nuclei as well when used in inverse kinematics with radioactive beams. The method has been successfully applied to investigate the isotopes $^{12,14}\text{Be}$ [9] and $^{8,9,11}\text{Li}$ [13].

The two-body (Core + n) and three-body (Core+ 2n) models within wave functions of different potentials have been used to investigate the ground state properties such as the proton, neutron and matter densities of many halo nuclei such as ^{22}N , ^{23}O , ^{14}Be , and ^{17}B , etc [14-18]. Also, the associated root mean square (rms) radii of these exotic nuclei have been studied by these models. The calculated results have been provided the halo structure for considered exotic nuclei.

In this work, the properties of the ground state for exotic ^8He and ^{12}Be nuclei, such as the neutron $[\rho^n(r)]$, proton $[\rho^p(r)]$, matter $[\rho_m(r)]$, the corresponding root-mean-square (rms) radii, and elastic charge form factors ($F_{ch}(q)$), will be investigated using the Hartree-Fock (HF) and Bear-Hodgson (BH) calculations. We will examine the reaction cross-sections (σ_R) for these nuclei using the Kox formula (KF) and Glauber model (GM).

2. Theory

The $\rho_m(r)$ of exotic nuclei is [19]:

$$\rho_m(r) = \rho_c(r) + \rho_h(r) \quad (1)$$

where $\rho_c(r)$ (core density) and $\rho_h(r)$ (halo density) are expressed as [19]:

$$\rho_c(r) = \frac{1}{4\pi} \sum_{n\ell j} N_c^{n\ell j} |R_{n\ell j}(r)|^2 \quad (2)$$

$$\rho_h(r) = \frac{1}{4\pi} N_h^{n\ell j} |R_{n\ell j}(r)|^2 \quad (3)$$

where $R_{n\ell j}(r)$ and $N^{n\ell j}$ denote the radial wave function and the occupation number of the orbit $n\ell j$, respectively.

The radial wave functions $R_{n\ell j}(r)$ taken from the solution to radial part of the Schrodinger equation using BH potential [20]:

$$\frac{d^2 R_{n\ell j}(r)}{dr^2} + \frac{2m}{\hbar^2} \left[\varepsilon_{n\ell j} - V(r) - \frac{\hbar^2 \ell(\ell+1)}{2m r^2} \right] R_{n\ell j}(r) = 0 \quad (4)$$

Where $\varepsilon_{n\ell j}$ is the single-particle binding energy and the $V(r)$ is the core potential given as [21]:

$$V(r) = V_0(r) + V_{so}(r) \mathbf{L} \cdot \mathbf{S} + V_c(r) \quad (5)$$

$V_0(r)$ is the central potential takes the following from [21]:

$$V_0(r) = \frac{-V_0}{1 + [e^{(r-R_0)/a_0}]} \quad (6)$$

$V_{so}(r)$ is the spin orbit potential [21]:

$$V_{so}(r) = V_{so} \frac{1}{r} \left[\frac{d}{dr} \frac{1}{1 + e^{(r-R_{so})/a_{so}}} \right] \quad (7)$$

$V_c(r)$ (for protons only) is Coulomb potential [21]:

$$V_c(r) = \begin{cases} \frac{Ze^2}{r} & \text{for } r > R_c \\ \frac{Ze^2}{R_c} \left[\frac{3}{2} - \frac{r^2}{2R_c^2} \right] & \text{for } r \leq R_c \end{cases} \quad (8)$$

and $V_c(r) = 0$ for neutrons.

The V_0 is potential depth [in Eq (6)] takes the BH form [22]:

$$V_0 = V_0^{n,p} \quad \text{for } -15 < \varepsilon_{n\ell j} < 0 \quad (9)$$

$$V_0 = V_0^{n,p} - \beta(\varepsilon_{n\ell j} + 15) \quad \text{for } \varepsilon_{n\ell j} < -15 \text{ MeV} \quad (10)$$

The potential depth for neutrons (V_0^n) and protons (V_0^p) whereas the β is constant.

The Skyrme force are given by [23]:

$$V_{Skyrme} = \sum_{i < j} V_{ij} = t_0(1 + x_0 P_\sigma) \delta(\vec{r}) + \frac{t_1}{2} (1 + x_1 P_\sigma) [\delta(\vec{r}) \vec{k}^2 + \vec{k}'^2 \delta(\vec{r})] + t_2 (1 + x_2 P_\sigma) \vec{k}' \cdot \delta(\vec{r}) \vec{k} \\ + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha(\vec{R}) \delta(\vec{r}) \\ + i t_4 \vec{k}' \cdot \delta(\vec{r}) (\vec{\sigma}_i + \vec{\sigma}_j) \times \vec{k}, \quad (11)$$

P_σ represents the space exchange operator, $\delta(\vec{r})$ denotes the delta function, \vec{k} indicates the relative momentum, $\vec{\sigma}$ is the vector comprising the Pauli spin matrices, and $t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3$, and α represent the parameters associated with the Skyrme force.

The charge, as well as the densities of protons or neutrons with the range of the Skyrme HF methodology, are articulated by [24]:

$$\rho_g(\vec{r}) = \sum_{\beta \in g} w_\beta \psi_\beta^+(\vec{r}) \psi_\beta(\vec{r}), \quad g = n, p, ch \quad (12)$$

where ψ_β is the wave function of a single particle for the state β and w_β denotes the probability of occupation for the state β .

The nucleus charge distributions can be obtained from the following folding relation [25]:

$$\rho_{ch}(r) = \int \rho_p(r) f_p(r' - r) dr', \quad (13)$$

where $\rho_p(r)$ and f_p are the proton density and one proton intrinsic charge distribution, respectively.

Where f_p takes the following form of Gaussian [26]:

$$f_p(r) = \frac{1}{(\sqrt{\pi} a_p)^3} e^{(-r^2/a_p^2)} \quad (14)$$

The core (R_c), matter (R_m), proton (R_p) and neutron (R_n) rms radii are obtained by [27]:

$$R_g = \langle r_g^2 \rangle^{1/2} = \left[\frac{\int r^2 \rho_g(r) dr}{\int \rho_g(r) dr} \right]^{1/2} \quad g = c, m, n, p, ch \quad (15)$$

In plane wave Born approximation (PWBA), the elastic charge form factor ($F_{ch}(q)$) is given by [28]:

$$F_{ch}(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) j_0(qr) r^2 dr, \quad (16)$$

Where $j_0(qr)$ and q are the Bessel function and the momentum transfer, respectively.

The σ_R using the GM [29] and KF [30] given, respectively, as:

$$\sigma_R = 2\pi \int [1 - T(b)] b db, \quad (17)$$

$$\sigma_R(E) = \pi r_0^2 \left[\left(A_p^{1/3} + A_t^{1/3} + a \frac{A_p^{1/3} A_t^{1/3}}{(A_p^{1/3} + A_t^{1/3})} - C(E) \right) \right]^2 \left(1 - \frac{B_c}{E_{cm}} \right) \quad (18)$$

3. RESULTS AND DISCUSSION

The ground-state properties like the nuclear densities and the rms radii for exotic neutron-rich nuclei ^8He and ^{12}Be have been investigated in the framework of the HF and BH calculations. The elastic $F_{ch}(q)$ of above selected exotic nuclei are evaluated using the PWBA. We use the KDE0 Skyrme parameterization within HF calculation in this work. The values of the KDE0 parameterization employed in our calculations are $t_0 = -2526.511$, $t_1 = 430.9418$, $t_2 = -398.3775$, $t_3 = 14235.5193$, $x_0 = 0.7583$, $x_1 = -0.3087$, $x_2 = -0.9495$, $x_3 = 1.1445$, $W_0 = 128.9649$, $\gamma = 0.1676$ [31]. The densities of both core and tail (halo) parts in HF and BH calculations are described by the radial wave functions of the HF and BH potentials, respectively. The density of the two-neutron (tail part) is added to that of the core part to obtain the matter density for halo nuclei ^8He and ^{12}Be .

We assumed that both ^8He ($J^\pi, T=0^+, 2$) and ^{12}Be ($J^\pi, T=0^+, 2$) have a structure of the core nuclei ^6He ($J^\pi, T=0^+, 1$) and ^{10}Be ($J^\pi, T=0^+, 1$), respectively plus the valence two neutrons ($J^\pi, T=0^+, 1$). The core nucleus ^6He is assumed to have four nucleons (two-neutron and two-proton) in the $1s_{1/2}$ orbit and the remaining two neutrons occupying $1p_{3/2}$ orbit, while the core nucleus ^{10}Be is assumed to have four nucleons (two-neutron and two-proton) in the $1s_{1/2}$ orbit and the remaining four neutrons and two protons occupying $1p_{3/2}$ orbit. The two valence neutrons of ^8He and ^{12}Be are assumed to be in the ZBM model space (consists of $1p_{1/2}$, $1d_{5/2}$ and $2s_{1/2}$). Calculations of the two valence neutrons occupation numbers occupying the model space of ZBM are performed using the ZWM [32] realistic interaction, which are carried out via the OXBASH code [33] and summarized in Table I. It is obvious from this table that the structure of the outer two neutrons for ^8He and ^{12}Be is mixed configuration with dominant $(1p_{1/2})^2$.

Table II displays the values of the BH parameters utilized in the present calculations for selected nuclei. The potential depth for neutrons (V_0^n) and protons (V_0^p) in core nuclei has been used the default of the NushellX@MSU program [34], whereas the V_0^n for valence neutron and other parameters provide the experimental ε of last neutron as well as the matter rms radii of exotic nuclei. The parameter β has been fixed at a value of 0.51[22]. The calculated ε for both protons and neutrons are presented in Table III together with the ε_{exp} [35] of the valence neutron.

The HF and BH calculations for core (R_c), matter (R_m), proton (R_p) and neutron (R_n) rms radii (in fm) of selected halo nuclei are presented in Tables IV and V. For comparison purposes, the corresponding experimental rms radii [36-39] are also given in these tables. From these tables we noted that the calculated results of our present study are agree well within the quoted error with the experimental results.

Table I: Occupation numbers of ZBM model space.

Model space	Interaction	Occupation numbers		
ZBM	ZWM	$(1p_{1/2})^2$	$(1d_{5/2})^2$	$(2s_{1/2})^2$
		1.503	0.420	0.077

Table II: The BH parameters.

Nuclei	$V_0^{p,n}$ (MeV)				V_{so} (MeV)	$a_0=a_{so}$ (fm)	$r_0=r_{so}$ (fm)	r_c (fm)
	Core	Halo						
		$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$				
^8He	70.352	39.11	60.88	57.30	7.0	1.392	0.725	1.2
^{12}Be	65.344	34.65	51.63	51.22	7.0	1.324	0.745	1.2
^4He	55.70				7.0	1.236	0.620	1.2
^9Be	51.33				7.0	1.236	0.620	1.2

Table III. The calculated ε .

Nuclei	$n\ell_j$	Proton		Neutron		$\varepsilon_{exp.}$ (MeV) [35]
		V_0 (MeV)	ε (MeV)	V_0 (MeV)	ε (MeV)	
^8He	$1s_{1/2}$	90.23	-54.001	90.89	-55.278	----
	$1p_{3/2}$	----	----	75.06	-24.251	----
	$1p_{1/2}$	----	----	39.11	-1.062	-1.062
	$1d_{5/2}$	----	----	60.88	-1.062	
	$2s_{1/2}$	----	----	57.30	-1.062	
^{12}Be	$1s_{1/2}$	84.33	-52.240	86.10	-55.717	----
	$1p_{3/2}$	70.19	-24.538	71.55	-27.196	----
	$1p_{1/2}$	----	----	33.54	-1.836	-1.836
	$1d_{5/2}$	----	----	50.48	-1.836	
	$2s_{1/2}$	----	----	49.51	-1.836	

Table IV: The calculated R_c and R_m rms radii and experimental ones.

Nuclei	R_c (fm)			R_m (fm)		
	HF	BH	Exp. [36,37]	HF	BH	Exp. [37,38]
^8He	2.13	2.07	2.30 ± 0.07	2.60	2.58	2.58 ± 0.02
^{12}Be	2.29	2.30	2.36 ± 0.04	2.75	2.73	2.73 ± 0.05

Table V: The calculated R_p and R_n rms radii and experimental ones.

Nuclei	R_p (fm)			R_n (fm)		
	HF	BH	Exp. [39]	HF	BH	Exp. [37,39]
^8He	2.05	1.79	1.88 ± 0.17	2.76	2.80	2.82 ± 0.03
^{12}Be	2.40	2.25	2.38 ± 0.16	2.91	2.95	2.86 ± 0.09

The calculated matter densities obtained by both HF (left part) and BH (right part) calculations for exotic ^8He and ^{12}Be nuclei and core ^6He and ^{10}Be nuclei along with the tail (two-neutron halo) part are shown in Fig. 1. The black, blue and dashed- red curves represent the core, tail part and matter densities, respectively. The experimental matter densities for ^8He [40] and ^{12}Be [38] is presented in this figure by grey area for comparison. Figs. 1(a) and (b) show the densities for ^8He , while Figs. 1(c) and (d) correspond to those for ^{12}Be . A common feature of the dash-dotted blue curves, which can be shown in Fig. 1, is the long tail behavior. Clearly, the dash-dotted blue curves obtained with both HF and BH calculations lie within the experimental uncertainties and agree well with the experiment.

Figs. 2 (a)-(d) summarize the calculated proton (black curve) and neutron (blue curve) densities in ^8He (upper panel) and ^{12}Be (lower panel) obtained by the HF and BH calculations. From these figures, it can be clearly seen that the neutron densities of ^8He and ^{12}Be have a long tail with respect to the proton densities. This means that ^8He and ^{12}Be are neutron-halo nuclei.

Figs. 3(a)-(d) compare the calculated results of matter densities for unstable ^8He and ^{12}Be (dashed red distributions) and stable ^4He and ^9Be (blue distributions) isotopes. From these figures, we can observe that, there is a difference in the behavior of the blue and dashed red distributions. This demonstrates a long tail in dashed red distributions and supports the halo structure of ^8He and ^{12}Be nuclei.

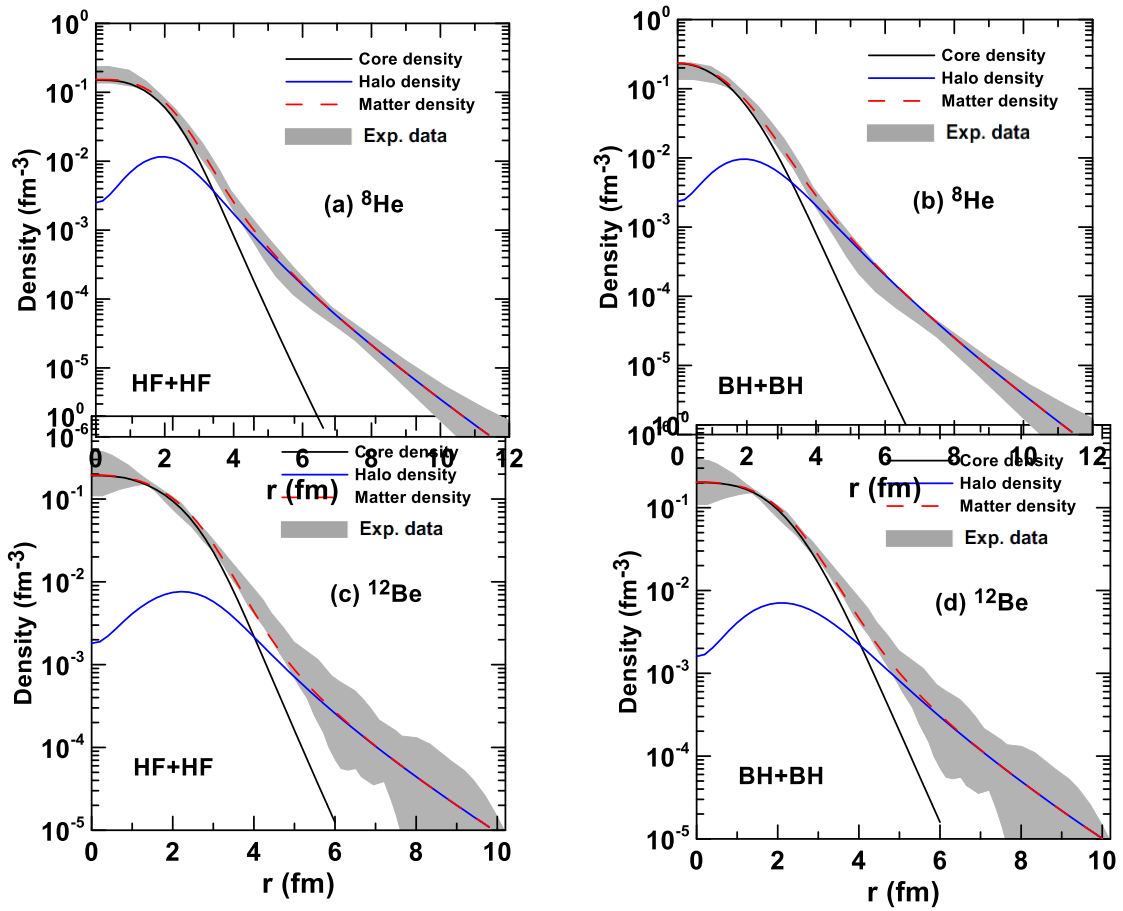


Fig. 1: The $\rho_c(r)$, $\rho_h(r)$ and $\rho_m(r)$ of ^8He and ^{12}Be .

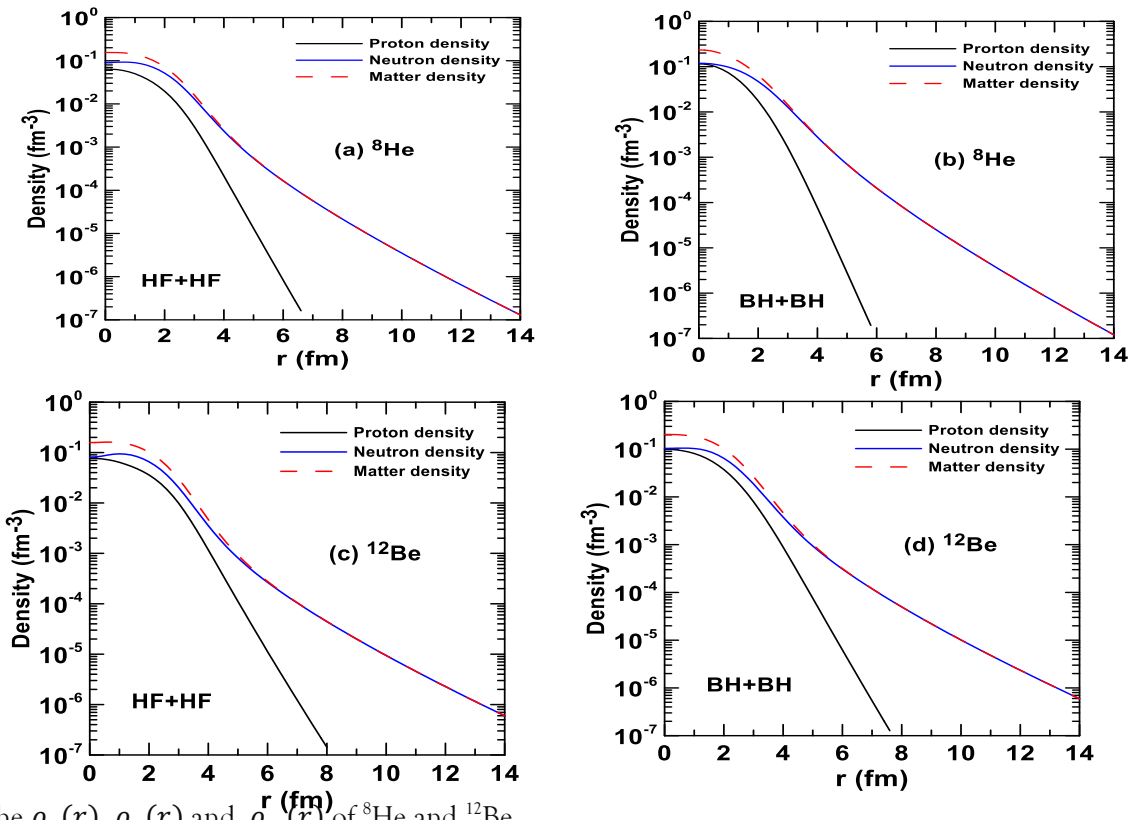


Fig. 2: The $\rho_p(r)$, $\rho_n(r)$ and $\rho_m(r)$ of ^8He and ^{12}Be .

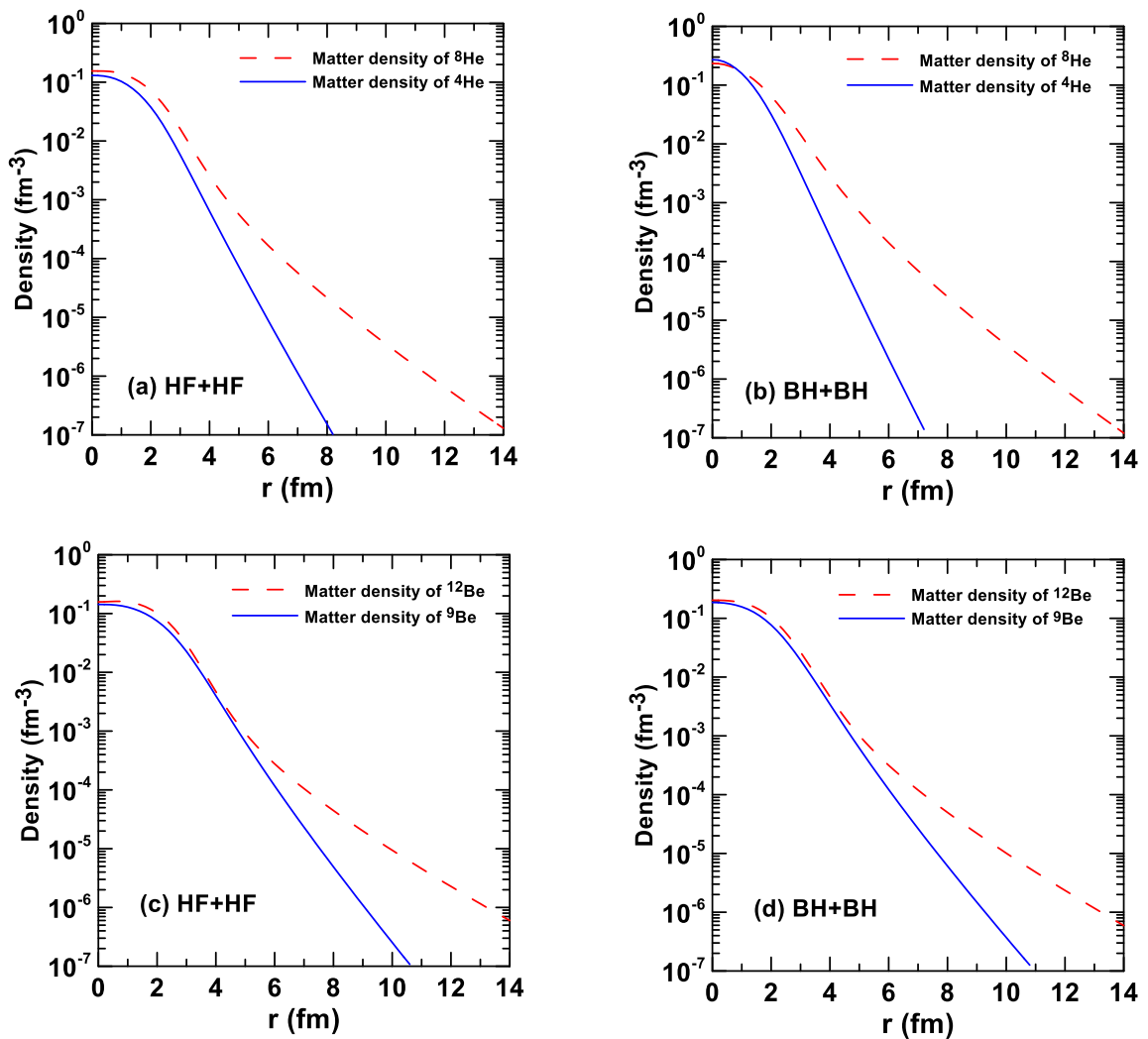
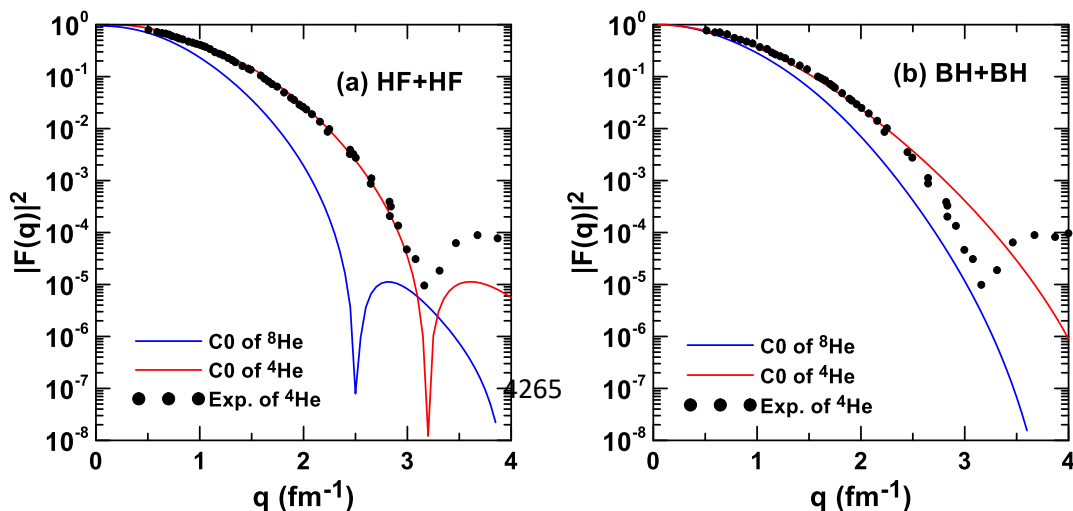


Fig. 3: The $\rho_m(r)$ of isotopes ${}^4,8\text{He}$ and ${}^9,12\text{Be}$.

Figs. 4(a)-(d) depicted the calculated results of $F_{ch}(q)$ for ${}^4,8\text{He}$ [Figs. 4(a) and 4(b)] and ${}^9,12\text{Be}$ [Figs. 4(c) and 4(d)] isotopes obtained by the PWBA. Therein, the blue and red curves refer to $F_{ch}(q)$ of unstable and stable isotopes, respectively. For comparison the experimental $F_{ch}(q)$ for stable isotopes ${}^4\text{He}$ [41] and ${}^9\text{Be}$ [42,43] are given by dotted symbols. The results of our calculations agree reasonably with the experiment data for stable isotopes. One can see from Figs. 4 (a), (c) and (d) there is one minimum in both blue and red curves in the whole considered q -range while those in Fig. 4(b) have no minimum in this q -range. The shift of the minima to smaller values of q is a common property of the blue curves. This is attributed to the proton densities enhancement in the peripheral region.



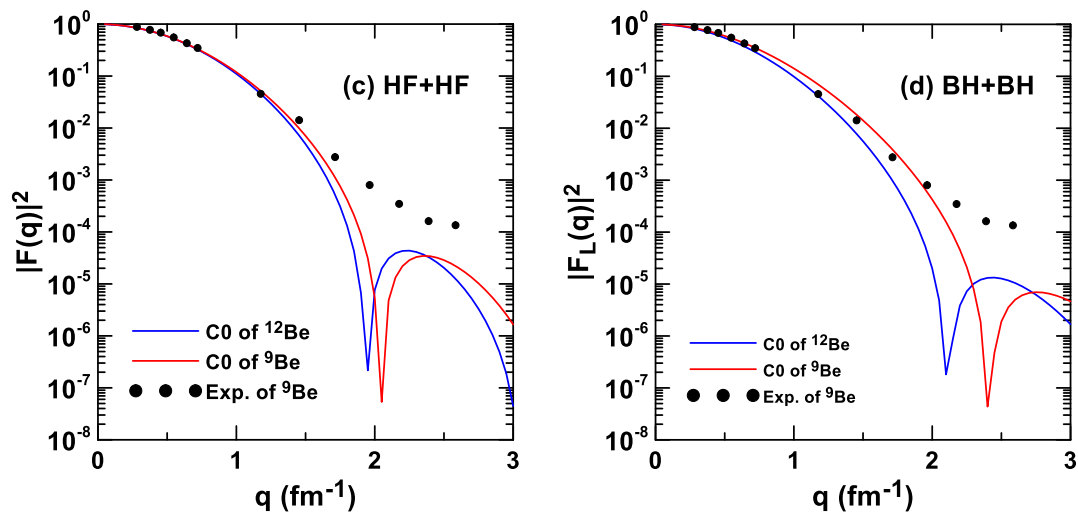


Fig. 4: The C0 form factors for ${}^4,8\text{He}$ and ${}^9,12\text{Be}$.

In this work, the σ_R of the exotic ${}^8\text{He}$ and ${}^{12}\text{Be}$ nuclei on target ${}^9\text{Be}$ at 790 MeV are studied by the KF and GM with the OLA and summarized in Table VI along with experimental results. The HO densities are used in GM calculations. It is demonstrated that the experimental data satisfactorily well by our calculations obtained by GM while the extracted σ_R by Kox formula closely agree with those experimental data within quoted error.

Table VI: The σ_R of the exotic ${}^8\text{He}$ and ${}^{12}\text{Be}$ nuclei on target ${}^9\text{Be}$.

Halo nuclei	Energy (MeV) [44]	σ_R (Cal.) (mb)		σ_R (Exp.) (mb) [44]
		KF	GM	
${}^8\text{He}$	790	762	753	757 ± 4
${}^{12}\text{Be}$	790	889	877	873 ± 22

4. SUMMARY AND CONCLUSIONS

The ground-state properties like the nuclear densities and the rms radii for exotic neutron-rich nuclei ${}^8\text{He}$ and ${}^{12}\text{Be}$ have been investigated in the framework of the HF and BH calculations. The evaluated results are compared with available experimental data. It found that a common feature of the neutron and matter densities for above selected exotic nuclei is the long tail behavior. The two valence neutrons of ${}^8\text{He}$ and ${}^{12}\text{Be}$ are assumed to be in the ZBM model space (consists of $1p_{1/2}$, $1d_{5/2}$ and $2s_{1/2}$). Calculations of the two valence neutrons occupation numbers occupying the model space of ZBM are performed using the ZWM realistic interaction, which are carried out via the OXBASH code. It has been obvious that the structure of the outer two neutrons for ${}^8\text{He}$ and ${}^{12}\text{Be}$ is mixed configuration with dominant $(1p_{1/2})^2$. The elastic $F_{ch}(q)$ of above selected exotic nuclei are evaluated using the PWBA and compared with those of their stable isotopes ${}^4\text{He}$ and ${}^9\text{Be}$. The shift of the minima to smaller values of q is a common property of the form factors for exotic nuclei. This is attributed to the proton densities enhancement in the peripheral region. In addition, the σ_R of the exotic ${}^8\text{He}$ and ${}^{12}\text{Be}$ nuclei on target ${}^9\text{Be}$ at 790 MeV are studied by the KF and GM with the OLA. It has been demonstrated that the experimental data satisfactorily well by our calculations obtained by GM while the extracted σ_R by KF closely agree with those experimental data within quoted error.

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