International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 19s, 2025
http://www.theaspd.com/ijes.php

Stratified Flow Over A Dipole And Conditions For The Non-Occurrence Of Blocking

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Abstract:

In the study on two-dimensional stratified flow in a channel, Dube (2002) and Yih (1960) proposed that the two-dimensional stratified flow over a barrier in a channel can be investigated by taking a suitable combination of sources, sink and doublets in place of barrier. Trustrum (1964) and then Dube (2023 & 2025) considered, however independently, the problem of two-dimensional channel flow over a barrier by applying an oseen-type approximation to the general flow and discussed the Long's hypothesis. Our work is to examine and find out relation between the pressure at infinity, Fourde number and the strength of dipole for a non-occurrence of blocking by assuming the dipole to be placed at the bottom of the channel with its axis parallel to it and directed against the uniform flow.

If the pseudo-velocity at infinity on the negative side (i.e. at) be not large enough, then there is apparently a possibility that a layer of the stratified fluid in the lower region of the channel may not be able to cross the diploe. This will result in what may be called the blocking of the incoming fluid by the diploe. This leads to a contradiction to the work of Trustrum (1964) that if the axis of the dipole be parallel to the channel wall then there is no possibility of blocking. So we restudy the problem of the stratified flow over a dipole placed at the bottom of the infinite channel with its axis parallel to the uniform flow at infinity. We shall also try to find out analytically relation between the pressure condition at infinity (on the negative side) and the strength of the dipole for the non-occurrence of blocking.

Nomenclature:

F	:	Froude number
F_0	:	Ordinary Froude number
$\bar{q}$	:	Velocity vector
$\bar{q}'$	:	Dimension of velocity of pseudo velocity
$\bar{F}$	:	External force other than gravitational force
$\bar{g}$	:	Acceleration due to gravity
p	:	Pressure
ρ	:	Density
ρ_0	:	Reference density

U	:	Horizontal velocity
U_0	:	Representative velocity
d	:	Reference length
g	:	Acceleration due to gravitation
ψ	:	Stream function
ψ'	:	Pseudo stream function
$\bar{k}$	:	Unit vector perpendicular to the plane of the motion
δ	:	Variation of height of the streamline.

Key Words: Steady two-dimensional flow of a stably stratified, Incompressible inviscid fluid towards a sink, pseudo-flow , pseudo-steam function, torricellian vacuum

Governing Equation, Boundary conditions and solution:

We consider a two-dimensional stratified flow over a dipole placed at the origin at the bottom of an infinite horizontal channel formed by $y = 0$ and $y = d$. The axis of the dipole is horizontal and directed against the flow and the fluid of the purely dipole flow has constant density equal to that of the lowest stratum of the stratified fluid. The dipole flow affects the stratified flow. Using the equation

$$\psi = \int \left(\frac{\rho}{\rho_0} \right)^{-1/2} d\psi' \qquad (1)$$

the physical flow is transformed into the pseudo-flow with uniform velocity at $x = -\infty$; the stream function for the perturbed pseudo-low in the channel in non-dimensional quantities is then given by

$$\left(\nabla^2 + \beta^2\right)\psi = \beta^2 y \ , \qquad (2)$$

($\beta = \dfrac{1}{F}$, F being the Froude number defined by $\left(\dfrac{U_0^2}{agd} \right)^{1/2}$)

The boundary conditions for ψ are set as

$$\begin{aligned} &\psi \to y \ \ as \ |x| \to \infty, \\ &\psi = 1 \ \ on \ \ y = 1 \quad \forall x, \\ &\psi = 0 \ \ on \ \ y = 0 \ \ x \neq 0. \end{aligned} \qquad (3)$$

Writing $\phi = \psi - y$, the equation (2) reduces to

$$\left(\nabla^2 + \beta^2\right)\phi = 0, \qquad (4)$$

with boundary conditions

$$\begin{aligned} &\phi \to y \;\; as \;\; |x| \to \infty, \\ &\phi = 1 \;\; on \;\; y = 1 \quad \forall x, \\ &\phi = 0 \;\; on \;\; y = 0 \quad x \neq 0. \end{aligned} \qquad (5)$$

The function ϕ then defines the perturbation in the flow, i.e. the stream function for the purely dipole flow without any external effect satisfies the Laplace equation.

In the absence of the dipole, the equation (4) admits the trivial solution $\phi = 0$, giving $\psi = y$, which represents the unperturbed uniform parallel pseudo-flow. In the perturbed pseudo-flow, the solution is, as shown by Drazin-Moore (1967), found in two types depending on β. When $\beta < \pi$, the solution does not contain wavy terms and when $\beta > \pi$, the solution contains both wavy and non-wavy terms.. It is important to note that when $\beta = 0$, no solution exists.

Assume $$\phi = \sum_{n=1}^{\infty} \phi_n(x) \sin n\pi y \;, \qquad (6)$$

the equation (4) gives $$\frac{d^2\phi_n}{dx^2} + \left(\beta^2 - n^2\pi^2\right)\phi_n = 0. \qquad (7)$$

The solution of this equation depends on the sign of $\left(\beta^2 - n^2\pi^2\right)$.

If $0 < \beta < \pi$, then $\left(\beta^2 - n^2\pi^2\right)$ can never be positive and so the solution is exponential type:

$$\phi_n = A_n' \exp\left[\left(n^2\pi^2 - \beta^2\right)^{1/2} x\right] + A_n \exp\left[-\left(n^2\pi^2 - \beta^2\right)^{1/2} x\right] \quad (8)$$

where A_n' and A_n are arbitrary constants; and if β lies in $N\pi < \beta < (N+1)\pi$ for some positive integer N, then

$\beta^2 - n^2\pi^2$ is positive for $n \le N$ and
negative for $n \ge N+1$.

Thus for $n \ge N+1$, the solution will be of the above exponential type while for $n \le N$, the solution will be of sinusoidal type

$$\phi_n = A_n' \sin\left[\left(\beta^2 - n^2\pi^2\right)^{1/2} x\right] + A_n \exp\left[\left(\beta^2 - n^2\pi^2\right)^{1/2} x\right] \qquad (9)$$

which is stationary wave.

Thus when $\beta > \pi$, waves occur. To avoid the solution in which waves may occur, we shall restrict the consideration to the case when $\beta < \pi$.

The above solution cannot be accepted, for it cannot satisfy the necessary boundary conditions for the flow over a dipole. Thus the above method of solution fails, and hence we solve it by different method.

Since the disturbance of the flow is caused by the dipole at the origin, therefore the solution must be that one having a dipole singularity at the origin. Drazin-Moore (1967) used the Dirac delta function for the dipole singularity at the origin and accordingly the boundary condition on $y = 0$ was set as $\phi = K\delta(x)$.

Following the Drazin-Moore, the solution of the equation (4) and having a dipole singularity at the origin is, for $\beta < x$, written as

$$\phi = -K\pi \sum_{n=1} \frac{n \sin nxy}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x|(n^2\pi^2 - \beta^2)^{1/2}\right], \qquad (10)$$

where K is some constant.

The perturbed pseudo-stream function ψ is then given by

$$\psi = y + \phi = -K\pi \sum_{n=1} \frac{n \sin nxy}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x|(n^2\pi^2 - \beta^2)^{1/2}\right]. \qquad (11)$$

It may be noted that it is through the constant K that the dipole strength is to be involved.

The relationship between K and the dipole strength can be established simply by arguing that the diploe flow in the neighborhood of the origin is not affected by the stratified flow, i.e. the flow in the neighborhood of the origin (dipole singularity) is purely dipole flow.

Thus in the neighborhood of the origin, the equation (11) will reduce , by dropping the fisrt term and also putting $\beta = 0$, to

$$\psi = -K\pi \sum_{n=1}^{\infty} \sin \pi y \ \exp(-n\pi\,|x|) = \frac{-1}{2} K \frac{\sin \pi y}{\cosh \pi x - \cos \pi y}. \qquad (12)$$

The above result simplifies to $\psi = -\dfrac{K}{\pi} \dfrac{y}{x^2 + y^2}$. (13)

Again, considering the purely dipole flow in the neighborhood of the origin, it is seen that the physical stream function ψ_1 for the dipole flow may be taken as

$$\psi_1 = -\mu_1 \frac{y}{x^2 + y^2}, \ (x, y \ \text{small}) \qquad (14)$$

where μ_1 is the physical dipole strength.

Since the density of the fluid in the dipole flow is constant, the pseudo-stream function for the dipole flow is given by

$$\psi_2 = -\mu \frac{y}{x^2 + y^2}, \tag{15}$$

where $\psi_2 = \frac{\psi_1}{Ud}$ and $\mu = \frac{\mu_1}{U}$ is the non-dimensional dipole strength. Here x, y are non-dimensional coordinates.

Now, in the neighborhood of the origin, the equations ((13) and (15) represent the same flow and so , we find

$$K = \mu\pi \ . \tag{16}$$

This is the relation between the constant K and the non-dimensional dipole strength μ . So, writing K in terms of μ , the equation (11) now becomes

$$\psi = y - \mu\pi^2 \sum_{n=1} \frac{n \sin nxy}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x| (n^2\pi^2 - \beta^2)^{1/2} \right]. \tag{17}$$

This gives the perturbed pseudo-stream function of the flow in the channel.

From the equation (17), we find the horizontal component of the velocity

$$u = \frac{\partial\psi}{\partial y} = 1 - \mu\pi^3 \sum_{n=1}^{\infty} \frac{n^2 \cos n\pi y}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x| (n^2\pi^2 - \beta^2)^{1/2} \right]. \tag{18}$$

On the lower boundary of the channel excepting the origin, i.e. on $y = 0, x \neq 0$, the equation (18) becomes

$$u(x,0) = 1 - \mu\pi^3 \sum_{n=1}^{\infty} \frac{n^2}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x| (n^2\pi^2 - \beta^2)^{1/2} \right]. \tag{19}$$

Now, every term of the summation $\sum_{n=1}^{\infty} \frac{n^2}{(n^2\pi^2 - \beta^2)^{1/2}} \exp\left[-|x| (n^2\pi^2 - \beta^2)^{1/2} \right]$ is positive and the value of the summation increases as $|x|$ decreases and becomes indefinitely large at the origin and tends to zero as $|x| = -\infty$. So , $u(x,0)$ decreases in the magnitude as $|x|$ decreases up to a certain value where $u(x,0)$ becomes zero; if $|x|$ increases, the magnitude of $u(x,0)$ increases attaining the maximum value 1 at $|x| = \infty$. This is because of the fact that on either side of the dipole, the fluid velocity of the dipole and that of the stratified fluid are opposite to each other. This is in fact, what is expected that on the lower boundary, the velocity of the fluid in the stratified flow decreases as $|x|$ decreases, and so $u(x,0)$ will be zero at some point on the lower boundary. The points on either side of the dipole where the velocity is zero, i.e. the stagnation points are given by the equation by putting $u(x,0)=0$ in the equation (19), we get

$$0 = 1-\mu\pi^3\sum_{n=1}^{\infty}\frac{n^2}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-|x|(n^2\pi^2-\beta^2)^{1/2}\right]. \qquad (20)$$

It is obvious from the equation (20) that the two stagnation points are symmetrical with respect to the $y-$ axis i.e. equidistance from the origin.

The positions of the stagnation points are obviously depends on μ and β . We shall briefly show how the positions of the stagnation points change with μ keeping β fixed.

Differentiating equation (20) with respect to μ , we get

$$0 = -\pi^3\sum_{n=1}^{\infty}\frac{n^2}{(n^2\pi^2-\beta^2)^{1/2}}\exp\left[-|x|(n^2\pi^2-\beta^2)^{1/2}\right]+\mu\pi^3\sum_{n=1}^{\infty}n^2\exp\left[-|x|(n^2\pi^2-\beta^2)^{1/2}\right]\frac{d|x|}{d\mu}$$

giving $$\frac{d|x|}{d\mu} = \frac{1}{\mu^2\pi^3\sum_{n=1}^{\infty}n^2\exp\left[-|x|(n^2\pi^2-\beta^2)^{1/2}\right]} > 0 . \qquad (21)$$

This shows that as μ increases, the stagnation points on the lower boundary shift away from the origin on either side by the same amount. This is what we expected.

To see this more clearly, we may consider the location of stagnation points in the limiting case when the stratification is very small, i.e. when β is very small.

When β is very small, it's second and higher power terms can be neglected as compared to π . By doing so, the equation (20) gives the stagnation points approximately as

$$0 = 1-\mu\pi^2\sum_{n=1}^{\infty}n.\exp(-n\pi|x|)$$
$$= 1-\mu\pi^2 e^{-\pi|x|}\left[1+2e^{-\pi|x|}+3e^{-2\pi|x|}+\ldots\right] = 1-\mu\pi^2 e^{-\pi|x|}\frac{1}{(1-e^{-\pi|x|})^2}$$

whence , $e^{-\pi|x|}(1-e^{-\pi|x|})^2 = \mu\pi^2$

or $\left(e^{\frac{1}{2}\pi|x|} - e^{-\frac{1}{2}\pi|x|}\right)^2 = \mu\pi^2$

or $4\sinh^2 \dfrac{\pi|x|}{2} = \mu\pi^2$.

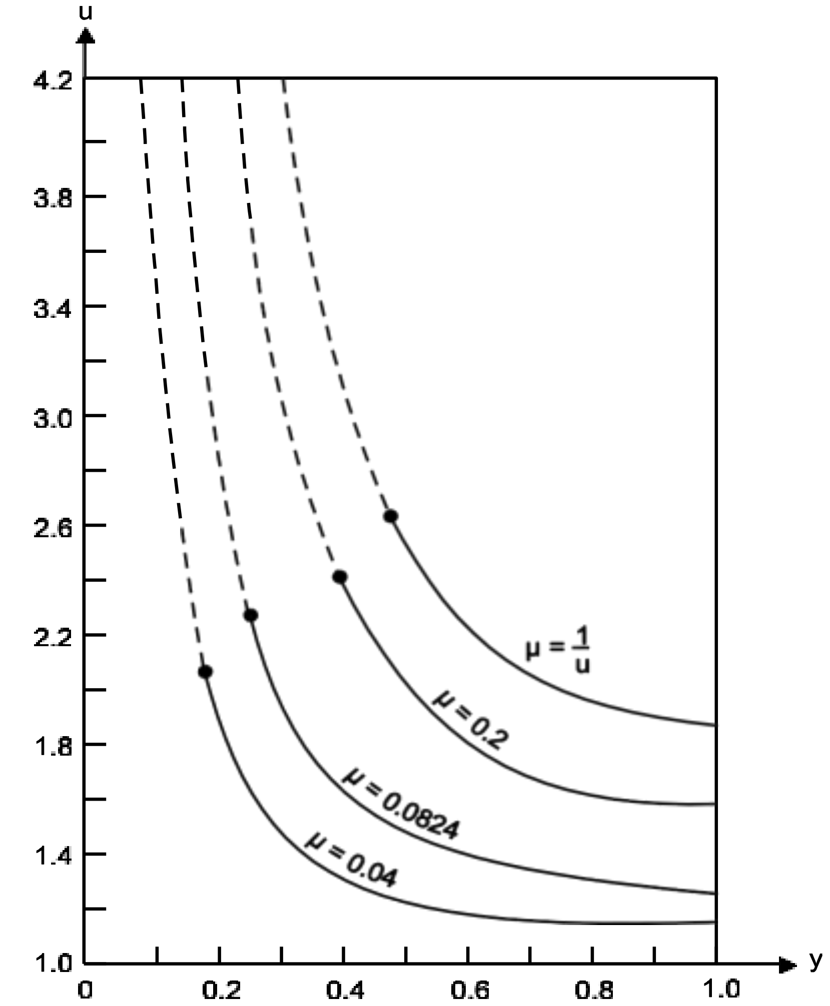

Fig. 1 Curves of u against y for different values of μ the continuous part is for the stratified flow and the dotted part is for the dipole flow.

Figure-1 establishes the continuous part for stratified flow and the dipole flow.

Therefore, when β is negligibly small, the stagnation points are given by

$$|x| = \frac{2}{\pi}\sinh^{-1}\left(\frac{\pi\sqrt{\mu}}{2}\right) . \qquad (22)$$

Thus, as μ increases, $|x|$ increases and vice-versa. Hence the stagnation points on the lower boundary $(y = 0)$ shifts away from the origin as the dipole strength μ increases. The same result is expected to hold for β not small and so even $\beta > \pi$ and not equal to any multiple of π .

It may be interesting to see also how the horizontal component of velocity behaves near the y -axis above the origin. From the equation (18), taking $\beta < 1$ and expanding the right hand side in powers of β and summing, we find in the limit when $x \to 0$,

$$u = 1 + \frac{1}{4}\mu\pi^2\left[\operatorname{cosec}^2\frac{\pi x}{2} + 2\frac{\beta^2}{\pi^2}\log\left(2\sin\frac{\pi y}{2}\right) + O(\beta^4)\right], 0 < y \le 1 . \qquad (23)$$

This gives the horizontal component of the pseudo-velocity when the pseudo-streamlines cross the y -axis above the origin. Curves of u against y are drawn in the graph and shown in the figure-1 for different values of μ .

Yih (1969) blocking may be defined as the phenomenon of stagnation of a layer of the fluid leading from an obstacle upstream to infinity. In the case of stratified flow, he pointed out that it is possible for the part of the fluid near the bottom to be blocked while the top part still flows, depending upon the ar-upstream velocity and the height of the obstacle

Extending the idea of blocking defined above, it may be possible that when μ (dipole strength) is large enough, the lower stratum of fluid coming from $x = -\infty$ may not be able to rise and cross the dipole, which may result to blocking. Blocking in the real sense may not occur; for, here the obstacle, if occurs, will invalidate the assumptions at $x = -\infty$ making thereby the flow analysis upset. However, on the hypothesis of the possibility of blocking, one can easily find a maximum value of μ up to which normal flow can ensure and flow analysis can be carried out.

If a fluid particle, coming along the lowest stratum of the stratified fluid, can move up to the top of the dipole, then obviously no question of blocking will arise and the condition for it being so is that the hydrodynamic pressure at the top of the dipole must not be less than the hydrostatic pressure there at. This condition naturally implies a certain relationship between μ and p_∞ (the hydrodynamic pressure at $x = -\infty$). In what follows, we try to find this relationship between μ and p_∞, and then deduce the possible maximum value of μ corresponding to a prescribed p_∞, when β is small.

In the pseudo-flow of the stratified fluid, we have the density H defined by

$$H = p + \frac{q^2}{2} + \frac{\rho y}{F_0^2} \qquad (24)$$

is constant on a pseudo-streamline.

Hence, considering a streamline

$$p+\frac{q^2}{2}+\frac{\rho y}{F_0^2}=\text{value of } H \text{ at } x=-\infty$$

$$=p_\infty+\frac{1}{2}+\frac{\rho\psi}{F_0^2} \quad (\because q=1, \text{ and } \psi=y \text{ at } x=-\infty)$$

So, for the streamline $\psi=0$, $p+\frac{q_0^2}{2}+\frac{y}{F_0^2}=p_\infty+\frac{1}{2}$ giving

$$p_0=p_\infty+\frac{1}{2}(1-q_0^2)-\frac{y}{F_0^2}. \tag{25}$$

Here, p_0 and q_0 are the pressure and the velocity at any x on the streamline $\psi=0$ at which $\rho=1$.

If the height of the region to which the dipole flow is confined be denote by y_0, then the hydrodynamic pressure p_0 at the top of the dipole is

$$p_0=p_\infty+\frac{1}{2}(1-q_0^2)-\frac{y_0}{F_0^2}, \tag{26}$$

Here q_0 refers to the velocity at the top of the dipole.

Again, if p_H be the hydrostatic pressure at the top of the dipole, then

$$p_H=\frac{\rho_m(1-y_0)}{F_0^2}, \tag{27}$$

where $\rho_m=1-\frac{\alpha}{2}$ is the mean density of the stratified fluid and α being the stratification constant.

Now , if $p_0<p_H$, there will be no chance for the fluid particles coming along the lower boundary of the channel to come up to the highest point of the dipole flow region and so the condition for the said fluid particles to cross the dipole , i.e. the condition for the non-occurrence of blocking is $p_0>p_H$.

Using the value of p_0 and p_H from the equations (26) and (27), we find the non-occurrence of blocking

$$p_\infty+\frac{1}{2}(1-q_0^2)-\frac{y_0}{F_0^2}>\frac{\rho_m(1-y_0)}{F_0^2}.$$

Since $p_m < 1$, the above inequality will satisfied if $p_\infty + \frac{1}{2}(1-q_0^2) - \frac{y_0}{F_0^2} \geq \frac{(1-y_0)}{F_0^2}$ which further gives

$$q_0 \leq \left(1-2p_\infty - 2F_0^{-2}\right)^{1/2} . \qquad (27)$$

At the top of the dipole, the fluid velocity is horizontal and so by (25), we have

$$q_0 = 1 - \mu\pi^3 \sum_{n=1}^{\infty} \frac{n^2 \cos n\pi y_0}{(n^2\pi^2 - \beta^2)^{1/2}} . \qquad (28)$$

This gives the relation between μ and p_∞ for the non-occurrence of the blocking for $\beta < \pi$.

When β is small, by Cesaro's sum, we have

$$\sum_{n=1}^{\infty} \frac{n^2 \cos n\pi y}{(n^2\pi^2 - \beta^2)^{1/2}} = \frac{1}{\pi} \sum_{n=1}^{\infty} n \cos n\pi y_0 \left[1 + \frac{1}{2}\frac{\beta^2}{n^2\pi^2} + \frac{3}{8}\frac{\beta^4}{n^4\pi^4} + \ldots \right].$$

Hence for β small enough, if we neglect terms of second and higher orders of β , the inequality (28) will reduce to

$$1 + \frac{1}{4}\mu\pi^2 \operatorname{cosec}^2 \frac{\pi y_0}{2} \leq \left(1 + 2p_\infty - 2F_0^{-2}\right)^{1/2}$$

or $$\mu \leq c \sin^2\left(\frac{\pi y_0}{2}\right) , \qquad (29)$$

where $$C = \frac{4}{\pi^2}\left[\left(1-2p_\infty - 2F_0^{-2}\right)^{1/2} - 1\right]. \qquad (30)$$

Since $\mu > 0$, the inequality (29) requires that C should be positive and therefore

$$p_\infty > F_0^{-2} . \qquad (31)$$

Also, when β is small enough so that second and higher powers of β can be neglected, then it is seen from the equation (27) that the streamline $\psi = 0$ cuts the y -axis at the point y_0 gives by the equation

$$y_0 = \mu\pi \sum_{n=1}^{\infty} n \sin n\pi y_0 = \frac{1}{2}\mu\pi \cot\left(\frac{\pi y_0}{2}\right) .$$

We have $$\mu = \frac{2}{\pi} y_0 \tan\left(\frac{\pi y_0}{2}\right) . \qquad (32)$$

Using this value of μ , the inequality (29) gives

$$y_0 \le \frac{\pi}{4} c \sin \pi y_0 . \qquad (33)$$

The trigonometric function $\sin \pi y_0$ continuously increases from $y_0 = 0$ to $y_0 = \frac{1}{2}$, attaining the maximum value 1 at $y_0 = \frac{1}{2}$.

Hence, the inequality (33) will be satisfied if $y_0 \le \frac{1}{2}$ and $c \ge \frac{2}{\pi}$.

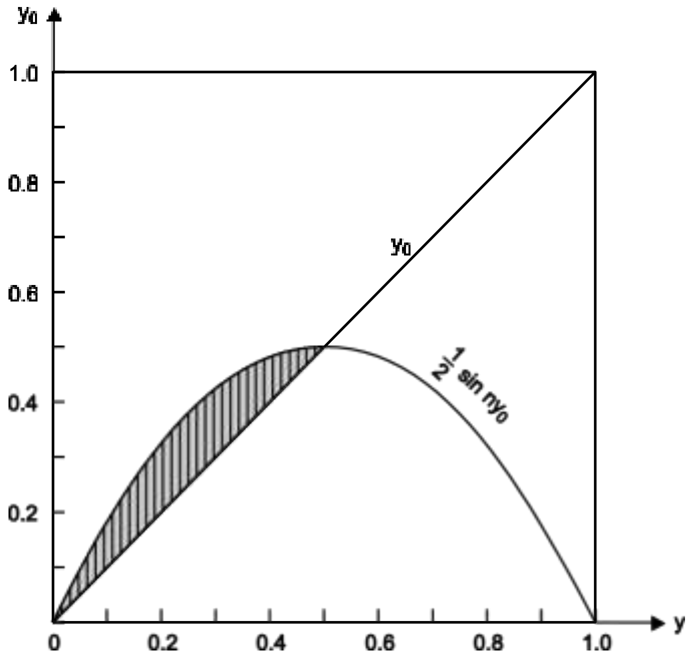

Fig. 2 Graph of the in equation $y_0 \le \frac{1}{2} \sin n\pi y_0$ where the shaded portion is the feasible region

This can also be verified from the graph shown in **figure-2** where the shaded portion is the required region from holding the inequality.

With the maximum value of y_0 (i.e. $y_0 = \frac{1}{2}$), the equation (32) gives $\mu = \frac{1}{\pi}$. This gives the possible maximum value of μ for the non-occurrence of blocking.

Thus the restrictions on μ and c for the non-occurrence of blocking are

$$\mu \le \frac{1}{\pi} \text{ and } c \ge \frac{2}{\pi} . \qquad (34)$$

The restriction on c , when applied to the equation (30), gives

$$\frac{4}{\pi^2}\left[(1+2p_\infty-2F_0^{-2})^{1/2}-1\right]\geq\frac{2}{\pi},$$

or $$1+2p_\infty-2F_0^{-2}\geq\left(\frac{\pi}{2}-1\right)^2,$$

or $$p_\infty\geq\frac{\pi^2}{8}+\frac{\pi}{2}+F_0^{-2}. \qquad (35)$$

Hence for the non-occurrence of blocking, p_∞ should be at least equal to $\frac{\pi^2}{8}+\frac{\pi}{2}+F_0^{-2}$, while μ should at most be equal to $\frac{1}{\pi}$.

Since F_0 does not contain the stratification constant, the inequality (35) gives the condition for the non-occurrence of blocking in the case of homogeneous fluid. Thus, for insufficient pressure at infinity (i.e. insufficient velocity and hence insufficient energy at infinity), the influence pf the dipole is felt far upstream and so its effect cannot be totally ignored. This indeed contradicts the Trustrum result.

Conclusion:

In the conclusion, we discuss the nature of change of flow with β and μ. For the flow not contain waves, β should be less than π and also for the non-occurrence of blocking $\mu\leq\frac{1}{\pi}$ and $p_\infty\geq\frac{\pi^2}{8}+\frac{\pi}{2}+F_0^{-2}$.

The case when $\beta=0$ implies $\alpha=0$ and so , in case $\beta=0$, the stratified fluid flow reduces to the homogeneous irrotational flow of constant density $\rho=1$ everywhere. In that case, there is no distinction between the pseudo-stream function and the natural stream function. The flow is simply described by the stream function ψ as

$$\psi=y-\mu\pi\sum_{n=1}^{\infty}\sin n\pi y.\exp(-n\pi\,|x|) \text{ (putting } \beta=0 \text{ in equation (17)}$$

$$=y-\frac{1}{2}\mu\pi\frac{\sin\pi y}{\cosh\pi x-\cos\pi y} \qquad (36)$$

When $\beta\neq 0$ but small enough, the flow pattern of the stratified fluid flow will not differ much from that of the homogeneous fluid flow given by the equation (36).

Again, for $\beta\neq 0$, we have

$$u = 1 - \mu\pi^3 \sum_{n=1}^{\infty} \frac{n^2 \cos n\pi y}{\left(n^2\pi^2 - \beta^2\right)^{1/2}} \exp\left[-|x|\left(n^2\pi^2 - \beta^2\right)^{1/2}\right] \text{ and}$$

$$v = \mu\pi^2 \sum_{n=1}^{\infty} n \sin n\pi y . \exp\left[-|x|\left(n^2\pi^2 - \beta^2\right)^{1/2}\right].$$

From these equations, it can be seen that the effect of β is to make the fluid move faster than that in the corresponding homogeneous fluid.

The streamline $\psi = 0$ demarcates the stratified fluid flow from the dipole flow region and so $\psi > 0$ defines the stratified flow while $\psi < 0$ defines the dipole flow.

When β is small, the boundary of the dipole flow region cannot taken as approximately given by (assuming μ small)

$$x^2 + y^2 = \mu \ . \tag{37}$$

Which is obtained by putting $\psi = 0$ in the equation (36) and then expanding and simplifying by retaining up to second order terms.

In the case when $\mu \neq 0$, the two stagnation points symmetrically placed on either side of the origin appear on the lower boundary (from equation (20)). As μ increases from 0 to $\frac{1}{\pi}$, the stagnation points shift away from the origin by equation (22) and accumulation of fluid occur near those points. This can also be verified from the stream pattern shown in **figure-3**. As a result of shifting away of the stagnation points, the shape of the demarcating curve (i.e. the part of the streamline $\psi = 0$) is found flattened extending more on both sides of the x -axis. This can be verified mathematically from the equation (22) , and is confirmed from the graph of the streamline pattern from **figure-3**.This is actually what is expected and is partly due to the influence of the stratified flow. When there is no possibility of blocking, the maximum height of the dipole flow region is $y = \frac{1}{2}$ and this corresponds to $\mu = \frac{1}{\pi}$.

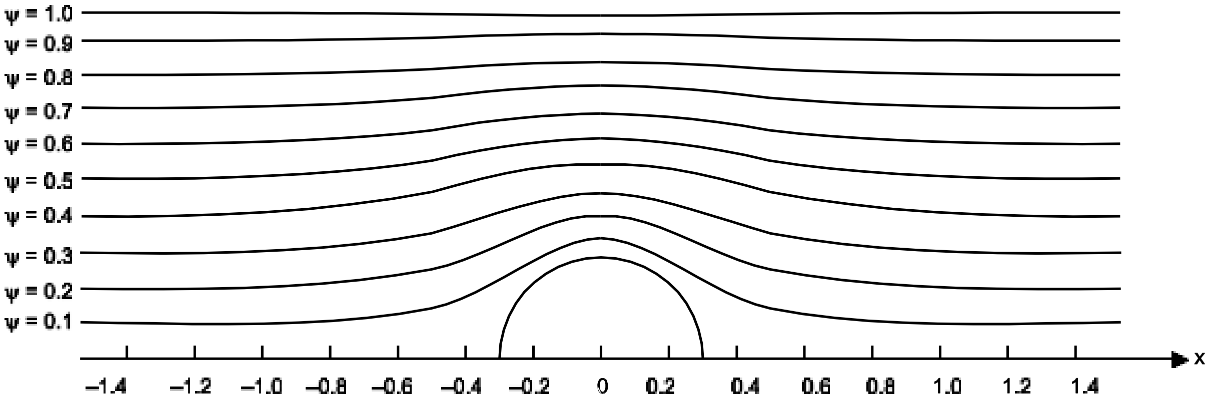

Fig. 3(a)

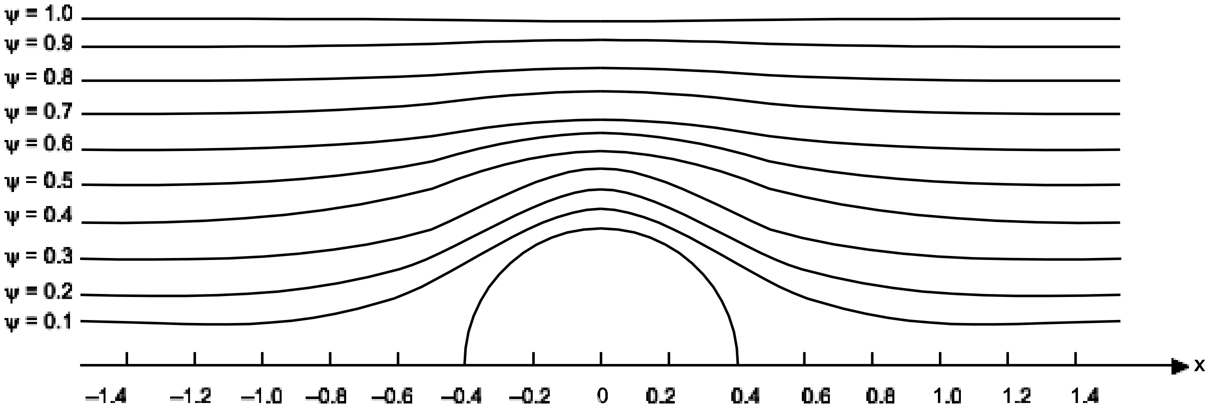

Fig. 3(b)

Fig. 3 Streamlines for the flow over a dipole of strength μ placed at

If $\mu > \frac{1}{\pi}$, the maximum pressure at infinity, i.e. $p_\infty = \frac{\pi^2}{8} + \frac{\pi}{2} + F_0^{-2}$ is not sufficient enough for the fluid particles on $\psi = 0$ on the left side of the dipole to rise against the gravity and to cross the y -axis above the dipole and so in that case there is a possibility of blocking of fluid, to occur on or near the lower boundary. This violates indeed the basic assumptions at infinity on the negative side, i.e. the pseudo-velocity is uniform at $x = -\infty$. Thus the pressure at infinity and the dipole strength are related for the non-occurrence of blocking in the flow field.

Applications of the work:

The study of stratified fluids also find applications in industries. The concept of solar pond and ocean thermal energy conversion (OTEC) may be mentioned. The intrusion of a heavy fluid into a lighter one occurs in the process of manufacturing glass. Idea of fluid flow in porous media is applicable to hydrology and is of vital interest to petroleum industries and paper industries. Axisymmetric flow of non-homogeneous fluids have also a bearing on such engineering devices as centrifuges and meteorological phenomena as tornados.

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