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Prediction of Agricultural GSDP of Assam using Cobb Douglas, Constant Elasticity of Substitution and Multiple Linear Regression models

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Abstract
This study predicts the agricultural Gross State Domestic Product (GSDP) of Assam, using statistical and econometric models for policy formulation in agricultural economics. We explore various models, including Cobb Douglas model, Constant Elasticity of Substitution model and Multiple Linear Regression model and found that the Multiple Linear Regression offers the best fit based on higher
R^2, lower MSE, and AIC, along with statistically significant t-values.
Our findings highlight the critical role of agricultural productivity in driving economic growth, enhancing the Gross State Domestic Product (GSDP), and supporting food security and employment. By integrating robust econometric models such as the Cobb-Douglas production function, the Constant Elasticity of Substitution (CES) model, and Multiple Linear Regression (MLR), this study provides valuable empirical insights.
Objectives:
1. *To predict the agricultural economy in the state using Machine Learning models.*
2. *To study the performance of selected model to assess the agricultural economy in the state.*

Methods: *The methods and models we have used here are Ordinary Least Squares (OLS) and Non- linear Curve Fitting methods are used to estimate the parameters of the models. OLS is applied in the Cobb-Douglas and Multiple Linear Regression models, while Non-linear Curve Fitting is used for the Constant Elasticity of Substitution (CES) model.*
Results: *Among the employed models, area and labour were identified as significant determinants of Assam's agricultural economic growth. Among the predictive models, the Multiple Linear Regression (MLR) model demonstrated the best fit, as indicated by its highest R^2 value, lowest Mean Squared Error (MSE), lowest root mean squared error (RMSE), lowest mean absolute percentage error (MAPE) and lowest Akaike Information Criterion (AIC).*
Conclusions: *This study employed Ordinary Least Squares (OLS) and Non-linear Curve Fitting techniques to estimate parameters for three production function models, Cobb-Douglas, Constant Elasticity of Substitution (CES), and Multiple Linear Regression (MLR). OLS was applied to the Cobb- Douglas and MLR models, while the CES model was estimated using Non-linear Curve Fitting due to its structural complexity.*
The analysis identified area and labour as key contributors to Assam's economic growth. Among the models, the Multiple Linear Regression (MLR) model outperformed the others in terms of predictive accuracy, as demonstrated by its highest R^2, lowest Mean Squared Error (MSE), lowest root mean squared error (RMSE), lowest mean absolute percentage error (MAPE) and lowest Akaike Information Criterion (AIC) values. These results highlight the effectiveness of MLR in capturing the relationship between agricultural inputs and economic output in Assam, making it the most suitable model for policy formulation and future forecasting efforts.
Key words: *Agricultural Productivity, agricultural Gross State Domestic Product (GSDP), Cobb Douglas, Constant Elasticity of Substitution, Multiple Linear Regression*

INTRODUCTION:

Understanding the relationship between agricultural inputs and outputs remains a foundational concern in agricultural economics. This relationship is typically captured through production functions mathematical models that quantify how different inputs such as land, labour, capital, fertilizer, and technology contribute to output, often measured in terms of yield or total production. For researchers and policymakers, identifying the most appropriate functional form is crucial for accurately evaluating productivity and guiding policy interventions. Among the various production functions, the Cobb- Douglas, Constant Elasticity of Substitution (CES), and Multiple Linear Regression (MLR) models are particularly influential. The Cobb Douglas production function, widely employed due to its log-linear form and interpretable elasticity coefficients, assumes constant returns to scale and unitary elasticity of substitution (Bhatti et al., 1996; Hossain et al., 2006; Prajneshu, 2008). The CES production function offers greater flexibility by allowing varying elasticities of substitution between inputs, though it is computationally intensive and nonlinear in estimation (Henningsen & Henningsen, 2012; Henningsen et al., 2019). Multiple Linear Regression, though not a production function in the strict sense, is often used to empirically estimate the separate contributions of agricultural inputs to output under less restrictive assumptions (Samiyu, 2021). These models have been extensively used to analyze the impact of input factors such as irrigation, fertilizer use, electricity, credit availability, rainfall, and high-yielding variety (HYV) seeds on agricultural productivity across various Indian states. Similarly, regression analyses have identified both significant and insignificant contributors to Agricultural GDP, depending on the structural and policy context (Reddy & Dutta, 2018; Kulsesthra & Agarwal, 2019). These findings underscore the importance of selecting appropriate models to reflect the realities of the agricultural production environment. While Cobb-Douglas provides simplicity and interpretability, CES offers a more nuanced substitution framework, and MLR ensures flexibility 1 with larger datasets. Each has strengths and limitations, and their application must be guided by data availability, research objectives, and economic context. In regions like Assam ranked 17th in Agricultural GDP among Indian states understanding the determinants of agricultural productivity is crucial for informed policy-making. The state's diverse agro-climatic conditions, reliance on monsoon rainfall, and variable input adoption patterns necessitate empirical evaluation using robust models. This study employs Cobb-Douglas, CES, and MLR approaches to model agricultural output in Assam, aiming to quantify input elasticities, estimate substitution patterns, and assess the contribution of individual inputs to economic performance. The integration of such modelling efforts not only enhances our understanding of agricultural productivity but also informs targeted interventions to stimulate rural development, food security, and inclusive economic growth in the region.

1. Objectives of the Study:

The aim of this paper is to predict the impact of three different models on agricultural economy and their performance evaluation with respect to Assam. Based on this the following objectives are formulated:

I. To predict the agricultural economy in the state using Machine Learning models.

II. To study the performance of selected model to assess the agricultural economy in the state.

3. METHODS AND MODELS:

The models used in this study include the Cobb Douglas Production Function, Constant Elasticity of Substitution, Multiple Linear Regression models. The Cobb-Douglas model is specified in its log-linear form, which enables the estimation of input elasticities directly from the regression coefficients. This functional form assumes a constant elasticity of substitution equal to one and provides a convenient framework to analyze returns to scale and factor productivity. The parameters were estimated using the Ordinary Least Squares (OLS) method in Python, and log transformations were applied to ensure linearity in parameters. This model is especially useful for its interpretability and widespread applicability in agricultural economics. The CES model generalizes the Cobb-Douglas function by relaxing the assumption of unitary elasticity of substitution between inputs. It accommodates varying degrees of substitutability, which is particularly relevant in the context of diverse agricultural environments and input interactions. The CES function was estimated using non-linear regression techniques, implemented through numerical optimization methods available in Python's scientific computing libraries. This model adds analytical depth by allowing a flexible substitution structure and provides insights into how changes in input mix affect output levels. The MLR model serves as a baseline comparative framework for analysing the relationship between agricultural output and input variables without imposing specific structural assumptions on production technology. It treats the output as a linear function of independent variables such as land area, labour, fertilizer use, irrigation, and capital inputs. Using OLS estimation, the MLR model helps assess the individual and joint statistical significance of explanatory variables.

In addition, performance evaluation metrics such as R^2 (Coefficient of Determination), Mean Squared Error (MSE), t-Statistic, Akaike Information Criterion (AIC), and Instantaneous Growth Rate have been incorporated for comprehensive model evaluation

3.1 Cobb Douglas Model:

The studies on production function were made firstly by Knut Wicksell (economist) in 1906. Then, Cobb-Douglas production function was developed by Charles W. Cobb (mathematician) and Paul H. Douglas (economist) in 1928. The Cobb-Douglas production function is widely used in economic studies. This function describes the economic output as a function of two factors, capital and labour. Cobb-Douglas production

function is used the modelling the substitution between capital input, labour services and technical change. This model implies the elasticity of substitution equals one. This function describes the economic output as a function of two factors, capital and labour. The Cobb-Douglas production function is given by
The General form of Cobb-Douglas production function is formulated as:

$$Q_i = AX_{1i}{}^{\beta 1}X_{2i}{}^{\beta 2}X_{3i}{}^{\beta 3} \ldots\ldots\ldots\ldots . X_{ki}{}^{\beta k}e^{\varepsilon i} \tag{1}$$

Where:

- Q_i is the ith observed output for the observation,
- A is a constant representing total factor productivity,
- X_{ji} represents the quantity of the j th input for the ith observation,
- βj are parameters indicating the output elasticity of each input,
- ϵi is the error term.

Taking log on both sides of eq(1)

$$\log Q_i = \log A + \beta_1 \log X_{1i} + \beta_1 \log X_{1i} \tag{2}$$

Transformation and Logarithms: Taking the logarithm of both sides of the equation, we obtain:

$$\log_e (Q_i) = \log_e (\text{ A}) + \beta_1 \log_e (X_{1i}) + \cdots \ldots\ldots + \beta_k \log_e (X_{ki}) + \epsilon_i \tag{3}$$

This equation ensures that the model satisfies the assumption of linear regression.
Where j=1, 2, ……………….,k
In matrix notation we obtain,

$$\begin{bmatrix} \log_e Q_1 \\ \log_e Q_2 \\ \vdots \\ \log_e Q_k \end{bmatrix}_{n\times 1} = \begin{bmatrix} 1 & \log_e X_{11} & \ldots & \log_e X_{1k} \\ 1 & \log_e X_{21} & \ldots & \log_e X_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \log_e X_{n1} & \ldots & \log_e X_{nk} \end{bmatrix}_{n\times(k+1)} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}_{(k+1)\times 1} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}_{n\times 1} \tag{4}$$

Which can be written as:

$$\begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_k \end{bmatrix}_{n\times 1} = \begin{bmatrix} 1 & Z_{11} & \ldots & Z_{1k} \\ 1 & Z_{21} & \ldots & Z_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & Z_{n1} & \ldots & Z_{nk} \end{bmatrix}_{n\times(k+1)} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}_{(k+1)\times 1} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}_{n\times 1} \tag{5}$$

From where we get,

$$W = Z\beta \tag{6}$$

And

$$\beta = (z/z)^{-1} z/W \tag{7}$$

Where,

$$\text{Var} - \text{Cov}(\beta) = \sigma^2 (z/z)^{-1} \tag{8}$$

And

$$\sigma^2 = \frac{\text{RSS}}{n - \text{no.of parameter}} \tag{9}$$

For brevity we have used only two explanatory variables namely Labour and Capital
L = X_{1i} and K = X_{2i} and $\beta_1 = \alpha, \beta_2 = \beta$
Then,

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L + \epsilon_i \tag{10}$$

Similarly, for Rice Yield as Output we can Consider Capital and Fertilizer as inputs. The significance of the model lies in the fact that α, β represents direct elasticities of Labour and Capital w.r.t output Q i.e. GSDP, α, β represents direct elasticities of Capital and Fertilizer.

3.1.1 Estimation of the Parameters of Cobb Douglas Model:
From the model above we get,

$$\varepsilon_{YK} = \frac{d\log Y}{d\log K} \tag{11}$$

After solving we get,

$=\frac{d\log Y}{dY}\frac{1}{1/k}\frac{dY}{dK}$

$= \frac{K}{Y}dK$ (12)

$= \mathrm{A}\alpha K^{\alpha-1}L^{\beta}\frac{L}{\mathrm{A}K^{\alpha}L^{\beta}}$

$=\alpha$

Similarly,

$\varepsilon_{YL}=\frac{d\log Y}{d\log L}$ (13)

After solving we get,

$=\frac{d\log Y}{dY}\frac{1}{1/L}\frac{dY}{dL}$

$=\frac{LdY}{YdL}$ (14)

$= \mathrm{A}K^{\propto}\beta L^{\beta-1}\frac{L}{\mathrm{A}K^{\alpha}L^{\beta}}$

$=\beta$

The Return to scale in CD production function is achieved by

- $\alpha+\beta>1$ (15)
- $\alpha+\beta<1$ (16)
- $\alpha+\beta=1$ (17)

3.2 CONSTANT ELASTICITY OF SUBSTITUTION (CES) PRODUCTION FUNCTION:

The Constant Elasticity of Substitution (CES) production function, initially introduced by Arrow, Chenery, Minhas, and Solow in 1961, is a generalized form of the Cobb-Douglas production function. Unlike Cobb-Douglas, CES imposes a constant elasticity of substitution across its isoquants, meaning the rate at which inputs can be substituted for one another remains uniform throughout. The CES production function assumes that the elasticity of substitution between any pair of inputs is consistent. Various forms of the CES function exist; for this study, the CES production function proposed by Kmenta in 1967 will be used, focusing on capital (K) and labor (L) inputs. The equation for this CES production function, characterized by a constant elasticity of substitution (CES), is expressed as:

$$\mathrm{Q}=\mathrm{A}[\boldsymbol{\delta k}^{-\rho}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{L}^{-\rho}]^{\frac{-\theta}{\rho}} \quad (A>0;0<\sigma<1;-1<\rho\neq 0) \quad (18)$$

Here, Q denotes the total output value, L represents labour input measured in person-years, K represents capital input measured in monetary terms, and δ,ρ, and σ are parameters. $A\in[0,\infty)$ represents the productivity(technologic progress level), $\delta\in[0,1]$ denotes the inputs' optimal distribution, $\rho\in[-1,0)\cup(0,\infty)$ represents the elasticity of substitution and $\vartheta\in(0,\infty)(\vartheta=KL>0)$ is the function's homogenous order of return to scale (degree of homogeneity) In the original form of CES, parameter ϑ was taken as $\vartheta=1$.If $\vartheta=1$, constant return to scale, if $\vartheta<1$, decreasing return to scale and if $\vartheta>1$, increasing return to scale.

The elasticity of substitution (EOS) of CES function is written as,

$$\sigma=\frac{\partial\ln\left(\frac{K}{L}\right)}{\partial\ln\left(\frac{MP_L}{MP_K}\right)} \quad (19)$$

$$=\frac{1}{1-\rho}\geq 0 \quad (20)$$

The logarithmic form of CES function is given by

$$\ln\ \mathrm{Q}=\ln\gamma-\frac{\mathrm{P}}{\rho}[\delta k^{-\rho}+(1-\delta)L^{-\rho}] \quad (21)$$

Uzawa (1962) and Mc Fadden (1963) tried to extend the CES function to n-input factor production function given by,

$$\frac{\partial Q}{\partial L}=(1-\delta)A^{-\rho}\left(\frac{Q}{L}\right)^{\rho+1} \quad (22)$$

$$Q_L=(1-\delta)A^{-\rho}\left(\frac{Q}{L}\right)^{\rho+1} \quad (23)$$

Taking the logarithm on both sides of Eq. (23) we get,

$$\ln\left(\frac{Q_L}{Q}\right) = \ln(1-\delta) + A^{-\rho} + (\rho+1)\ln\left(\frac{Q}{L}\right) \quad (24)$$

Eq. (24) represents the relationship between agricultural GSDP, labour and capital invest.

3.2.1 Parameter Estimation of CES Production Function

The CES (Constant Elasticity of Substitution) production function is nonlinear with respect to its parameters, which prevents it from being easily transformed into a linear form for traditional linear estimation methods. To estimate the parameters of the CES production function, nonlinear fitting techniques are typically employed. This estimation process assumes that the input variables are either non-stochastic or, if they are stochastic, that they are independent of the disturbance term (Hoff, 2004).

Generally, there are two common approaches for estimating CES parameters: the linear Taylor series approximation and the nonlinear least squares method. The linear Taylor series method can be used with respect to the parameter ρ, providing a simplified approach for estimation.

3.2.2 Estimating the CES function using Kmenta Approximation:

The production function can be written in the form as (Kmenta, 1967)

$$\ln Q_i = \ln\gamma - \frac{1}{\rho}\ln\left[\delta K_i^{-\rho} + (1-\delta)L_i^{-\rho}\right] + u_i \quad (25)$$

The parameters of the CES production function can be estimated from Eq. (34) using nonlinear least squares techniques, which are supported by various computer programs. Alternatively, a simplified approach involves linearizing the CES function with respect to the parameter ρ, which makes use of ordinary least squares estimation. This linearization is achieved through a Taylor series expansion around ρ=0. By ignoring higher-order terms beyond the second order, the Taylor expansion approximates the CES function with a linear model, allowing for simpler estimation

$$\ln Q_i = \ln\gamma + \vartheta\delta\ln K_i + \vartheta(1-\delta)\ln L_i + u_i \quad (26)$$

Let us define the new variable as

$Y^* = \ln Q_i, X_{1^*}{=}\ln K_i\ X_{2^*} = \ln L_i$

$\beta_0 = \ln\gamma, \beta_1 = \vartheta\delta, \beta_2 = \vartheta(1-\delta)$

Then Eq (35) can be written as

$$Y^* = Y^* + \beta_1 X_1^* + \beta_2 X_2^* \quad (27)$$

The method of ordinary least squares helps to estimate the following parameters

$$Y = e^{\beta 0}, \delta = \frac{\beta 1}{\beta_1+\beta_2}, \vartheta = \beta + \beta \quad (28)$$

ρ is related to the elasticity of substitution (σ) between the two inputs. Specifically, $\sigma = \frac{1}{1-\rho}$. The value of ρ can range from $-\infty$ to 1 and, where $\rho = 1$ corresponds to a Cobb-Douglas production function (infinite substitution), and $\rho = -\infty$ indicates perfect complements.

3.3 Multiple Linear Regression:

Sir Francis Galton (1822-1911) was a pioneering figure who introduced the concept of regression toward the mean, laying foundational ideas for regression analysis, particularly in the context of heredity. Following him, Karl Pearson (1857-1936), a prominent statistician, developed the Pearson correlation coefficient, which became a crucial measure of linear relationships, providing essential principles for understanding regression analysis. Later, Ronald A. Fisher (1890-1962) advanced the field with the introduction of the analysis of variance (ANOVA) and maximum likelihood estimation, both critical tools for statistical inference and regression analysis. George E.P. Box (1919-2013) further contributed through the Box-Jenkins methodology in time series analysis, emphasizing the importance of regression models in experimental design and statistical modelling across various fields.

A Nobel laureate, Clive W.J. Granger (1934-2009), made significant contributions to econometric methods, including cointegration and error correction models, which enhance the applications of regression analysis in economic research. Meanwhile, David A. Freedman (1938-2008) critically examined the interpretation of regression results and highlighted the limitations of regression analysis in establishing causal relationships, influencing contemporary understanding of statistical inference. Herman Wold (1909-1992) was instrumental in the development of structural equation models, extending traditional regression analysis and facilitating a deeper understanding of complex variable relationships. William H. Greene has also played a key role in advancing regression techniques, particularly in panel data analysis and econometric modelling, enhancing empirical research methodologies. Finally, James W. Cooper is known for his application of regression analysis across various disciplines, including economics and environmental science, demonstrating the versatility and importance of regression methods in diverse fields.

3.3.1 Parameter Estimation of Multiple Linear Regression:

The model we have used in the study is

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 X_{i5} + \beta_6 X_{i6} + \varepsilon_i \quad (29)$$

With the assumptions,

(i) ε_i is normally distributed

(ii) $E(\varepsilon_i) = 0$

(iii) $E({\varepsilon_i}^2) = {\sigma_s}^2$

(iv) $E(\varepsilon_i \varepsilon_j) = 0$ for $\text{i} = 1993$ to 2023

In general, the model is

$$\text{Y} = \beta_0 + \beta_1 X + \epsilon \quad (30)$$

Where,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} x_{21} & x_{31} & \cdots & x_{k1} \\ x_{22} & x_{32} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & \vdots \\ x_{2n} & x_{3n} & \cdots & x_{kn} \end{bmatrix}$$

Using the method of least squares and the sample regression equation is

$$Y_i = b_0 + b_1 X_i + e_i \quad (31)$$

The sum of squared residuals is

$$\text{ESS} = \sum e_i^2 = \sum (Y_i - b_0 + b_1 X_i)^2 \quad (32)$$

From partial derivatives we get

$$\frac{\partial \text{RSS}}{\partial b_0} = -2\sum_i (Y_i - b_0 + b_1 X_i)(-1) = 0 \quad (33)$$

$$\frac{\partial \text{RSS}}{\partial b_1} = -2\sum_i (Y_i - b_0 + b_1 X_i)(-X_i) = 0 \quad (34)$$

Dividing each of these equations by -2 we get the normal equations as

$$\text{n}b_0 + b_1 \sum X_i = \sum Y_i \quad (35)$$

$$b_0 \sum X_i + b_1 \sum X_i^2 = \sum X_i Y_i \quad (36)$$

From these normal equations solving we get,

$$b_0 = Y - b_1 X$$

And

$$b_1 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} \quad (37)$$

4. Results:

Table 4.1.1: The Regression Results of the model:

Taking logarithmic form of the model, we have estimated the parameters Capital and Labour are

R square	0.972
Adjusted R square	0.961
F Statistic	2457

	Coefficients	Standard Error	t Stat	P-value
Intercept	-5.7761	1.1151	-5.1799	3.252E-06
ln K	0.45	1.167	2.809	0.018
Ln L	0.55	0.73	2.127	0.049

The model coefficient of determination R^2 (0.972) and adj R^2 (0.961) shows that the model has a very high fitting precision. Cobb-Douglas model parameters are obtained as follows:

Table 4.1.2: Cobb Douglas model parameters:

$\hat{A}$	$\hat{\propto}$	$\hat{\beta}$	$\propto^2+\hat{\beta}$
2	0.45	0.55	1

From the table above we can observe that $\propto^2+\hat{\beta}=1$ which indicates constant return to scale in terms of productivity of output.

The Cobb Douglas Production Function is:

$$Q_{Cobb(GSDP)} = 2K^{0.45}L^{0.55} \quad (38)$$

The elasticity of capital α is 0.45. This shows that 1% increase in capital lead to 0.45% increase in GSDP. Similarly, the elasticity of labour β is 0.55 which indicates that 1% increase in labour lead to 0.55% increase in GSDP.

Residual analysis of Cobb Douglas Production Function:

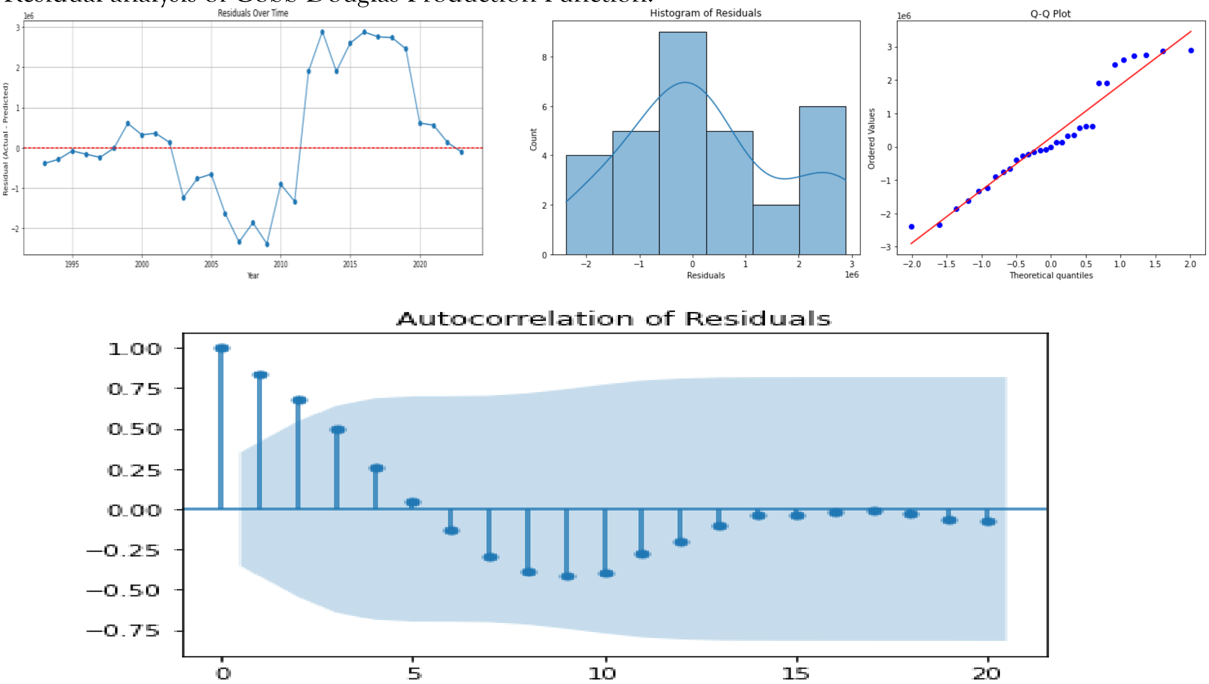

Fig 4.1: Residual analysis

From the figure above we can infer that the residuals are scattered randomly around the red zero line, without any systematic pattern. The randomness supports linearity and suggests that the functional form is appropriate. Normality assumption of residuals is mostly satisfied.

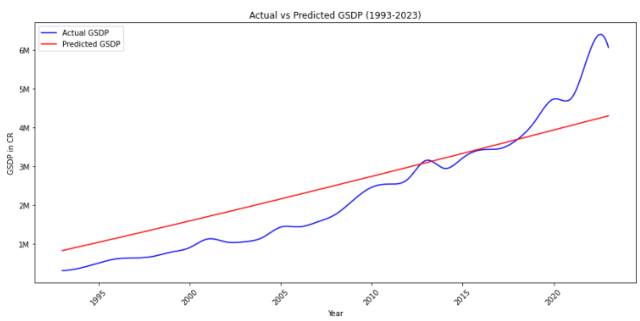

Fig 4.2: Actual and Predicted GSDP for Cobb Douglas Production Function

In the figure above the actual GSDP shows a nonlinear upward trend over time with clear cyclical or seasonal fluctuations. The curve appears to accelerate, suggesting that the economy is growing at an increasing rate, especially in the later periods. Predicted GSDP in the model appears linear and smooth, lacking the curvature and volatility of the actual series. It underestimates GSDP in the later years and overestimates in the earlier periods. This indicates that the Cobb-Douglas model, as implemented, captures the trend but fails to account for cyclical patterns or nonlinear growth acceleration.

Table 4.2.1: Parameter Estimation of CES Production Function:

δ	(1-δ),	θ	ρ	σ	γ	−θ/ρ
0.45	0.55	0.9	0.9	10	e^0	-1

The CES Production Function is:

$Q_{CES(GSDP)}$=0.93(0. 45$K^{0.9}$ + 0.55$L^{0.9}$) (39)

Capital contributes 45% and labour 55% to the CES production process. Labour is slightly more significant than capital in determining output. A positive, small value of ρ indicating limited substitutability between capital and labor. The closer $\rho \rightarrow 0$, the more the CES approaches the CobbDouglas case. Elasticity of substitution $\sigma = 10$ greater than 1 implies that capital and labour are more substitutable than in the Cobb-Douglas case ($\sigma = 1$). $\gamma = e^0 = 1$ scales the CES function, reflecting minor efficiency loss (as A = 0.93). In the Kmenta approximation, $\frac{-\theta}{\rho} - 1$ this ratio influences how the log − linearized CES form bends. A value of -1 is standard and expected when $\theta = \rho$ Labour slightly outweighs capital in production importance. Substitutability is slightly elastic, but close to Cobb-Douglas. Efficiency factor (∼ 0.93) implies minor productivity losses. The function is well-suited for modeling GSDP when labour and capital have different, but complementary roles.

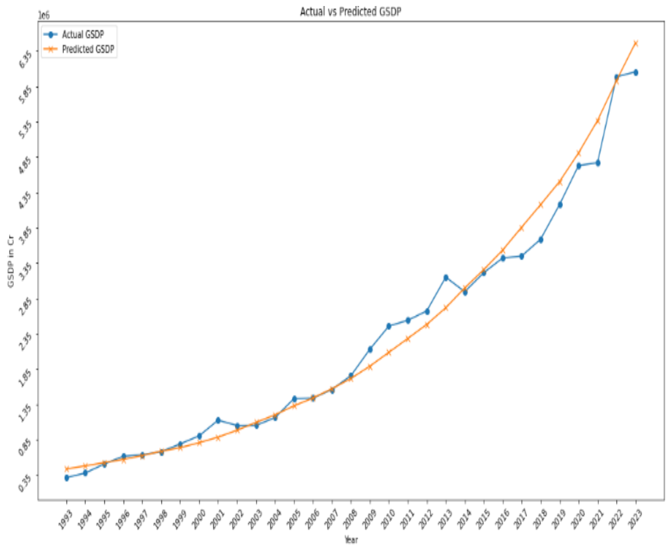

Fig 4.3: Actual GSDP and Estimated GSDP obtained from CES Production Function:

The resulting graph above illustrates both the actual and estimated GSDP values, following the Kmenta approach. The CES model captures the relationship between output (GSDP) and input factors capital and labour. The predicted GSDP values (orange line) closely follow the trend of the actual GSDP values (blue line) across the time period, indicating that the CES model provides a good fit for the underlying production process. While minor deviations are present in certain years, the model tracks both the long- term growth trend and short-term fluctuations in GSDP reasonably well. The divergence towards the end of the series may reflect nonlinearities, external shocks, or time-varying factors not captured by the static CES framework. However, the overall structural relationship appears robust. This alignment confirms that the CES production function, with its constant elasticity parameter (σ), is effective in modelling the substitution possibilities between labour and capital.

Table 4.3.1: Parameter Estimation of Multiple Linear Regression:

R square	0.982
Adjusted R square	0.985
F Statistic	247.4

Parameters	Coeff.	Std. Err.	t	P>|*t*|
Constant	-4.0899	1.362	-2.000	0.008
area	2.1519	0.636	3.382	0.004
farm	2.5263	1.402	0.802	3.090
labour	4.3974	0.958	4.589	0.000
capital	-0.0133	0.022	-0.609	0.551

The equation of the model is

$$GSDP=-4.089+2.151\times area+0.6601\times labour \quad (40)$$

From the Multiple Linear Regression model, we can see that labour and area are significant at 0.05 level of significance. For every one unit increase in area, holding labour constant, the GSDP is expected to increase by 2.151 units. Similarly, for every one unit increase in labour, holding area constant, the GSDP is expected to increase by 0.6601 units.

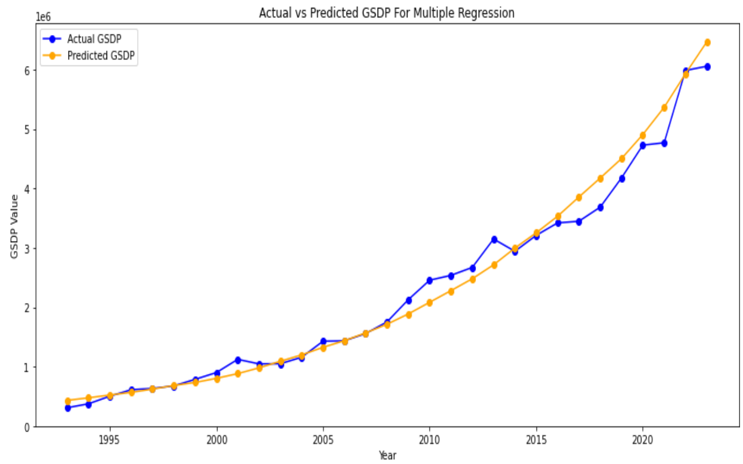

Fig 4.4: Actual GSDP and Estimated GSDP from Multiple Regression Model:

The estimated GSDP values closely track the actual GSDP values across most years, indicating that the model fits the data well. Minor deviations are observed, which may be due to external shocks, structural changes, or variables not included in the model. For the period 1993 to 2000, the model predicts modest and stable growth, consistent with actual GSDP, reflecting the gradual liberalization period. Mid-2000's to 2015 both actual and estimated GSDP show accelerated growth, possibly due to capital expansion and improved labour productivity. Slight divergence in certain years after 2015 suggests either policy interventions, exogenous shocks (e.g., demonetization, pandemic), or increasing non-linearities not captured by Multiple Linear Regression.

Evaluation measures of the models:

The linear part of the model estimated by Cobb Douglas Production function, Multiple Linear Regression and for non-linear estimation we have used CES Production Function (Zhang 2003). Plot obtained in Fig 6.3.5. shows the fit statistics of Cobb Douglas Production function, CES Production Function, Multiple Linear Regression and Log Linear Regression Model.

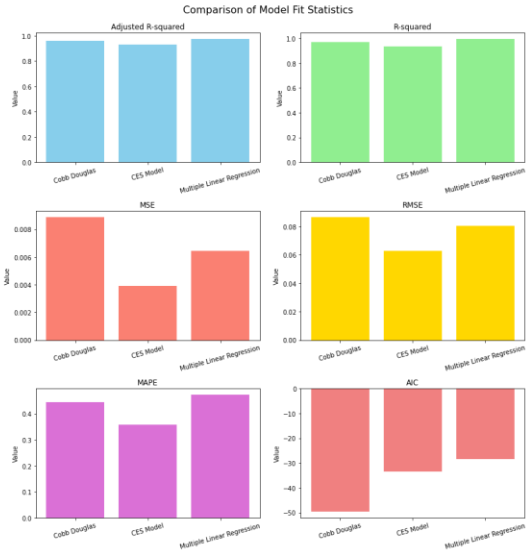

Fig 4.5: Predictive performance of the models

CONCLUSION:

Based on the comparative evaluation of three functional forms Cobb-Douglas, Constant Elasticity of Substitution (CES), and Multiple Linear Regression (MLR) for estimating Agricultural GSDP in Assam, the Multiple Linear Regression model emerges as the most accurate and robust estimator. It consistently outperforms the other models across all key statistical indicators, demonstrating the lowest values for Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). These metrics underscore its superior goodness- of-fit and predictive accuracy. For future research and policy development, these models can be further enhanced by incorporating dynamic elements such as time trends, climatic variability, and technological progress. Additionally, applying these models to panel data across districts would allow for the exploration of spatial heterogeneity in production responses. This, in turn, can lead to more nuanced and targeted agricultural development strategies tailored to the specific needs and conditions of different regions within the state.

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