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Comparing Randomised and Systematic Designs for Optimal Input Mapping in Business, Marketing, and IT Education Field Trials

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Abstract
Randomisation is a fundamental principle for en- suring unbiased treatment effects in experimental designs across various domains, including business, marketing, and information technology (IT) education. However, the choice between ran- domised and systematic designs must align with the experiment's goals, particularly in large-scale field trials. This study investi- gates the suitability of these designs when mapping optimal input levels across a structured grid for educational interventions. A simulation study employing Bayesian hierarchical models and geographically weighted regression (GWR) revealed that, for extensive trials, randomised and systematic designs produce comparable results when fitting linear models or ignoring spatial variation. Conversely, for quadratic models, especially when spatial variation is significant, systematic designs outperform randomised designs in terms of achieving lower true mean squared errors (MSE) for coefficient estimation. These findings suggest systematic designs may offer enhanced robustness and reliability for designing and analyzing large-scale interventions in business, marketing, and IT education contexts, where precise spatial mapping and optimization are critical.

1. INTRODUCTION

The concept of incorporating randomisation within experi- mental designs has long been recognised as a critical tool for enhancing the validity and reliability of statistical inference in agricultural research. Fisher (1934) was among the first to systematically articulate the benefits of randomisation, demon- strating how its application, within structured frameworks such as block designs or Latin squares, can control for extrane- ous sources of variability and ensure unbiased estimation of treatment effects (Verdooren 2020). Over time, agricultural researchers have adopted a variety of experimental designs that leverage these principles, including completely randomised designs (CRD), randomised complete block designs (RCBD), split-plot arrangements, and Latin square configurations (Pe- tersen 1994). These methodologies have proven indispensable for controlled field experiments aimed at detecting treatment differences with minimal confounding.

In recent years, however, the context of agricultural experi- mentation has evolved significantly, particularly with the emer- gence of on-farm experimentation (OFE). Unlike traditional small-plot trials conducted under highly controlled research conditions, OFE empowers farmers to conduct large-scale field trials within the operational realities of their own farms (Evans et al. 2020). This participatory approach is designed to help farmers evaluate new management practices, optimise inputs, and reduce uncertainties by generating evidence specific to their unique environmental, economic, and operational condi- tions (Cook et al. 2013).

When OFE trials are designed to compare categorical man- agement strategies or to identify superior-performing vari- eties, conventional randomised designs remain highly effec- tive (Pringle et al. 2004; Selle et al. 2019). Randomisation mitigates spatial biases and facilitates robust statistical tests, thereby ensuring reliable conclusions. However, in the context of precision agriculture (PA), where the focus often shifts from comparing discrete treatments to generating continuous spatial maps of optimal input levels (e.g., nitrogen application rates), traditional randomised approaches may not be ideal (Pringle et al. 2004).

A key limitation arises from the operational constraints of variable-rate applicators (VRA), which require pre- defined prescription maps prior to application (Piepho et al. 2011). When treatment levels are assigned randomly across a field, the spatial distances between different treatment plots vary ir- regularly, complicating interpolation of treatment-response re- lationships and introducing spatial uncertainty. Consequently, systematic designs—where treatments follow a structured, pre- dictable spatial arrangement—offer a practical alternative by facilitating smoother interpolation and more reliable construc- tion of treatment response maps. Despite these advantages, systematic designs have received relatively limited attention in the OFE literature, with randomisation often regarded as a default requirement.

The statistical analysis of systematic designs for OFE presents unique challenges. The true optimal treatment level at any given location is inherently unknown, and spatial heterogeneity in soil properties, microclimate, and other en- vironmental factors can cause treatment effects to vary con- tinuously across the landscape. To address this complexity, Cao et al. (2022) proposed a Bayesian spatial framework incorporating random parameters that vary locally to capture spatial trends in treatment response. While this approach offers flexibility and theoretical rigor, its computational demands and the prerequisite of advanced statistical expertise may limit its accessibility to farmers and practitioners.

Alternatively, geographically weighted regression (GWR) has emerged as a more practical technique for capturing spatially varying treatment effects in OFE (Rakshit et al. 2020). By applying localised regression models within moving spatial windows, GWR can account for continuous spatial variation without the computational overhead of full Bayesian approaches. Simulation studies by Evans et al. (2020) have shown that GWR effectively distinguishes yield variability arising from applied treatments versus background environ- mental factors. Nonetheless, many of these studies were lim- ited by their reliance on randomised designs and simplistic, often linear, models of treatment response.

Other research efforts have explored the use of systematic designs, such as chessboard arrangements, to enhance the spatial resolution of treatment-response mapping (Alesso et al. 2021). Findings suggest that systematic designs often outper- form randomised designs in terms of spatial prediction ac- curacy, particularly when treatments vary continuously across fields. However, practical considerations such as machinery- induced smoothing of treatment boundaries and the oversim- plification of response functions (e.g., assuming linearity) can introduce biases if not properly addressed (Pringle et al. 2004). Understanding the true nature of nutrient-response relation- ships is also essential for improving OFE methodologies. Nu- merous studies have highlighted that yield responses to inputs like fertilisers often follow non-linear patterns influenced by complex soil-plant interactions, nutrient interactions, and site- specific factors (Marschner 2011; Glynn 2007). Linear models, while convenient, may oversimplify these dynamics and fail to capture critical thresholds, plateaus, or diminishing returns (Piepho and Edmondson 2018; Liben et al. 2019). Quadratic or higher-order polynomial models provide a more realistic framework for representing these non-linearities but require careful application to avoid overfitting, particularly in spatially complex OFE settings.

In this study, we investigate whether randomisation is es- sential for large-scale strip trials aimed at generating treatment response maps for OFE. Using comprehensive simulation scenarios, we compare systematic and randomised experi- mental designs across linear and quadratic treatment-response functions. We further evaluate the ability of GWR to re- cover spatially varying treatment effects under different design configurations, spatial correlation structures, and degrees of treatment-response complexity. Our simulations reveal that, contrary to conventional wisdom, systematic designs often yield superior spatial prediction performance, particularly for non-linear responses. Moreover, we show that the commonly not always produce optimal results. Instead, selecting a fixed bandwidth informed by the experimental design's spatial struc- ture enhances the accuracy of treatment effect estimation.

2. METHODS

This segment outlines the full and comprehensive systematic framework applied in the simulation-based inquiry of spatially notified agricultural trials. It comprises of three primary com- ponents. First, Section ?? introduces the basic and fundamental statistical model used to represent yield results across plots. Section ?? then details the incorporation of spatial reliance into treatment effect modeling through ordered and methodical covariance matrices. Lastly, Section ?? (described elsewhere) elaborates on the implementation of Geographically Weighted Regression (GWR), a local modeling technique applied to approximate spatially varying coefficients.

A. Fundamental Statistical Framework

In precision farming and field-based agronomic exper- iments, trial units are generally organized into a two-dimensional array, commonly forming a grid with r rows and c columns. This layout produces a total of n = r *c unique empirical units or plots. Each unit is identified by a coordinate s_i R^2, which signifies its spatial position at the center of the plot.

Let y(si) represent the observed outcome (for instance, grain yield, biomass, or another trait of interest) at the ith location. The observations are modeled applying a mixed- effects linear formulation:

$$\mathbf{Y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}, \qquad (1)$$

where **Y** is an n *1 vector of responses, **X** and **Z** are design matrices for the fixed and random effects, respectively. Vectors **b** and **u** contain the corresponding coefficients, and e denotes residual noise capturing unexplained variation.

The model assumes that random components **u** and **e** follow a multivariate normal distribution and are mutually uncorrelated:

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{e} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbf{\Sigma}_u & 0 \\ 0 & \mathbf{\Sigma}_e \end{pmatrix} \right), \qquad (2)$$

where **Σu** and **Σe** denote the covariance matrices for random effects and error conditions, respectively. This model serves as the statistical foundation upon which spatial dependencies and treatment heterogeneity are later layered.

A. Modeling Spatial Correlation in Treatment Effects

Accurate analysis of field trial data in agricultural research necessitates the incorporation of spatial structures, as envi- ronmental and management conditions typically vary across locations. To accommodate this, our model directly integrates spatial dependence within the treatment effect parameters using a hierarchical framework that allows for both fixed and location-specific influences.

The observed response at each spatial site si is modeled conditionally using the formulation:

$$\begin{aligned} y(s_i) \mid \mathbf{u}_i, \vartheta_u, \sigma_e &\sim f\left(\sum_{m=1}^{l} b_m x_m(s_i) + \sum_{j=1} u_j(s_i) z_j(s_i), \sigma_e^2 \right), \\ \mathbf{u}_i \mid \vartheta_u &\sim \mathrm{N}\left(\mathbf{0}, \mathbf{V}_u(\vartheta_u)\right), \\ e(s_i) &\sim \mathrm{N}\left(0, \sigma_e^2\right), \end{aligned} \qquad (3)$$

where $x_m(s_i)$ and $z_j(s_i)$ denote known covariates evaluated at site si associated with deterministic and random effects, respectively. The corresponding coefficients are b_m for fixed effects and $u_j(s_i)$ for spatially varying components. The random vector ui follows a multivariate normal distribution with mean zero and covariance matrix Vu(θ_u), which is governed by the hyperparameter vector θ_u. To capture correlations between treatment-specific random components, we factor the covariance matrix **Vu** as:

$$\mathbf{V}_u = \mathbf{B}(\sigma_u)\mathbf{R}_u\mathbf{B}(\sigma_u), \qquad (4)$$

where $\mathbf{B}(\sigma_u)$ is a diagonal matrix with standard deviations for individual treatment effects, and $\mathbf{R}_u$ denotes the inter- treatment correlation matrix. This correlation structure is sam- pled from the LKJ distribution (Lewandowski et al. 2009), which offers a flexible prior for modeling varying degrees of correlation. The shape parameter ϵ within the LKJ distribution controls the spread around the identity matrix; higher values encourage near-independence among effects, whereas smaller values allow stronger associations.

To introduce dependence across spatial locations, the full covariance for all random effects across the grid is expressed using the Kronecker product:

$$\mathbf{\Sigma}_u = \mathbf{V}_s \otimes \mathbf{V}_u, \qquad (5)$$

where **Vs** is the spatial covariance matrix encoding location- to-location dependence. The Kronecker product enables si- multaneous modeling of variation across treatment levels and spatial positions, resulting in a cohesive representation that accounts for both types of heterogeneity.

a) Separable AR1 Spatial Structure.: A practical and ef- ficient method for modeling spatial autocorrelation is through the use of a separable first-order autoregressive (AR1) struc- ture. This approach assumes that spatial dependencies diminish geometrically as the physical distance increases along the row and column directions of the grid (Butler et al. 2017). The spatial covariance matrix in this context is constructed as:

$$\mathbf{V}_s = \mathbf{AR1}(\rho_c) \otimes \mathbf{AR1}(\rho_r), \qquad (6)$$

where ρc represents the decay parameter for correlation in the column-wise (horizontal) dimension, and ρr governs the decay in the row-wise (vertical) dimension. Each **AR1**( ) matrix captures the auto-dependence in its respective axis.

The separability assumption simplifies computation consid- erably, especially when dealing with large spatial grids, as it allows the overall matrix operations to be decomposed into lower-dimensional components. Furthermore, this structure fits naturally with the typical plot arrangements used in field ex- periments, making it a computationally viable and statistically appropriate model for representing spatial interactions across agricultural plots.

b) Mate´rn Covariance Structure.: To capture more so- phisticated and multifaceted or less regular spatial depen- dencies, the Mate´rn covariance model provides a extremely flexible alternative (Cressie and Huang 1999; Selle et al. 2019). The spatial covariance between two sites separated by a distance d is specified as

$$V_s(d) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}\, d}{r}\right)^{\nu} K_\nu\left(\frac{\sqrt{2\nu}\, d}{r}\right), \qquad (7)$$

where $\bar{\sigma}^2$ is the variance component, r is the scale (range) parameter dictating how quickly correlation decays, and ν is the smoothness coefficient. The function Kν( ) is the modified Bessel function of the second kind, and Γ( ) indicates the gamma function. The option of ν determines the differentia- bility of the spatial process, with smaller values permitting more irregular spatial surfaces.

The Mate´rn model is especially beneficial in agricultural settings where spatial variability may arise from complicated environmental gradients, soil shifts, or management zones. Its adaptability makes it appropriate for both seamless and effortless and swiftly varying spatial fields.

By using either the AR1 or Mate´rn structures for **V**s, this modeling approach permits for a realistic representation of field-level heterogeneity, thereby enhancing inference on spatial treatment effects.

c) Interpretation and Implications.: Incorporating spatial correlation through structured covariance matrices allows each plot's response to benefit from surrounding observations. This "borrowing of strength" ensures more stable and accurate estimation of treatment effects, even when treatments are not repeated within a local neighborhood (Panten et al. 2010; Piepho et al. 2011). These spatial smoothing mechanisms are vital in agricultural research, where spatial variation is inevitable due to natural and management-induced field het- erogeneity.

C. Geographically Weighted Regression (GWR)

Geographically Weighted Regression (GWR) is a powerful spatial analysis method designed to model spatially vary- ing relationships between response and explanatory variables (Rakshit et al. 2020). Unlike traditional global regression models that assume constant coefficients across space, GWR allows model parameters to vary by location, thereby capturing local patterns and spatial heterogeneity in the data.

Formally, the GWR model for a response variable y(si) observed at spatial location si is defined as:

$$y(s_i) = \beta_0(s_i) + \sum_{j=1}^{k} \beta_j(s_i) z_j(s_i) + \varepsilon_i, \qquad (8)$$

where $z_j(si)$ are the explanatory variables at location si, βj(si) are the corresponding location-specific coefficients, and the residual error term εi is assumed to follow a normal distribution N (0, τ^2).
Parameter estimation in GWR is typically performed using locally weighted least squares:

$$\hat{\boldsymbol{\beta}}(s) = \mathbf{Z}^\top\mathbf{W}(s)\mathbf{Z} \quad \mathbf{Z}^\top\mathbf{W}(s)\mathbf{Y}, \qquad (9)$$

where **W**(s) is a spatial weighting matrix centered at location s, constructed using a kernel function that down-weights observations farther from s. The design matrix **Z** contains the explanatory variables for all observations, and **Y** is the vector of response values.
Several kernel functions are commonly used to define spatial weights, including Gaussian, exponential, bi-square, and tri- cube forms (Gollini et al. 2015). In this study, we adopt the Gaussian kernel for its smooth decay properties. Among all model tuning parameters, the kernel bandwidth—defining the spatial scale of influence—is generally the most critical. A smaller bandwidth increases local sensitivity, while a larger bandwidth favors smoother spatial estimates.
Bandwidth selection is typically guided by model fit criteria, with the corrected Akaike Information Criterion (AICc) being a popular choice:

$$\text{AICc} = 2n\log(\tau^2) + n\log(2\pi) + n\frac{n + \text{tr}(\mathbf{S})}{n - 2 - \text{tr}(\mathbf{S})}, \quad (10)$$

where **S** is the smoothing matrix, with its ith row computed as

$$\mathbf{S}_i = \mathbf{Z}_i \ \mathbf{Z}^\top\mathbf{W}(s_i)\mathbf{Z} \quad \mathbf{Z}^\top\mathbf{W}(s_i). \qquad (11)$$

In practical field experiments, bandwidth may also be de- termined using prior knowledge of experimental layout. For example, the window size can be fixed to ensure represen- tation of all treatment levels within the local neighborhood, which enhances model interpretability and numerical stability (Rakshit et al. 2020).
All GWR analyses presented in this study were conducted using the GWmodel package in R (Lu et al. 2014; Gollini et al. 2015), which offers comprehensive tools for spatial regression modeling in agricultural and environmental applications.

3. SIMULATION STUDY

To rigorously assess the performance of Geographically Weighted Regression (GWR) under different experimental designs, we conducted a comprehensive simulation study. This study aims to compare the effectiveness of randomized and systematic (strip-based) designs for estimating spatially varying treatment effects. By using simulated data, we gain full control over the data-generating process, ensuring that estimation performance can be evaluated without interference from unknown or unmeasured confounders (Piepho et al. 2013).

A. Experimental Scenarios

The simulation study considers multiple factorial scenarios that vary key design and model parameters:
- **Design Type:** Randomized versus systematic (strip-wise)
- **Response Function:** Linear and quadratic forms of yield
- **Coefficient Correlation:** Low versus high correlation among spatially varying treatment coefficients.
- **Spatial Covariance Structure:** Identity (no spatial structure), separable autoregressive (AR1) structure, and Mate´rn covariance.
- **Bandwidth Configuration:** Fixed bandwidths of 5 and 9, and data-driven bandwidth selected by AICc minimiza- tion.

In the systematic layout, a fixed bandwidth of 5 ensures that all five nitrogen treatments (0, 35, 75, 105, 140 kg/ha) are represented in each local regression window. For randomized layouts, a wider window of size 9 is used to maintain treatment diversity due to random clustering effects.

B. Simulation Setup and Spatial Configuration

The synthetic field was designed as a grid with 20 columns and 93 rows, yielding a total of 1860 plots. Nitrogen treatments were assigned to plots either randomly or in a strip-wise (systematic) fashion, allowing for direct comparison across design strategies. Simulated yields were generated using a predefined spatial model.

C. Yield Models and Coefficient Structures

Two distinct forms of treatment-response relationships were considered:

a) Linear Response.: A linear yield model was specified using coefficients b0 = 65 (intercept) and b1 = 0.05 (slope), consistent with empirical studies such as the Las Rosas maize trial (Rakshit et al. 2020; Cao et al. 2022). Spatial variation in the coefficients was introduced with standard deviations σu0 = 5 and σu1 = 0.01, and the residual error was modeled as εi (0, 1).

Spatial dependency was incorporated using either the AR1 model (Equation (??)) with correlation parameters ρc = 0.15 and ρr = 0.5, or the Mate ́rn model (Equation (??)) with σ2 = 1, r = 1, and ν = 1.5. The location-specific coefficients were constructed as

$$\beta_0(s_i) = b_0 + u_0(s_i), \quad \beta_1(s_i) = b_1 + u_1(s_i), \qquad (12)$$

where $u_0(s_i)$ and $u_1(s_i)$ are spatially correlated random fields.

b) Quadratic Response.: To model non-linear yield re- sponses, a quadratic function was used with coefficients b0 = 65, b1 = 0.05, and b2 = −0.0003. Spatial variation in the quadratic term was modeled using σu2 = 0.0001. The coefficients were generated as:

$$\begin{aligned} \beta_0(s_i) &= b_0 + u_0(s_i), \\ \beta_1(s_i) &= b_1 + u_1(s_i), \\ \beta_2(s_i) &= b_2 + u_2(s_i). \end{aligned} \qquad (13)$$

D. Yield Computation

The yield for each plot was computed as:

$$y(s_i) = \begin{cases} \beta_0(s_i) + \beta_1(s_i)N_i + \varepsilon_i, & \text{for linear model} \\ \beta_0(s_i) + \beta_1(s_i)N_i + \beta_2(s_i)N_i^2 + \varepsilon_i, & \text{for quadratic model,} \end{cases} \qquad (14)$$

where Ni is the nitrogen level at location si. Examples of the resulting yield curves under each scenario are presented in Figure 3.

E. Evaluation Criteria

The key objectives of the simulation were:

• To assess how effectively GWR recovers true, spatially varying treatment effects under varying spatial dependen- cies.

• To evaluate the sensitivity of GWR to bandwidth selection under different experimental designs.

• To compare the statistical efficiency of systematic versus randomized trial designs for precision agricultural infer- ence.

By maintaining constant spatial parameters across simula- tion runs, we ensure that observed differences in model perfor- mance are attributable solely to design strategy and modeling choices, thereby enabling a rigorous and fair assessment of best practices for spatially informed experimental design in on-farm trials.

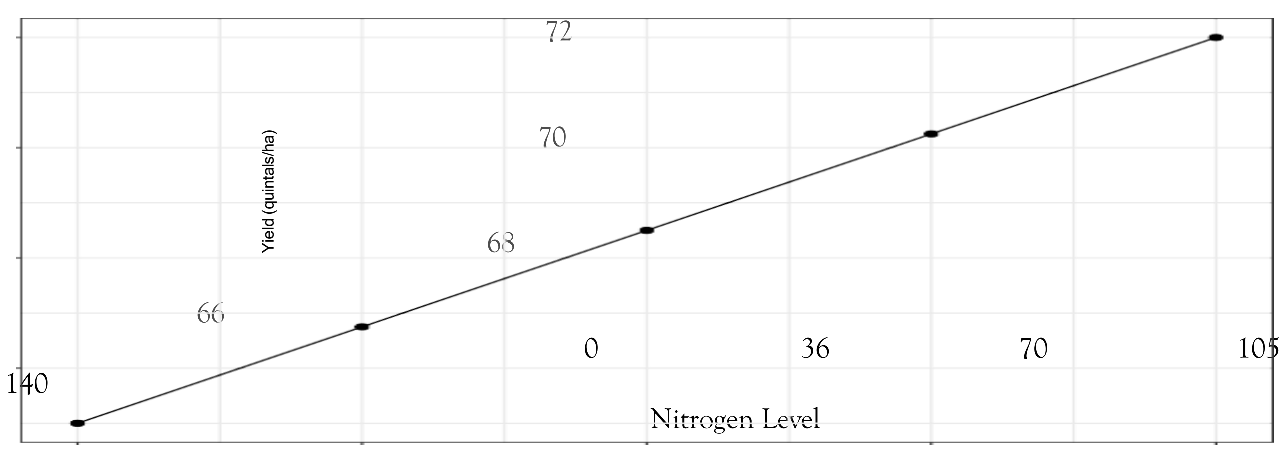

Fig. 1. Linear relationship between crop yield and nitrogen levels,
Where y = 65 + 0.05x

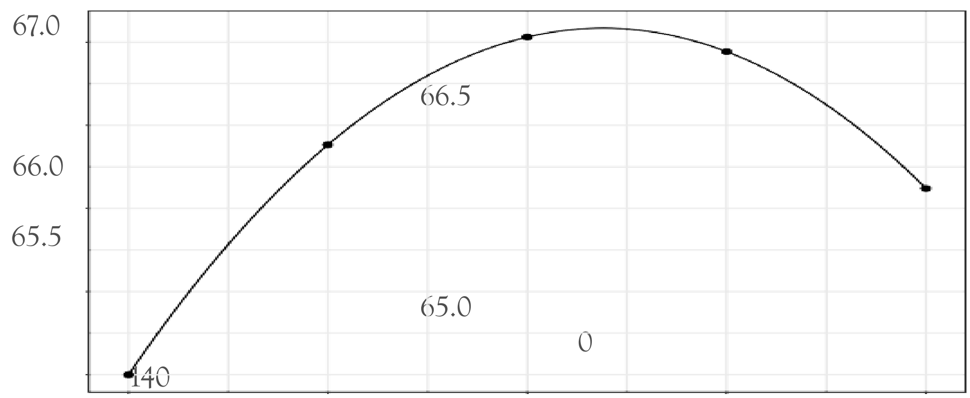

Fig. 2. Quadratic relationship between crop yield and nitrogen levels,
Where y = 65 + 0.05x — 0.003x2.

Fig. 3. Noise-free linear and quadratic relationships between crop yield and nitrogen levels.Sample preparation

4. RESULTS

The simulation study was repeated 100 times to methodi- cally evaluate the performance of randomised and structured experimental designs under both linear and quadratic response conditions. This segment presents the comparative analysis of the estimation accuracy for spatially varying coefficients ob- tained through GWR. The evaluation is organized as follows: Subsection IV-A presents the Mean Squared Error (MSE) outcomes, highlighting how estimation accuracy is affected by design option, bandwidth option, spatial correlation, and parameter structure. Subsection IV-B reports results from an ANOVA assessing the significance of the experimental aspects. Lastly, Subsection ?? inspects the effectiveness and behaviour of bandwidth choice via AICc

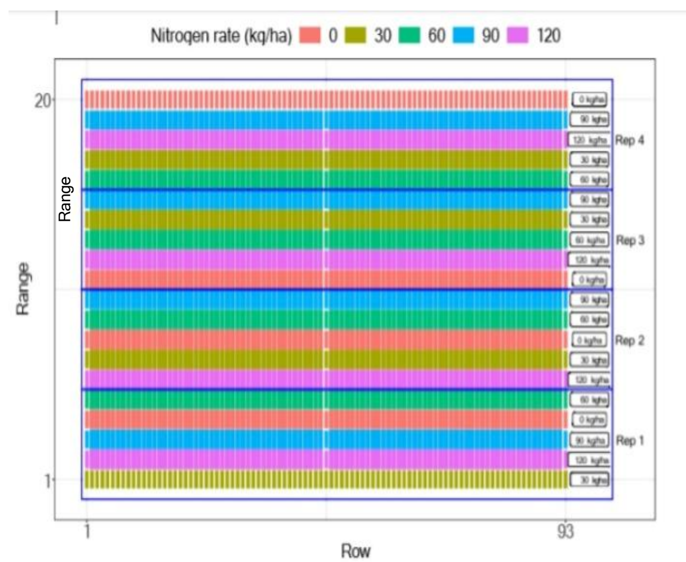

Fig. 5. Treatments are methodically designated into huge and massive strips in each replicate block

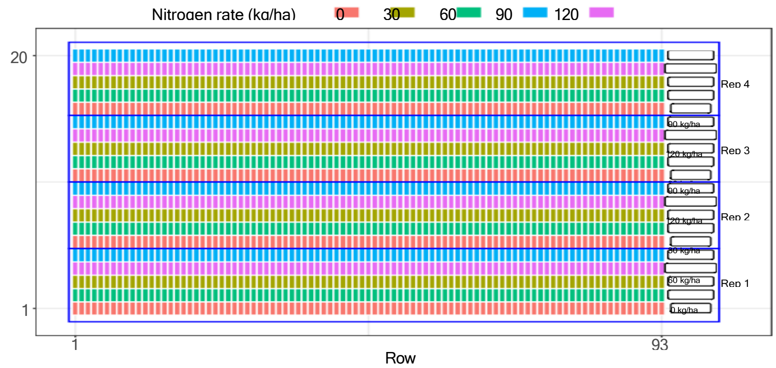

Fig. 6. The nitrogen treatments with five levels (0, 35, 70, 105 and 140 kg/ha) randomly (??) and methodically (??) allocated into strips in each replicate block

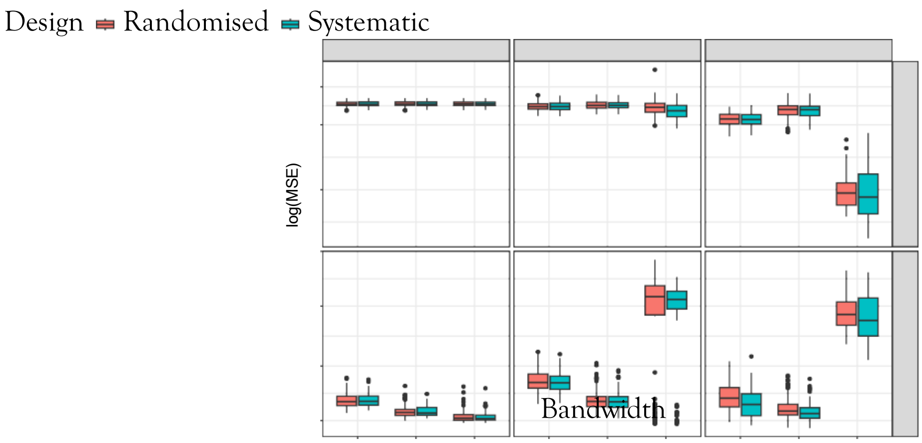

Fig. 7. Log-MSE boxplots for GWR estimates under linear response with varying bandwidths and spatial covariance structures.

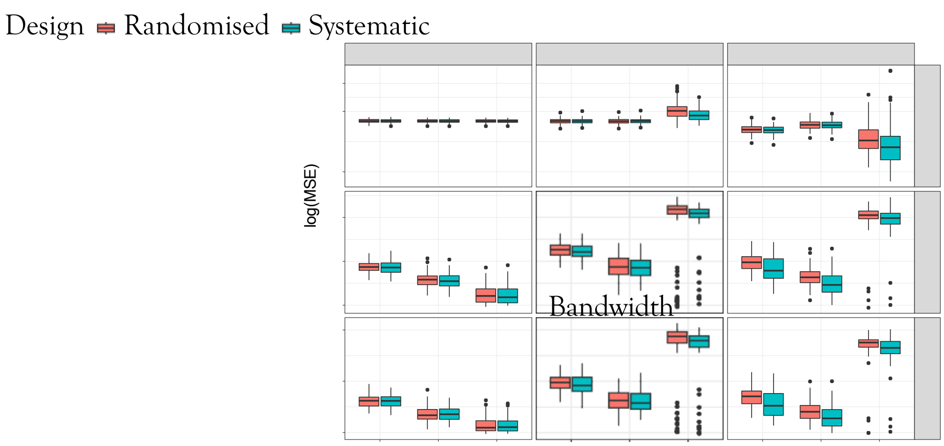

Fig. 8. Log-MSE boxplots for GWR estimates under quadratic response with varying bandwidths and spatial covariance structures.

A. Mean Squared Error Analysis

To quantify estimation accuracy, we computed the true Mean Squared Error (MSE) of spatially varying coefficients across all grid locations in the area. The MSE was calculated by comparing the true coefficients, $\beta =$

b+u, to the calculated coefficients, $\hat{\beta} = \hat{b} + \hat{u}$, squaring the difference for each plot, and averaging across the whole and complete area. Results are visualised in Figures ?? and ??, where the term ' ' NS '' signifies situations without spatial variation ($V_s = I_{n \times n}$), ' ' AR1 '' refers to the separable autoregressive structure with $\rho_c = 0.15$and $\rho_r = 0.5$, and ' ' Matern '' matches to the Mate´rn covariance structure with $\nu = 1.5$. Due to the generally small magnitude of the coefficient MSEs, natural logarithmic transformations were applied to improve graphical interpretation.

For situations supposing a linear response, GWR performed similarly under both randomised and structured designs. As displayed in Figure ??, the estimation accuracy of the in- tercept (β_0) and slope (β_1) exhibited no notable difference between the two designs, regardless of bandwidth option or the presence of spatial covariance structures. This discovery remained uniform and unchanging under both low ($\epsilon = 1$) and high ($\epsilon = 0.1$) parameter correlation situations (see Figure ??). Across all conditions, bandwidth choice established on AICc produced the smallest MSEs, followed by corrected bandwidths of 9 and 5, respectively.

In contrast, under a quadratic response model, distinct performance differences appeared between the two designs. As depicted in Figures ?? and ??, methodical designs demonstrated superior performance when estimating $\beta_1$and $\beta_2$compared to randomised designs, especially when spatial correlation structures were present. Notably, while AICc- selected bandwidths produced the minimum MSEs for the intercept β_0, they did not consistently yield the most exact and true estimates for the slope and quadratic conditions. In these instances, corrected bandwidths; specifically bandwidth 9; delivered better performance for estimating spatially varying treatment effects.

These outcomes highlight the interaction between band- width, spatial covariance structure, and response model in- tricacy. When spatial variation was absent, AICc bandwidths consistently supplied the most exact and true estimates across all coefficients. However, in the presence of spatial reliance, especially with AR1 AR1 or Mate´rn structures, corrected bandwidths; especially bandwidth 9; were more effective for precisely estimating spatially varying slopes and quadratic conditions. For the intercept, AICc-based bandwidth option remained ideal in spatially associated settings.

Overall, these results demonstrate that structured and orderly designs linked with carefully selected bandwidths im- prove estimation accuracy for non-linear treatment responses, especially when spatial reliance is present.

B. Analysis of Variance (ANOVA)

A complete and thorough ANOVA was conducted to further evaluate the influence of experimental design, bandwidth, spatial covariance, coefficient type, and parameter correlation on GWR estimation accuracy. The analyses were performed individually for the linear and quadratic response situations, considering both chief effects and second-order interactions among the five primary aspects:

- Design type (randomised or structured)
- Bandwidth (corrected 5, corrected 9, or AICc-selected)
- Spatial covariance structure (V_s)
- Coefficient identity (β_0, β_1, or β_2)
- Parameter correlation intensity (ϵ)

The ANOVA outcomes are summarised in Table ??. For the linear response, no notable difference was noted between randomised and methodical and organized designs, corrobo- rating the MSE results. In contrast, for the quadratic response, design type and its interactions with bandwidth and coefficient conditions were statistically notable ($p < 0.001$), confirming the benefit of structured and orderly designs in correctly estimating non-linear treatment effects.

Interestingly, the intensity of parameter correlation (ϵ) and its interactions did not considerably affect estimation accuracy in either response situation, indicating that GWR performance is relatively insensitive to the magnitude of correlation among spatially varying parameters.

Bandwidth option and its interactions were consistently no- table across both response models, stressing the essential role of bandwidth in local regression performance. Furthermore, spatial covariance structure and its interaction with coefficient conditions considerably affected estimation accuracy, espe- cially in the quadratic response situation.

C. Bandwidth Selection Using AICc

An additional aspect of the study investigated the behaviour of bandwidth selection via the AICc criterion. The simulations revealed a consistent tendency for the AICc-selected band- width to converge towards minimal values (close to 1) when

TABLE I

SUMMARY OF ANOVA RESULTS FOR LINEAR AND QUADRATIC RESPONSES. SIGNIFICANT EFFECTS (p < 0.05) ARE HIGHLIGHTED IN BOLD.

Factor/Interaction	Linear		Quadratic	
	Df	p-value	Df	p-value
Design	1	0.094	1	**<0.001**
Bandwidth	2	**<0.001**	2	**<0.001**
Covariance (V_S)	2	**<0.001**	2	**<0.001**
Coefficients (β)	1	**<0.001**	2	**<0.001**
Correlation (ϵ)	1	0.171	1	0.898
Design × Bandwidth	2	0.216	2	**<0.001**
Design × Covariance	2	0.522	2	0.013
Design × Coefficients	1	0.094	2	**<0.001**
Bandwidth × Covariance	4	**<0.001**	4	**<0.001**
Bandwidth × Coefficients	2	**<0.001**	4	**<0.001**
Covariance × Coefficients	2	**<0.001**	4	**<0.001**

spatial covariance was incorporated into the model, irrespec- tive of design type or response complexity. This suggests that under spatially correlated conditions, GWR preferentially re- lies on highly localised information for coefficient estimation. Conversely, in scenarios where spatial covariance was ex- cluded (i.e., Vs = In×n), the selected bandwidth expanded significantly, often encompassing nearly the entire row of the field (Figure 11). This reflects the absence of spatial structure, prompting GWR to utilise broader spatial windows to improve estimation stability.

These findings underscore the adaptive nature of AICc bandwidth selection and its sensitivity to underlying spatial structures, highlighting its utility for optimising GWR perfor- mance in spatially heterogeneous agricultural trials.

5. DISCUSSION

Randomised designs have traditionally been favoured in agronomic and biometric research for on-farm experimentation (OFE), due to their ability to mitigate biases and ensure statistical validity. However, the findings of this simulation study indicate that systematic designs can offer comparable or superior performance to randomised designs when the goal is to generate spatially varying treatment effect maps using GWR. The relative performance of these designs was found to be influenced primarily by the nature of the treatment response and the spatial covariance structure, while the corre- lation among treatment coefficients exhibited negligible impact on estimation accuracy. Importantly, both response type and expected spatial structure are factors that farmers can evaluate prior to trial implementation, providing practical guidance for design selection.
Systematic designs demonstrated a clear advantage when the underlying treatment-response relationship was quadratic. In these scenarios, systematic layouts facilitated more reliable estimation of spatially varying coefficients, particularly under spatially correlated conditions. Conversely, when the response was linear, both designs performed similarly, suggesting that design choice is less critical in such cases. However, con- sidering the inherent variability of treatment responses across

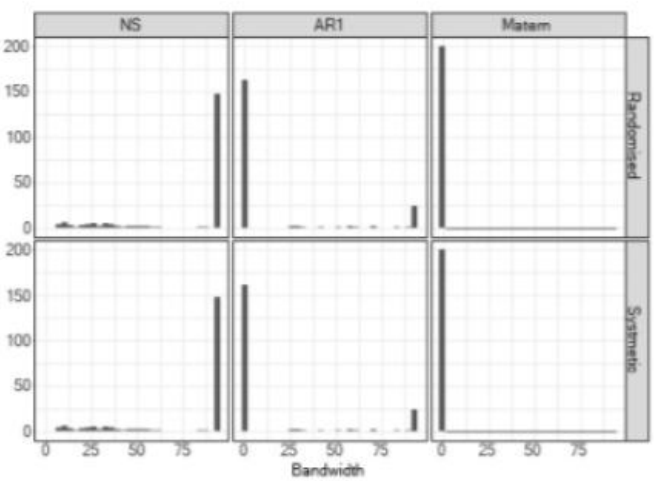

Fig. 9. Histogram of optimal bandwidth for linear response

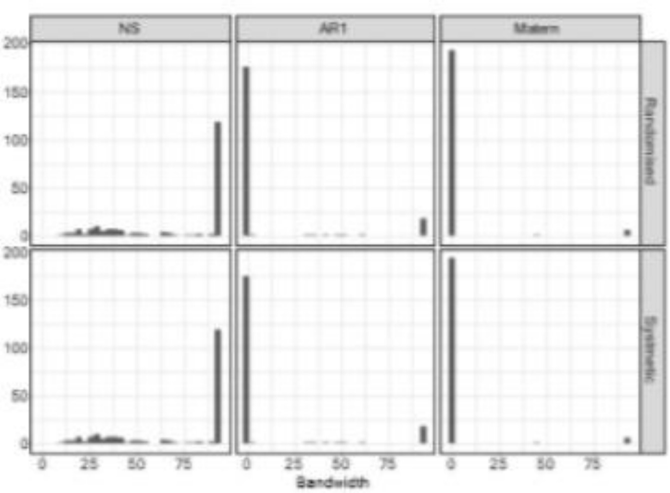

Fig. 10. Histogram of optimal bandwidth for quadratic response.

Fig. 11. Histogram of optimal bandwidth found by AICc for linear and quadratic response.

large agricultural fields, as noted by Rakshit et al. (2020), a systematic design offers greater flexibility by safeguarding against potential non-linear relationships.

The presence and nature of spatial covariance further influenced design performance. In scenarios devoid of spatial au- tocorrelation, differences between randomised and systematic designs were minimal, as expected, given that spatial independence negates the influence of plot arrangement. However, when spatial structure was introduced—particularly through AR1 AR1 or Mate ́rn covariance models—systematic designs consistently outperformed randomised counterparts, especially under quadratic response assumptions. The greatest design advantage was observed under Mate ́rn covariance conditions, where systematic layouts enhanced estimation accuracy for both linear and quadratic responses. Given the scale and inherent heterogeneity of modern OFE fields (add relevant ref- erence), the likelihood of negligible spatial variability is low, reinforcing the practical recommendation to favour systematic designs in applied settings.

The simulation results also highlighted limitations in band- width selection via AICc. Although AICc-minimising bandwidths are theoretically optimal in a likelihood framework, they frequently skewed towards extreme values—either highly local (bandwidth 1) or overly broad (bandwidth 93, matching the number of rows). In practice, these bandwidths produced higher MSEs compared to fixed bandwidths in- formed by experimental design structure (e.g., 5 or 9). This suggests that AICc-based bandwidth selection is prone to over- fitting or underfitting in the context of OFE, particularly when only one treatment observation per grid is available.

Fixed bandwidths, carefully chosen to encompass all treatment levels within a GWR window, offer a more reliable alternative by ensuring complete representation of treatment effects in local regressions, a necessity for accurate interpolation, especially under non-linear responses.

It is important to acknowledge that this study did not exhaustively explore all possible design structures or sources of spatial heterogeneity. Alternative designs, such as che- querboard or wave patterns, which have been proposed for OFE (Bramley et al. 1999), were not considered. Addition- ally, spatial zones defined by topographical or environmental gradients were excluded from the analysis. While GWR in- herently adjusts for localised spatial variation by estimating a global template model with location-specific refinements, pronounced zonal effects may influence model performance in practice. Future research should investigate the interaction between systematic designs, alternative layouts, and zone- based heterogeneity to provide more comprehensive guidance for OFE implementation.

In summary, this study advocates for the broader adoption of systematic designs in OFE, particularly when anticipating spatial correlation or potential non-linear treatment responses. Coupled with fixed, design-informed bandwidth selection, this approach enhances the reliability of spatially varying treatment effect estimation using GWR, offering practical benefits for both researchers and farmers engaged in precision agriculture.

6. CONCLUSION

Randomised designs have traditionally been favoured by agronomists and biometricians for on-farm experimentation (OFE), owing to their robustness against bias. However, this simulation study demonstrates that systematic designs can of- fer superior performance under specific conditions, particularly when the objective is to generate spatially varying treatment effect maps for large-scale strip trials.

The results indicate that systematic designs yield lower mean squared errors (MSE) for estimated coefficients and pro- vide more robust inference, especially when spatial variation is present or when non-linear (quadratic) treatment responses are expected. In contrast, when spatial variation is negligible or when a strictly linear treatment-response relationship is assumed, the performance differences between systematic and randomised designs are minimal, and either approach may be adopted without compromising estimation quality.

Given the scale and heterogeneity typical of modern OFE, coupled with the practical advantages of systematic designs in implementation and their enhanced compatibility with spatial modelling techniques such as GWR, we recommend system- atic designs for large OFE strip trials aimed at constructing spatially varying treatment maps. Systematic layouts offer greater flexibility for post-experiment statistical analysis while ensuring reliable estimation of treatment effects across diverse spatial conditions.

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