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Modelling Ecological Risk Assessment Using Temporal Intuitionistic Fuzzy Sets for Corrosion Impact Assessment

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Abstract:
This study proposed a novel division operation over Intuitionistic Fuzzy Sets to assess its decision-making properties. The study extends Intuitional Fuzzy Sets (IFS) by integrating temporal properties to perform operational functions in order to enhance the decision-making process. We modeled different causes of corrosion using Intuitionistic Fuzzy Set (IFS) to predict the corrosion type within four chosen villages (D_1, D_2, D_3, D_4). We estimated the normalized Hamming distances among different corrosion profiles and its causes to quantify which type of corrosion pose serious threat to the village. The results show that TIFSs effectively resolve uncertainties in decision-making, particularly in cases where traditional fuzzy set methods produce ambiguous outcomes, where FS failed to distinguish between wind and water corrosion for village $D3$, the TIFS-based approach provided a clear classification. Our findings suggest that TIFS-based framework is robust to address the ecological risks paving way for management interventions. However, fuzzy sets representing membership values and intuitionistic fuzzy sets for connection and non-connection value fail to yield consistent outcomes.
Keywords: *Intuitionistic Fuzzy Sets, Temporal Fuzzy Sets, Temporal Intuitionistic Fuzzy Sets (TIFS), Environmental management, Ecological Modelling, Corrosion.*

1. INTRODUCTION

Fuzzy sets are commonly applied in Engineering, Science, Health, Technological sectors. [21] developed the concept of fuzzy sets, a subdivision of crisp sets by maximizing its membership values from the binary options of 0 or 1 to any value within the continuous interval [0,1]. In this framework, function$\mu_A$map the elements from a non-empty set X to $[0, 1]$ for any $x \in X, \mu_a(x)$ represents the degree of membership, while 1 – $\mu_a(x)$ indicates the degree of non -membership degrees. K. T. Atanassov [3] proposed IFS with membership and non-membership degrees. If $x \in X$, an IFS is defined by the functions $\mu_a(x)$ and $\nu_a(x)$ such that $0 \leq \mu_a(x) + \nu_a(x) \leq 1$.

The concept of TIFS was first proposed by [4, 5] in 1991, as an advancement of temporal fuzzy sets integrated with IFSs. TIFSs enable accurate estimation of the real time processing with the ability to trace the changes of the item over time. Temporal intuitionistic fuzzy sets are defined using the entire operations along with operators for IFSs. In 1999, [19] introduced several measures to compare IFSs. Later, [9] defined level operators, max-min insinuation operator, and $P_{\alpha,\beta}, Q_{\alpha,\beta}$ operator [8] for the TIFSs in 2009.[20] introduced a new level operator in 2014. Studies conducted by [7] focused on various detachment appraise and inclusion measures for TIFSs in 2016.The concept of multi-parameter temporal IFSs was initially proposed by [10] in 2016. In 2018, [13] proposed an entropy measure for temporal TIFs, and introduced fuzzification function for TIFSs in 2019.

Temporal IFS of Second Type (TIFSST) theory was initially proposed by [14] in 2016. After its introduction, [15, 16] established the concept of Certain Level Operator and various distances measures among TIFSST in 2019. [1] investigated on the aggregation of infinite chains of IFS data in 2020. [11, 12] developed the morphological operations on TIFS. [17] introduced Interval Valued Temporal Neutrosophic Fuzzy Sets (IVTNFS)and measured its efficacy on E-management [18].

Figure-1 demonstrates the list of authors investigated on the Temporal IFSs. Although IFSs may sometimes failed to differentiate membership and non-membership values, TIFS play an effective role in decision-making by providing reliable results. They allow non-zero hesitation degree during every assessment. In this article, section 1 introduces the IFSs and its chronology, section 2 offers key definitions related to this study. Section 3 examines the properties of TIFS, with the new division operator, and Section 4 concludes the paper.

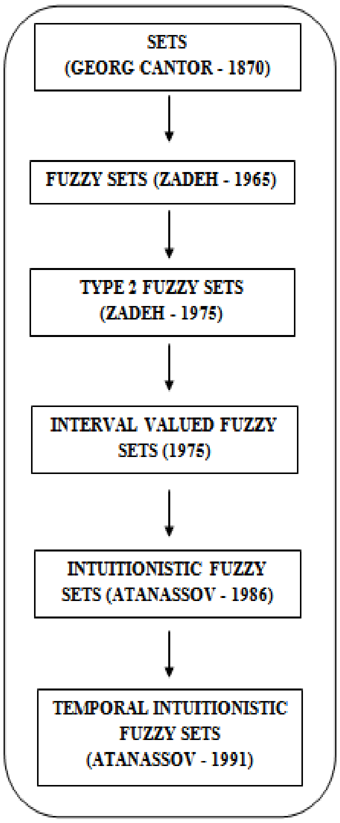

Figure 1 Research Timeline and Contributions to the Development of Temporal Intuitionistic Fuzzy Set (TIFS) Operators in Environmental Assessment

2. PRELIMINARIES

Here, we provide some essential definitions that are relevant to this study.

Definition: 2.1 [19] Let us consider set Uis non-empty. An object of the kind is defined like a fuzzy set (FS) S within U.

$$S = \{< m, \phi_S(m) > | m \in U\}$$

wherever the degree of membership value of the element $m \in U$ is defined by the function$\phi_S: U \rightarrow [0,1]$ and for each $m \in U$ with the condition $0 \leq \phi_S(m) \leq 1$.

Definition 2.2 [3] An object of the following kind is an IFSs S in U.

$$S = \{< m, \phi_S(m), \chi_S(m) > | m \in U\}$$

The degree of membership grade and the grade of non-membership part $m \in U$ are defined by the functions$\phi_S(m): U \rightarrow [0,1]$ and $\chi_S(m): U \rightarrow [0,1]$, respectively, and for each $m \in U$.

$$0 \leq \phi_S(m) + \chi_S(m) \leq 1$$

The standard fuzzy set can be articulated as$S = \{< m, \phi_S(m), 1 - \phi_S(m) > | m \in U\}$

Definition 2.3[4] The following object of the form is known as Temporal Intuitionistic Fuzzy Sets (TIFS).

$$S = \{< (k, y), \phi_S(k, y), \chi_S(k, y) > | (k, y) \in U \times P\}$$

Where

i) $S \subset U$ is a fixed set.

ii) $\phi_S(k, y) + \chi_S(k, y) \leq 1$ For every $< k, y > \in U \times P$

Where the functions $\phi_S(k)$: U $\to [0, 1]$ and $\chi_S(k)$: U $\to [0, 1]$, respectively, for all $k \in U$ with the period $y \in P$

Definition: 2.4 Let U be universe and consider two IFSs defined on U

$$S = \{< k, \phi_S(k), \chi_S(k) > | k \in U\}$$

And

$$T = \{< k, \phi_T(k), \chi_T(k) > | k \in U\}$$

Then, the division operation defined as the following form [2]

$$R = \{< \phi_S(k).\chi_T(k) + \phi_T(k).\chi_S(k), \phi_S(k).\phi_T(k) + \chi_S(k).\chi_T(k) > | k \in U\}$$

2.5 [Distance between Temporal Intuitionistic Fuzzy Sets

Let A and B be two Temporal Intuitionistic Fuzzy Sets (TIFSs) defined on the universe [6] $U = \{k_1, k_2, k_3, \dots k_n\}$, we define the following distance measures between them as:

2.5.1 Hamming Distance

$$= \frac{1}{2}\sum_{k=1}^{n} \left|\mu_{c_k}(k, y) - \mu_{v_k}(k, y)\right| + \left|\nu_{c_k}(k, y) - \nu_{v_k}(k, y)\right| + |\pi_{c_k}(k, y) - \pi_{v_k}(k, y)|$$

2.5.2 Normalized Hamming Distance

$$= \frac{1}{2n}\sum_{k=1}^{n} \left|\mu_{c_k}(k, y) - \mu_{v_k}(k, y)\right| + \left|\nu_{c_k}(k, y) - \nu_{v_k}(k, y)\right| + |\pi_{c_k}(k, y) - \pi_{v_k}(k, y)|$$

2.5.3 Euclidean Distance

$$= \sqrt{\frac{1}{2}\sum_{k=1}^{n} \left|\mu_{c_k}(k, y) - \mu_{v_k}(k, y)\right| + \left|\nu_{c_k}(k, y) - \nu_{v_k}(k, y)\right| + |\pi_{c_k}(k, y) - \pi_{v_k}(k, y)|}$$

2.5.4 Normalized Euclidean Distance

$$= \sqrt{\frac{1}{2n}\sum_{k=1}^{n} \left|\mu_{c_k}(k, y) - \mu_{v_k}(k, y)\right| + \left|\nu_{c_k}(k, y) - \nu_{v_k}(k, y)\right| + |\pi_{c_k}(k, y) - \pi_{v_k}(k, y)|}$$

3 TEMPORAL INTUITIONISTIC FUZZY SETS FOR DIVISION OPERATION

This section introduces a division operation for TIFS.

Definition: 3.1 Consider two Temporal Intuitionistic Fuzzy Sets (TIFS) defined on a universe U.

$$A = \{< (k, y), \mu_A(k, y), \nu_A(k, y) > | (k, y) \in U \times P\}$$

and

$$B = \{< (k, y), \mu_B(k, y), \nu_B(k, y) > | (k, y) \in U \times P\}$$

Then, the division operation over TIFS is define as the following form C

$$= \left\{\begin{matrix} < \mu_A(k, y)\nu_B(k, y) + \nu_A(k, y)\mu_B(k, y), \\ \mu_A(k, y)\mu_B(k, y) + \nu_A(k, y)\nu_B(k, y) \end{matrix} \middle| (k, y) \in U \times P\right\}$$

Let $A, B \subset U$. For each element $k \in U$ instance $y \in P$, denote the membership and non-membership grades by $\mu_A(k, y)$ & $\nu_A(k, y)$, respectively. The operation between A and B gives a new set C.

Theorem: 3.2 A ÷ B = C is a TIFS if A &B be any two TIFSs

Proof: We will demonstrate that the sum of membership and non-membership degrees does not exceed 1.

$= \left(\mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y)\right) + \left(\mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y)\right)$
$= \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y) + \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y)$
$= \mu_A(k,y)(\nu_B(k,y) + \mu_B(k,y)) + \nu_A(k,y)(\mu_B(k,y) + \nu_B(k,y))$
$= (\mu_A(k,y) + \nu_A(k,y)) + (\mu_B(k,y) + \nu_B(k,y))$
From A, B ∈ TIFS follows that

$$\mu_A(k,y) + \nu_A(k,y) \le 1$$

and

$$\mu_B(k,y) + \nu_B(k,y) \le 1$$

Therefore, $\left(\mu_A(k,y) + \nu_A(k,y)\right)\left(\mu_B(k,y) + \nu_B(k,y)\right) \le 1$
Hence, $A \div B = C$ is a TIFSs.

Theorem: 3.3 The division process is associative if A, B and C are any three TIFSs

Proof: Let us assume two TIFSs in a universe U

$$A = \{< (k,y), \mu_A(k,y), \nu_A(k,y) > |(k,y) \in U \times P\}$$

and

$$B = \{< (k,y), \mu_B(k,y), \nu_B(k,y) > |(k,y) \in U \times P\}$$

Then, the division operation over TIFS is define as the following form C

$$= \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

Again use the division function that we have between B and A.

$$= \left\{ \begin{matrix} < \mu_B(k,y).\nu_A(k,y) + \nu_B(k,y).\mu_A(k,y), \\ \mu_B(k,y).\mu_A(k,y) + \nu_B(k,y).\nu_A(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

$$= \left\{ \begin{matrix} < \mu_A(k,y)\nu_B(k,y) + \nu_A(k,y)\mu_B(k,y), \\ \mu_A(k,y)\mu_B(k,y) + \nu_A(k,y)\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

Consequently, division operator is commutative for any two temporal Intuitionistic fuzzy sets.

Theorem: 3.4 Associative qualities are satisfied by the division operation if A, B, and C are any three temporal Intuitionistic fuzzy sets.

Proof: Let us assume A and B are two TIFSs defined on a universe U

$A = \{< (k,y), \mu_A(k,y), \nu_A(k,y) > |(k,y) \in U \times P\}$,

$$B = \{< (k,y), \mu_B(k,y), \nu_B(k,y) > |(k,y) \in U \times P\}$$

and

$$C = \{< (k,y), \mu_C(k,y), \nu_C(k,y) > |(k,y) \in U \times P\}$$

Use the division function that we have between A and B

$$= \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

To find $(A \div B) \div C$

$$= \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\} \div \mathrm{C}$$

$$\begin{aligned} = \{< & \mu_A(k,y).\mu_B(k,y).\nu_c(k,y) + \mu_A(k,y).\nu_B(k,y).\nu_c(k,y) + \\ & \mu_A(k,y).\nu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\mu_B(k,y).\mu_C(k,y) + \\ & \nu_A(k,y).\nu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\mu_B(k,y).\nu_C(k,y), \\ & \mu_A(k,y).\mu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\nu_B(k,y).\nu_C(k,y) > |(k,y) \in U \times P\} \end{aligned}$$

$$\begin{aligned} = \{< & \mu_A(k,y)(\mu_B(k,y) + \nu_B(k,y))\nu_c(k,y) + \\ & \left(\mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y)\right)\mu_C(k,y). \\ & \nu_A(k,y)(\nu_B(k,y).\mu_C(k,y) + \mu_B(k,y).\nu_C(k,y)), \\ & \mu_A(k,y).\mu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\nu_B(k,y).\nu_C(k,y) > |(k,y) \in U \times P\} \end{aligned}$$

Using the division function between B and C, to find $A \div (B \div C)$

$$= \left\{ \begin{matrix} < \mu_B(k,y).\nu_C(k,y) + \nu_B(k,y).\mu_C(k,y), \\ \mu_B(k,y).\mu_C(k,y) + \nu_B(k,y).\nu_C(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

To find $A \div (B \div C)$

$$= A \div \left\{ \begin{matrix} < \mu_B(k,y).\nu_C(k,y) + \nu_B(k,y).\mu_C(k,y), \\ \mu_B(k,y).\mu_C(k,y) + \nu_B(k,y).\nu_C(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$

$$= \{< \mu_A(k,y).\mu_B(k,y).\nu_c(k,y) + \mu_A(k,y).\nu_B(k,y).\nu_c(k,y) + \mu_A(k,y).\nu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\mu_B(k,y).\mu_C(k,y)$$
$$\nu_A(k,y).\nu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\mu_B(k,y).\nu_C(k,y),$$
$$\mu_A(k,y).\mu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\nu_B(k,y).\nu_C(k,y) > |(k,y) \in U \times P\}$$

$$= \{< \mu_A(k,y)(\mu_B(k,y) + \nu_B(k,y))\nu_c(k,y) + \left(\mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y)\right)\mu_C(k,y).$$
$$\nu_A(k,y)(\nu_B(k,y).\mu_C(k,y) + \mu_B(k,y).\nu_C(k,y)),$$
$$\mu_A(k,y).\mu_B(k,y).\mu_C(k,y) + \nu_A(k,y).\nu_B(k,y).\nu_C(k,y) > |(k,y) \in U \times P\}$$

We conclude that, $(A \div B) \div C = A \div (B \div C)$ from $A \div (B \div C)$ and $(A \div B) \div C$. Thus, the associative property is satisfied by the division operation between any three temporal intuitionistic fuzzy sets.

Theorem 3.5 A monoid is formed by the operation division based on the set of all TIFSs.

Proof: $A \div B = C$ forms a TIFS, if A & B be two TIFSs with groupoid structure.

If A, B & $C$are any three TIFSs then the division operation is a semi-group and meets the associative qualities. We'll demonstrate that a neutral element exists.

Assuming X is a neutral element, we get $A \div X = A$.

Consequently,

$$A \div X = \left\{ \begin{matrix} < \mu_A(k,y).\nu_X(k,y) + \nu_A(k,y).\mu_X(k,y), \\ \mu_A(k,y).\mu_X(k,y) + \nu_A(k,y).\nu_X(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$$
$$= \{< (k,y), \mu_A(k,y), \nu_X(k,y) > |(k,y) \in U \times P\}$$
$$= A$$

The system of equations we have is as follows:

$$\mu_A(k,y).\nu_X(k,y) + \nu_A(k,y).\mu_X(k,y) = \mu_A(k,y)$$
$$\mu_A(k,y).\mu_X(k,y) + \nu_A(k,y).\nu_X(k,y) = \nu_A(k,y)$$

1st case: $\mu_A(k,y) = 0$.

Hence

$$\nu_A(k,y).\nu_X(k,y) = \nu_A(k,y)$$

i.e. $\nu_X(k,y) = 1$ and $\mu_X(k,y) = 0$

Thus, the neutral element is

$$X = \{((k,y), 0, 1)|(k,y) \in U \times P\} \equiv \bar{0}$$

2nd case: $\mu_A(k,y) \neq 0$.

Hence,

$$\frac{\left(\nu_A(k,y)(1 - \nu_A(k,y))\right)}{\mu_A(k,y)} = \mu_A(k,y),$$

$$\mu_A(k,y).\nu_X(k,y) + \nu_A(k,y)\frac{\left(\nu_A(k,y)(1 - \nu_A(k,y))\right)}{\mu_A(k,y)} = \mu_A(k,y).\mu_A(k,y)$$

$$\mu_A{}^2(k,y).\nu_x(k,y) + \nu_A{}^2(k,y) - \nu_A{}^2(k,y).\nu_x(k,y) = \mu_A{}^2(k,y)$$

$$\nu_x(k,y)(\mu_A{}^2(k,y) - \nu_A{}^2(k,y)) = \mu_A{}^2(k,y) - \nu_A{}^2(k,y)$$

$\nu_X(k,y) = 1$ and $\mu_X(k,y) = 0$.

Hence the neutral component is again the above set

$$X = \{((k,y), 0, 1) \mid < k, y > \in U \times P\} \equiv \bar{0}$$

Division, for example, is a monoid in TIFSs. We shall now demonstrate that divide does not create a group as there is no opposite element.

Assume that element X is the opposite of constituent A.
i.e. A ÷ X = $\bar{0}$
$= \left\{\begin{matrix} < \mu_A(k,y).\nu_X(k,y) + \nu_A(k,y).\mu_X(k,y), \\ \mu_A(k,y).\mu_X(k,y) + \nu_A(k,y).\nu_X(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\}$
$= \{((k,y),0,1) | < k,y > \in U \times P\}$
Therefore, we have the following system of equations:

$$\mu_A(k,y).\nu_X(k,y) + \nu_A(k,y).\mu_X(k,y) = 0$$
$$\mu_A(k,y).\mu_X(k,y) + \nu_A(k,y).\nu_X(k,y) = 1$$

1st case: $\mu_A(k,y) = 0$. Hence,
$\nu_A(k,y).\mu_X(k,y) = 0$ and $\nu_A(k,y).\nu_X(k,y) = 1$,
But
$\nu_A(k,y) \leq 1$ and $\nu_X(k,y) \leq 1$,
Therefore,

$$\nu_A(k,y) = \nu_X(k,y) = 1$$

thus so,

$$\mu_A(k,y) = \mu_X(k,y) = 0.$$

Therefore, the opposite element of $\{< (k,y),0,1 > | < k,y > \in U \times P\}$ is same.
i.e. $\{< (k,y),0,1 > | < k,y > \in U \times P\}$
2nd case: $\mu_A(k,y) \neq 0$.
Hence,
$\nu_x(k,y) = \frac{\nu_A(k,y).\mu_A(k,y)}{\mu_A(k,y)}$,
But,

$$\nu_x(k,y) \geq 0$$

Therefore,

$$\nu_A(k,y).\mu_x(k,y) = 0.$$

From case 1: $\nu_A(k,y) = 0$
i.e. $\mu_A(k,y).\mu_X(k,y) = 1$
but,
$\mu_A(k,y) \leq 1$ and $\mu_X(k,y) \leq 1$
Therefore,

$$\mu_A(k,y) = \mu_X(k,y) = 1$$

Therefore $\nu_X(k,y) = 0$ and so the opposite element of $\{< (k,y),1,0 > | < k,y > \in U \times P\}$ is $\{< (k,y),0,1 > | < k,y > \in U \times P\}$.
From case 2:
$\nu_A(k,y) \neq 0$ (i.e) $\mu_x(k,y) = 0$.
Therefore
$\nu_A(k,y).\nu_X(k,y) = 1$,
But, $\nu_A(k,y) \leq 1$ and $\nu_X(k,y) \leq 1$
So,

$$\nu_A(k,y) = \nu_X(k,y) = 1$$

As a result,
$\mu_A(k,y) = 0$.
This contradicts the second case requirement that, $\mu_A(k,y) = 0$.Thus, our hypothesis proved as false, and the common situation has no competing elements. The TIFSs (division) forms a monoid structure rather than a group.

Theorem 3.6 For each of TIFS A and B, we have the following, assuming X is a nonempty set:

$$\boldsymbol{A \div B = \neg A \div \neg B}.$$

Proof: Let A and B be any two TIFS, then division of A and B is

$$A \div B = \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\} \text{........ (i)}$$

and the complement of $A \div B$ is

$$= \left\{ \begin{matrix} < \nu_A(k,y).\mu_B(k,y) + \mu_A(k,y).\nu_B(k,y), \\ \nu_A(k,y).\nu_B(k,y) + \mu_A(k,y).\mu_B(k,y) > \end{matrix} \middle| < k,y > \in U \times P \right\}$$

$$= \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\} \text{........ (ii)}$$

From equation (i) and (ii)

$$A \div B = \neg A \div \neg B.$$

Theorem 3.7 Assuming X is a nonempty set, we have the following for each TIFS A and B

$$\boldsymbol{\neg (A \div B) = \neg A \div \neg B}.$$

Proof: Let A and B be any two TIFS, then the division operation of A and B is

$$A \div B = \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\} \text{........ (i)}$$

and the complement of $A \div B$ is

$$= \left\{ \begin{matrix} < \nu_A(k,y).\mu_B(k,y) + \mu_A(k,y).\nu_B(k,y), \\ \nu_A(k,y).\nu_B(k,y) + \mu_A(k,y).\mu_B(k,y) > \end{matrix} \middle| (k,y) \in U \times P \right\}$$

$$= \left\{ \begin{matrix} < \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) \end{matrix} \middle| (k,y) \in U \times P \right\} \text{........(iii)}$$

The complement of A and B defined as

$$\neg \mathrm{A} = \{< m,s >, \nu_\mathrm{A}(m,s), \mu_\mathrm{A}(m,s) > | < m,s > \in U \times P\}$$

and

$$\neg \mathrm{B} = \{< m,s >, \nu_\mathrm{B}(m,s), \mu_\mathrm{B}(m,s) > | < m,s > \in U \times P\}$$

Apply the division operator among the complement of A and B we have

$$\neg A \div \neg \mathrm{B} = \{< k,y >, \nu_\mathrm{A}(k,y), \mu_\mathrm{A}(k,y) > | < k,y > \in U \times P\} \div \{< k,y >, \nu_\mathrm{B}(k,y), \mu_\mathrm{B}(k,y) > | < k,y > \in U \times P\}$$

$$= \left\{ \begin{matrix} < k,y >, \nu_A(k,y).\mu_B(k,y) + \mu_A(k,y).\nu_B(k,y), \\ \nu_A(k,y).\nu_B(k,y) + \mu_A(k,y).\mu_B(k,y) > \end{matrix} \middle| < (k,y) > \in U \times P \right\}$$

$$= \left\{ \begin{matrix} < k,y >, \mu_A(k,y).\nu_B(k,y) + \nu_A(k,y).\mu_B(k,y), \\ \mu_A(k,y).\mu_B(k,y) + \nu_A(k,y).\nu_B(k,y) > \end{matrix} \middle| < (k,y) > \in U \times P \right\} \text{.......(iv)}$$

From equation (iii) and (iv)

$$\neg (A \div B) = \neg A \div \neg B.$$

4. Application of TIFSs in Ecological Administration

In this section, we are exploring the applications of TIFSs in the ecological management systems. Let A represent different causes for corrosion in village settings, C denote various types of corrosion, and V signify a different village. Understanding corrosion and its underlying causes allows us to identify, which kind of causes are strongly associated with types of corrosion in village settings. Each type of corrosion has a unique cause that indicates a certain level of connection, denoted as μ, and a level of disconnection, represented as ν. We will apply TIFSs to develop a decision making to this ecological problem. This approach essentially consists of three key phases:

(i) Identification of the source;

(ii) Development of ecological knowledge based on TIFSs principles;

(iii) The administration is assessed using the n-Hamming detachment among the sets C and V.

We first established a theoretical scenario for this problem before demonstrating its application. The distance, denoted as d_{NH}, between a village &types of corrosion in relation to their causes is defined,

$$= \frac{1}{2n}\sum_{k=1}^{n}\left|\mu_{c_k}(m,s) - \mu_{v_k}(m,s)\right| + \left|\nu_{c_k}(m,s) - \nu_{v_k}(m,s)\right| + \left|\pi_{c_k}(m,s) - \pi_{v_k}(m,s)\right|$$

where $c_i \in C$ and $v_i \in V$ and n represents total number of causes. Each village assumed to have a specific type of corrosion, if the distance, $d_{TIFSnHD}(C,V)$, between the cause and the village be minimal.

4.1 Investigational exemplar

Let us choose four villages and it's denoted as a set $D = \{D_1, D_2, D_3, D_4\}$ that are surveyed for different corrosion types denoted as C in the corrosion type we choose water corrosion its refer to degradation of land by flow of water and the second corrosion is tunnel corrosion likely it refers the degradation of soil leading to the formation of tunnels, the next corrosion is gully corrosion it can lead to soil loss, increased sedimentation in waterways and decreased agricultural productivity and the final corrosion is wind corrosion it refers the degradation of land caused by wind, which have the set of causes. Further, we analyzed the environmental factors like farming, graze, poor management rude, flow of water, reduced plants in terms of TIFSs.

A	Farming	Graze	Poor Management	Flow of Water	Reduced Plants
Wind Corrosion	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.3,0.5,0.2)	(0.5,0.3,0.2)	(0.5,0.4,0.1)
Tunnel Corrosion	(0.2,0.7,0.1)	(0.9,0,0.1)	(0.7,0.2,0.1)	(0.6,0.3,0.1)	(0.7,0.2,0.1)
Gully Corrosion	(0.5,0.3,0.2)	(0.3,0.5,0.2)	(0.2,0.7,0.1)	(0.2,0.6,0.2)	(0.4,0.4,0.2)
Water Corrosion	(0.1,0.7,0.2)	(0.3,0.6,0.1)	(0.8,0.1,0.1)	(0.1,0.8,0.1)	(0.1,0.8,0.1)

Table 1 - IFS-Based Representation of Environmental Conditions Observed Across Selected Villages

Based on survey data, Table 1 lists each type of corrosion along with its causes using Temporal Intuitionistic Fuzzy Set (TIFS) values. The causes A_i in Table 1 are characterized by three values: membership, non-membership, and uncertainty boundary. Pro control measures, we assume that the survey outcome be gathered from the village and analyze; the findings are displayed as obtained results in table 1.

C	Farming	Grazing	Rude Plan	Water Flow	Reduced Plants
Wind Corrosion	(0.6,0.2,0.2)	(0.4,0.4,0.2)	(0.2,0.7,0.1)	(0.5,0.3,0.3)	(0.3,0.5,0.2)
Tunnel Corrosion	(0.3,0.5,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.7,0.1,0.2)
Gully Corrosion	(0.2,0.6,0.2)	(0.3,0.3,0.4)	(0.7,0.1,0.2)	(0.3,0.6,0.1)	(0.3,0.5,0.2)
Water Corrosion	(0.4,0.4,0.2)	(0.4,0.2,0.4)	(0.1,0.6,0.3)	(0.5,0.4,0.1)	(0.4,0.5,0.1)

Table 2. IFS-Based Representation of Environmental Conditions Observed Across Selected Villages

Utilizing the previously mentioned normalized Hamming distance of FS (only for membership values), we calculated the distance (normalized hamming) among every village in the (Table-2) and each one of corrosion type in first table concerning every cause, resulting in the following tables presented below.

$d_{nHFS}(C,A)$	Wind Corrosion	Tunnel Corrosion	Gully Corrosion	Water Corrosion
D_1	**0.08**	0.11	0.18	0.1

D_2	0.19	**0.08**	0.13	0.17
D_3	**0.06**	0.13	0.1	**0.06**
D_4	0.18	0.17	**0.06**	0.18

Table 3 Normalized Hamming Distances Between Villages and Corrosion Types Using Membership Values (FS-Based)

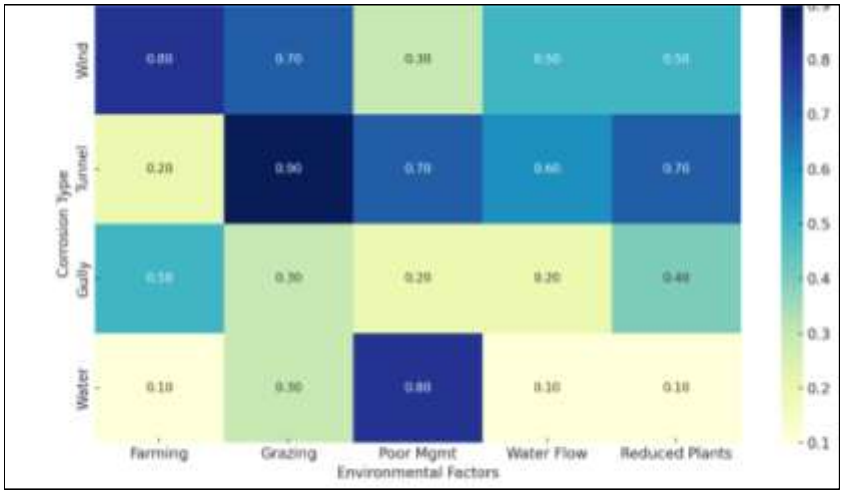

Figure 2 Membership degree for Corrosion Types across villages

According to the table 3, the village D_1 needs to be manage for wind corrosion, the town D_2 should focus on tunnel corrosion management, the town D_3 is concentrate wind as well as water corrosions, and town D_4 must manage gully corrosions (Fig. 2). There is an uncertain situation in village D_3 because, there is an uncertain situation to predict the problem that is wind and water corrosions are same values; therefore, we move to temporal Intuitionistic fuzzy sets concepts.

$d_{nHTIFS}(C, A)$	Wind Corrosion	Tunnel Corrosion	Gully Corrosion	Water Corrosion
D_1	0.19	0.26	0.38	0.22
D_2	0.41	0.2	0.3	0.36
D_3	0.13	0.28	0.26	0.2
D_4	0.43	0.38	0.2	0.42

Table 4. Decision Outcomes for Corrosion Management Based on Minimum Distance Analysis Using TIFS

RESULTS AND DISCUSSIONS

This study assesses the application of TIFS to classify the villages based on largely affected by its corrosion type and causative factors. We systematically determined which kind of corrosion demands immediate attention by estimating the hamming distance between corrosion causes and corrosion condition observed in four village.

Comparison of Hamming Distances

We used normalized Hamming distance on FS, accounting only membership values. Table 3 demonstrates the computed distances, where lower values indicate a stronger association between a village and a specific type of corrosion. Based on these results, village D1 exhibited the highest proximity to wind corrosion, D2 to tunnel corrosion, D3 to both wind and water corrosion (with equal distance values), and D4 to gully corrosion. However, D3 showed identical distance values for wind and water corrosion, making it difficult to determine the dominant corrosion type. To address this limitation, we extended the analysis using TIFSs, which incorporate both membership and non-membership values along with time-dependent factors (Fig. 3).

Table 4 presents the revised hamming distances computed using Temporal IFSs. This approach provided more precise results, resolving the ambiguity in village D3. The refined outcomes confirmed that D1 remains most affected by wind corrosion, D2 by tunnel corrosion, D3 by wind corrosion (resolving previous uncertainty), and D4 by gully corrosion.

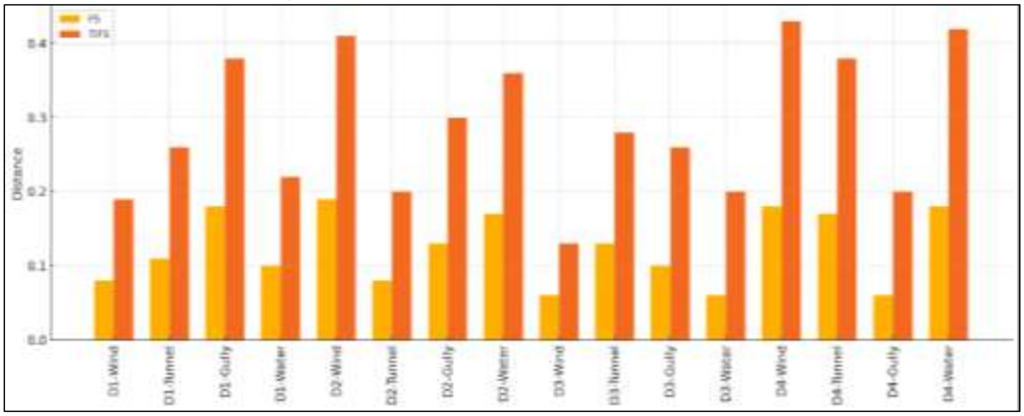

Figure 3. Normalized Hamming Distances Between Villages and Corrosion

Effectiveness of TIFS-Based Analysis

The comparison between the FS-based and TIFS-based distance measures demonstrates that TIFSs offer more reliable decision-making support in ecological assessments. Unlike standard fuzzy set methods focusing only membership values, TIFSs uses both non-membership and hesitation degrees to achieve comprehensive risk assessment. This distinction is particularly significant in real-world environmental management, where corrosion effects fluctuate over time and require dynamic assessment models. Additionally, our results confirm that the proposed TIFS-based framework improves decision-making by providing a structured methodology to assess the severity of corrosion issues in different villages. This approach enhances predictive accuracy with targeted ecological interventions using temporal and intuitionistic fuzzy sets.

Implications for Ecological Management

The findings suggest that local authorities should prioritize mitigation efforts based on the identified corrosion risks (Figure 4). For instance, village D1 requires immediate measures to address wind corrosion, such as afforestation and soil stabilization. Similarly, village D2 must implement erosion control strategies for tunnel corrosion, while D4 should focus on preventing gully formation through better land management. The case of D3 highlights the advantage of TIFS-based methods in resolving decision-making uncertainties that arise in conventional fuzzy set approaches.

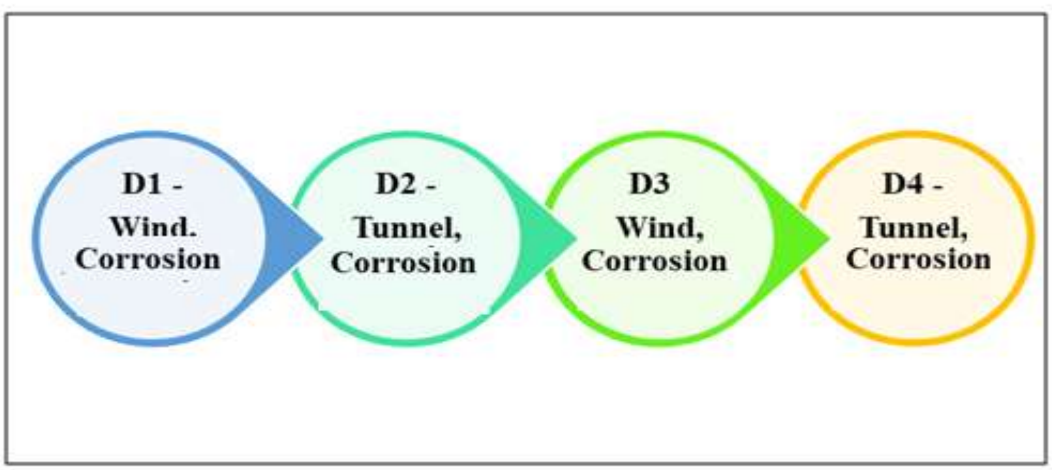

Figure 4 Relationship Between Villages and Corrosion Types Based on TIFS-Driven Environmental Analysis

5. CONCLUSION AND FUTURE WORK

This study demonstrates that Temporal Intuitionistic Fuzzy Sets provide accurate and reliable approach for identifying and managing corrosion risks in villages. TIFSs improve decision-making in uncertain environments using membership and non-membership values with hesitation degrees. Our results confirm that TIFSs outperform traditional fuzzy set methods by resolving ambiguities and capturing dynamic changes in ecological conditions. This method enables local authorities to prioritize mitigation efforts effectively. Future research will extend this approach to larger datasets and explore additional distance measures to further refine corrosion risk assessments.

Future research will focus on several key areas to enhance the applicability and accuracy of this approach. First, we plan to extend this model to larger datasets, incorporating additional villages and a broader range of corrosion types to improve generalizability. Second, we will explore alternative distance measures, such as the Euclidean and Hausdorff distances, to compare their effectiveness against the normalized Hamming distance used in this study. Third, integrating machine learning techniques with TIFSs could enhance predictive capabilities, enabling automated decision-making for ecological management. We will apply this framework to real-time environmental monitoring systems will allow for dynamic updates in risk assessment, making it more responsive to changing ecological conditions. Finally, future studies will explore the applicability of TIFSs in other domains, such as climate change analysis, resource allocation, and sustainable land management. These advancements will further establish TIFSs as a powerful decision-making tool for complex environmental and industrial challenge

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