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Power Mean Cordial Labeling Of Some Graphs

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Abstract
Let $G = (V, E)$ be a graph with p vertices and q edges. Let f be a function from $V(G)$ to $\{1,2,\dots,q+1\}$. Each edge e=uv is labelled with

$$f^*(uv) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

or

$$f^*(uv) = \begin{cases} 1 & \text{if } \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Then f^* is called a power mean cordial labelling of G if $|e_f(i) - e_f(j)| \le 1, \forall\, i, j \in \{0,1\}$ where $e_f(i)$ and $e_f(j)$ denotes the number of edges labeled with i and j respectively. A graph that admits power mean cordial labeling is called power mean cordial graph. In this paper power mean cordiality of some graphs are discussed.

Key words: *cordial, mean, , mean cordial, power mean cordial, path, cycle, star, ladder*
Subject Classification: 05C78

1. INTRODUCTION

Graphs considered here are simple, finite, connected and undirected. The vertex set and edge set of a graph are $V(G)$ and $E(G)$ respectively. The concept of cordial labeling was introduced by Cahit in the year 1987. The concept of mean labeling was introduced by S. Somasundaram and Raja Ponraj. The concept of power mean labeling was introduced by P. Mercy and S. Somasundaram. Motivated by above concepts, we introduced a new type of labeling called power mean cordial labeling.

Definition 1.1. Let G be a (p, q) graph. Let f be a function from $V(G)$ to $\{1,2,\dots,q+1\}$. Each edge e = uv is labelled with

$$f^*(uv) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

or

$$f^*(uv) = \begin{cases} 1 & \text{if } \left\lfloor \left(f(u)^{f(v)} f(v)^{f(u)}\right)^{\frac{1}{f(u)+f(v)}} \right\rfloor \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Then f^* is called a power mean cordial labelling of G if $|e_f(i) - e_f(j)| \le 1, \forall\, i, j \in \{0,1\}$ where $e_f(i)$ and $e_f(j)$ denotes the number of edges labeled with i and j respectively. A graph that admits power mean cordial labeling is called power mean cordial graph.

Example 1.2. A graph that admits a power mean cordial labeling which is given below:

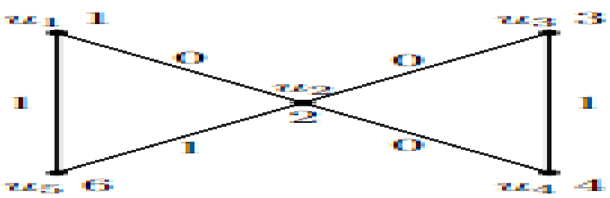

Figure 1 **Theorem 1.3.** Let P_n be a path graph with n vertices. Then P_n is a power mean cordial graph if and only if $n \ge 2$.

Proof. Let $P_n = \{u_1, u_2, \ldots, u_n\}$ be a path of length n.
Clearly P_n has $n-1$ edges.
Now we define a function $f: V(P_n) \to \{1,2,\ldots,q+1\}$.
The vertex labeling is defined as,
$f(u_i) = i, 1 \leq i \leq n$
Let the induced edge labelings are as follows:
For $1 \leq i \leq n-1$,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_i)^{f(u_{i+1})} f(u_{i+1})^{f(u_i)} \right)^{\frac{1}{f(u_i)+f(u_{i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Case (i): n is odd
If n is odd then P_n has $n-1$edges which is even.
Therefore the number of edges labeled with zero is given by $e_f(0) = \frac{n-1}{2}$.
Similarly the number of edges labeled with one is given by $e_f(1) = \frac{n-1}{2}$.
Here $|e_f(0) - e_f(1)| \leq 1$.
Thus P_n admits power mean cordial labeling if n is odd.
Case (ii): n is even
Then P_n has $n-1$ edges which is odd
Now the number of edges labeled with zero is given by $e_f(0) = \frac{n}{2}$.
Similarly the number of edges labeled with one is given by $e_f(1) = \frac{n}{2} - 1$.
Here $|e_f(0) - e_f(1)| \leq 1$.
Thus P_n admits power mean cordial labeling if n is even.
Therefore $P_n, n \geq 2$ satisfies power mean cordial labeling.
Hence $P_n, n \geq 2$ is a power mean cordial graph.
Example 1.4. P_8 is a power mean cordial graph which is given below:

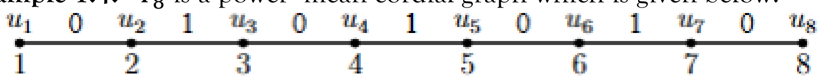

Figure 1 **Theorem 1.5.** Let $C_n$be a cycle graph with n vertices. Then C_n is a power mean cordial graph if and only if $n \geq 3$.
Proof. Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$ be the vertex set of cycle C_n.
Let $E(C_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_n u_1\}$.
Define $f: V(C_n) \to \{1,2,\ldots,q+1\}$.
$f(u_i) = i, 1 \leq i \leq n$
The induced edge labelings are as follows:
For $1 \leq i \leq n-1$,

$$f^*(u_i u_{i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_i)^{f(u_{i+1})} f(u_{i+1})^{f(u_i)} \right)^{\frac{1}{f(u_i)+f(u_{i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

If n is odd,

$$f^*(u_n u_1) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_n)^{f(u_1)} f(u_1)^{f(u_n)} \right)^{\frac{1}{f(u_n)+f(u_1)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

If n is even,

$$f^*(u_n u_1) = \begin{cases} 1 & \text{if } \left| \left(f(u_n)^{f(u_1)} f(u_1)^{f(u_n)} \right)^{\frac{1}{f(u_n)+f(u_1)}} \right| \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Case (i): n is odd
Here the number of edges labeled with zero is given by $e_f(0) = \frac{n+1}{2}$.
Similarly the number of edges labeled with one is given by $e_f(1) = \frac{n-1}{2}$.
Here $|e_f(0) - e_f(1)| \leq 1$.
Thus C_n admits power mean cordial labeling if n is odd.
Case (ii): n is even

Here the number of edges labeled with zero is given by $e_f(0) = \frac{n}{2}$
Similarly the number of edges labeled with one is given by $e_f(1) = \frac{n}{2}$
Here $|e_f(0) - e_f(1)| \leq 1$.
Therefore $C_n, n \geq 3$ satisfies power mean cordial labeling.
Hence $C_n, n \geq 3$ is a power mean cordial graph.
Example 1.6. C_6 is a power mean cordial graph which is given below:

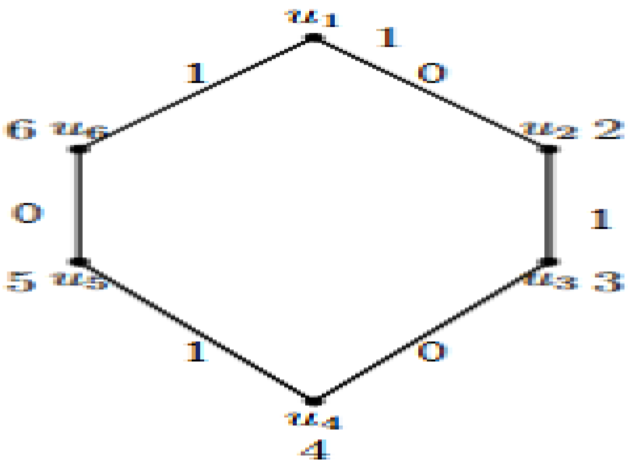

Figure 2 Theorem 1.7. The graph $S(L_n)$ formed by inserting one new vertex along each edge of a ladder graph is L_n a power mean cordial graph.
Proof. Let L_n be a ladder connecting two paths $\{v_1, v_2, \ldots, v_n\}$ and $\{u_1, u_2, \ldots, u_n\}$.
Let $G = S(L_n)$ be a graph obtained by subdividing all the edges of L_n.
Let $\{u'_i, v'_i : 1 \leq i \leq n-1\}$ be the vertices which subdivide the edges u_iu_{i+1}, v_iv_{i+1} respectively.
Let $w_i: 1 \leq i \leq n$ be the vertex which subdivide the edge u_iv_i.
Now $V(S(L_n)) = \{u_i, v_i, w_i : 1 \leq i \leq n\} \cup \{u'_i, v'_i : 1 \leq i \leq n-1\}$ and
$E(S(L_n)) = \{\{u_iu'_i, v_iv'_i, v'_iv_{i+1}, u'_iu_{i+1}, : 1 \leq i \leq n-1\} \cup \{u_i w_i, v_iw_i : 1 \leq i \leq n\}$
Now we define a function for labeling the vertices $f: V(S(L_n)) \to \{1,2,\ldots,q+1\}$ by
$f(v_i) = 6i - 5, 1 \leq i \leq n$
$f(u_i) = 6i - 3, 1 \leq i \leq n$
$f(v'_i) = 6i - 2, 1 \leq i \leq n-1$
$f(u'_i) = 6i - 1, 1 \leq i \leq n-1$
$f(w_i) = 6i - 4, 1 \leq i \leq n$
Now the induced edge labelings are follows:
For $1 \leq i \leq n-1$,

$$f^*(v_iv'_i) = \begin{cases} 1 & \text{if } \left\lceil \left(f(v_i)^{f(v'_i)} f(v'_i)^{f(v_i)}\right)^{\frac{1}{f(v_i)+f(v'_i)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(v'_iv_{i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(v'_i)^{f(v_{i+1})} f(v_{i+1})^{f(v'_i)}\right)^{\frac{1}{f(v'_i)+f(v_{i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(u_iu'_i) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_i)^{f(u'_i)} f(u'_i)^{f(u_i)}\right)^{\frac{1}{f(u_i)+f(u'_i)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(u'_iu_{i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u'_i)^{f(u_{i+1})} f(u_{i+1})^{f(u'_i)}\right)^{\frac{1}{f(u'_i)+f(u'_i)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(u_iw_i) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_i)^{f(w_i)} f(w_i)^{f(u_i)}\right)^{\frac{1}{f(u_i)+f(w_i)}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(v_iw_i) = \begin{cases} 1 & \text{if } \left[\left(f(v_i)^{f(w_i)} f(w_i)^{f(v_i)}\right)^{\frac{1}{f(v_i)+f(w_i)}}\right] \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Therefore $e_f(0) = 3n-2$ and $e_f(1) = 3n-2$.
Here $|e_f(0) - e_f(1)| \leq 1$.
Therefore $S(L_n)$, $n \geq 2$ satisfies power mean cordial labeling.
Hence $S(L_n)$, $n \geq 2$ is a power mean cordial graph.
Example 1.8. Subdivision of power mean cordial labeling of L_5 is given below:

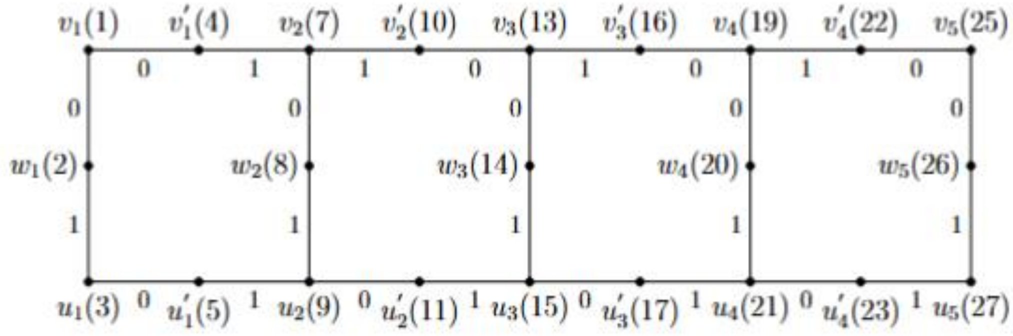

Figure 3 **Theorem 1.9.** The star graph $K_{1,n}$ admits power mean cordial labeling for all $n \geq 2$.
Proof. Let $K_{1,n}$ be a star graph with vertex set $V(K_{1,n}) = \{v_0, v_i : 1 \leq i \leq n\}$ and edge set $E(K_{1,n}) = \{v_0v_i : 1 \leq i \leq n\}$.
Define $f: V(K_{1,n}) \to \{1,2,\dots,q+1\}$.
Let $f(v_0)= 1$.
$f(v_i)= i+1$, $1 \leq i \leq n$
The induced edge labelings are as follows:
For $1 \leq i \leq n-3$,

$$f^*(v_0 v_{2i-1}) = \begin{cases} 1 & \text{if } \left[\left(f(v_0)^{f(v_{2i-1})} f(v_{2i-1})^{f(v_0)}\right)^{\frac{1}{f(v_0)+f(v_{2i-1})}}\right] \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

For $1 \leq i \leq n-4$,

$$f^*(v_0 v_{2i}) = \begin{cases} 1 & \text{if } \left[\left(f(v_0)^{f(v_{2i})} f(v_{2i})^{f(v_0)}\right)^{\frac{1}{f(v_0)+f(v_{2i})}}\right] \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Therefore $e_f(0) = n-4$ and $e_f(1) = n-3$.
Here $|e_f(0) - e_f(1)| \leq 1$.
Therefore $K_{1,n}$, $n \geq 2$ satisfies power mean cordial labeling.
Hence $K_{1,n}$, $n \geq 2$ is a power mean cordial graph.

Example 1.10. Star graph $K_{1,7}$ which admits power mean cordial labeling is given below:

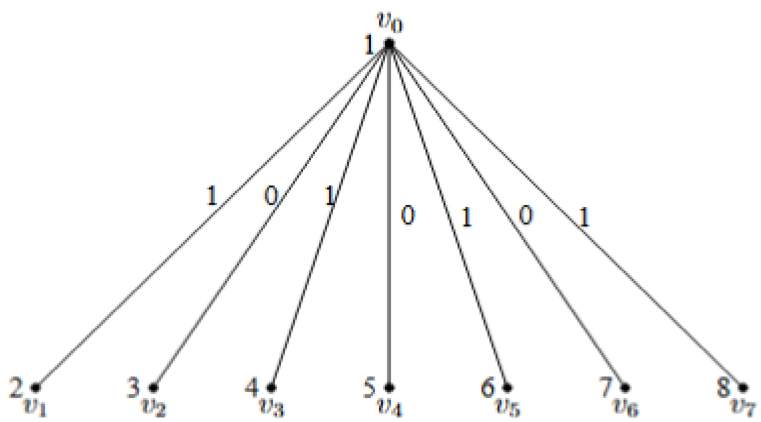

Figure 4 **Theorem 1.11.** The ladder graph L_n, $n \geq 3$ is a power mean cordial graph for all $n \geq 3$.

Proof.

Let $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and

$E(L_n) = \{v_i v_{i+1}, u_i u_{i+1}, : 1 \le i \le n-1\}\{u_i v_i : 1 \le i \le n\}$.

Define $f: V(L_n) \to \{1,2,\dots,q+1\}$.

$f(u_i) = 2i-1, 1 \le i \le n$

$f(v_i) = 2i, 1 \le i \le n$

Now the induced edge labelings are as follows:

For $1 \le i \le n-3$,

$$f^*(u_{2i-1}u_{2i}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_{2i-1})^{f(u_{2i})} f(u_{2i})^{f(u_{2i-1})}\right)^{\frac{1}{f(u_{2i-1})+f(u_{2i})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(u_{2i}u_{2i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_{2i})^{f(u_{2i+1})} f(u_{2i+1})^{f(u_{2i})}\right)^{\frac{1}{f(u_{2i})+f(u_{2i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(v_{2i}v_{2i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(v_{2i})^{f(v_{2i+1})} f(v_{2i+1})^{f(v_{2i})}\right)^{\frac{1}{f(v_{2i})+f(v_{2i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(v_{2i-1}v_{2i}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(v_{2i-1})^{f(v_{2i})} f(v_{2i})^{f(v_{2i-1})}\right)^{\frac{1}{f(v_{2i-1})+f(v_{2i})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(u_{2i-1}v_{2i}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_{2i-1})^{f(v_{2i})} f(v_{2i})^{f(u_{2i-1})}\right)^{\frac{1}{f(u_{2i-1})+f(v_{2i})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f^*(u_{2i}v_{2i+1}) = \begin{cases} 1 & \text{if } \left\lceil \left(f(u_{2i})^{f(v_{2i+1})} f(v_{2i+1})^{f(u_{2i})}\right)^{\frac{1}{f(u_{2i})+f(v_{2i+1})}} \right\rceil \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Therefore $e_f(0) = n+1$ and $e_f(1) = n+2$.

Here $|e_f(0) - e_f(1)| \le 1$.

Therefore $L_n, n \ge 3$ satisfies power mean cordial labeling.

Hence $L_n, n \ge 3$ is a power mean cordial graph.

Example 1.12. L_5 which admits power mean cordial labeling is given below:

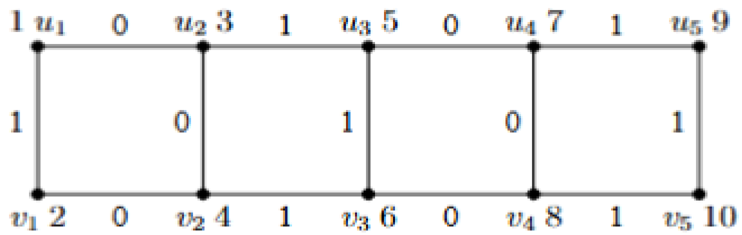

Figure 5

CONCLUSION

In this paper we introduced the concept power mean cordial labeling and here we discussed the power mean cordial labeling behavior of few graphs.

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