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Environmental Transportation Optimization using Modified Earth Mover's Distance Methodology

G. Padma[1], K. Srinivasa Rao[2], N. Ravi Shankar[3]
[1, 3] Department of Mathematics, GSS, GITAM (Deemed to be University), Visakhapatnam, India
[2] Department of Operations, GSB, GITAM (Deemed to be University), Visakhapatnam, India
[*] Corresponding author: Munkhchimeg.O@Humanities.mn

Abstract: The respective study reflects novel phenomenon with optimization for transportation issues within environmental, sciences using the "Intuitionistic Trapezoidal fuzzy number"s (ITFNs) and also an modified "Earth Mover's Distance" (EMD) ranks mechanism. Through capturing the uncertainty and also vagueness in the transportation demands and costs for ITFNs provide more accurate of a modelling framework. EMD enables the comparison and robust ranking of the ITFNs, facilitates informed productivity enhancement and decision-making of the transportation systems. The applications consisting: supply chain management", waste management, sustainable logistics and emergency response, finally contributes towards an efficient and also environmentall friendly "supply chain management" (SCM) and logistics practices.

Keywords: Earth Mover's distance, Intuitionistic trapezoidal fuzzy, numbers, Ranking function, uncertainty, Transportation problems.

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1. Introduction

The problem of transportation [1] is subjected to be longstanding issues for operations research and also process with management science, seeks for the optimization for resource distribution and minimizes costs. However, the traditional solution derives often rely within the crisp data, inherent uncertainty neglectes and ambiguity present within the real-world scenarios [2-10]. This limitation have spurred for the development of the "fuzzy set theory", that introduced through "Lotfi A. Zadeh" in the year of 1965 [13], that reflects a framework for the mathematical for addressing uncertainty and modeling imprecision. Building based on the foundation, researchers have derives an "intuitionistic fuzzy optimization process", pioneered through Angelov [15], this have gained by prominence in the recent years. However, "Intuitionistic Trapezoidal fuzzy number's (ITFNs) have significantly emerged as the powerful tool to capture complex uncertainties and integrated both non-membership and membership degrees [11]. Nevertheless, the comparing ITFNs and ranking remains as challenge, as the conventional process often fail for addressing the subtleties of inherent within the "Intuitionistic Fuzzy Set"s. This bridge the gap, the study proposes with a novel aspect that leverages with the "Earth Mover's Distance" (EMD) as an ranking process for ITFNs within transportation issues. Finally developing in the computer vision of [12], EMD and this measures the dissimilarity among distributions of probability, serves a robust process of mechanism to quantifying the ITFN differences. Through integrating of the ITFNs with a modified EMD, the research depicts a robust aspect of framework to solve transportation problems that remarkably characterized through uncertainty and enables effective informed decision-making phenomenon.

The respective papers have organized bellow: Section 2 depicts preliminaries on the fuzzy concepts, consisting with ""Intuitionistic Trapezoidal fuzzy number"s" and arithmetic operations. Section 3 reflects mathematical formulation of transportation model using linear programming. Calculations of "Earth Mover's Distance" (EMD) for GITFNs and also the modified EMD for GITFNs in transportation problems are reflected within section 4. Farther in Section 5, proposed solution to the transportation problem using modified EMD for ranking GITFNs is presented. Section 6 represents a numerical example to apply the methodology for the transportation problem with "intuitionistic trapezoidal fuzzy" numbers using EMD. Comparative study presented in Section 7. The paper concludes with results and discussion in Section 8.

2. Preliminaries

This respective part covers basic concepts of the fuzzy sets including "intuitionistic trapezoidal fuzzy" numbers and arithmetic phenomenon [16].

Definition 2.1: Fuzzy Set

Let U to be as a universal set. A fuzzy set of $\widetilde{A}$ of U is significantly defined through the membership function "$f_{\widetilde{A}}: U \rightarrow [0,1]$", wherein "$f_{\widetilde{A}}(x)$" reflects membership degree of x within $\widetilde{A}$. The respective fuzzy set $\widetilde{A}$ have reflects as: "$\tilde{A} = \{(x, f_{\tilde{A}}(x))/x \in U\}$."

Definition 2.2: "Intuitionistic Fuzzy Set"

The "Intuitionistic Fuzzy Set" $\tilde{A}$ within the U determines through "$\tilde{A} = \{(x, f_{\tilde{A}}(x), g_{\tilde{A}}(x))/x \in U\}$" wherein "$f_{\tilde{A}}$, and $g_{\tilde{A}}$ are functions from U to [0, 1]" represents effective membership and also a non-membership of "x in U, respectively", such that : "$0 \leq f_{\tilde{A}}(x) + g_{\tilde{A}}(x) \leq 1$, "for the all "$x \in U$".

Definition 2.3: "Intuitionistic Fuzzy Number"

The "Intuitionistic Fuzzy Set" "$\tilde{A} = \{(x, f_{\tilde{A}}(x), g_{\tilde{A}}(x))/x \in U\}$" known as "Intuitionistic Fuzzy Number" on the real line as **R** if this is depicts

1. "Intuitionistic fuzzy normality" "$(\exists\, z \in \boldsymbol{R}, f_{\tilde{A}}(z) = 1 \text{ and} g_{\tilde{A}}(z) = 0)$", 2. "Intuitionistic fuzzy convexity" "$(f_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq Min\ (f_{\tilde{A}}(x), f_{\tilde{A}}(y))$" also "$g_{\tilde{A}}(\lambda x + (1-\lambda)y) \leq Max\ (g_{\tilde{A}}(x), g_{\tilde{A}}(y))$", wherein "$x, y \in U, \lambda \in [0,1]$", 3. "$f_{\tilde{A}}(x)$ and $g_{\tilde{A}}(x)$" are effectively piecewise for continuous value of real-valuedoptimization, 4. Support for the $\tilde{A}$is bound.

Definition 2.4: "Intuitionistic Trapezoidal Fuzzy Number"

An "Intuitionistic Fuzzy Number" $\tilde{A}$ is known as "Intuitionistic Trapezoidal fuzzy number" (ITFN) is denoted through "$\tilde{A} = (a_1, a_2, a_3, a_4)(a'_1, a_2, a_3, a'_4)$" to the function of membership as "$f_{\tilde{A}}$ and non-membership function $g_{\tilde{A}}$ "defined by

$$\text{“}f_{\tilde{A}} = \begin{cases} 0, x < a_1 \\ \frac{x-a_1}{a_2-a_1},\ a_1 \leq x \leq a_2 \\ 1, a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4},\ a_3 \leq x \leq a_4 \\ 0, a_4 < x\text{"} \end{cases} \quad \text{and "}g_{\tilde{A}} = \begin{cases} 0, x < a'_1 \\ \frac{x-a_2}{a'_1-a_2}, a'_1 \leq x \leq a_2 \\ 1, a_2 \leq x \leq a_3 \\ \frac{x-a_3}{a'_4-a_3}, a_3 \leq x \leq a'_4 \\ 0. a'_4 < x\text{"} \end{cases}$$

Definition 2.5: Generalized of the ITFN

Generalized "Intuitionistic Trapezoidal fuzzy number" is reflected through

"$\tilde{A} = (a_1, a_2, a_3, a_4; \omega_a)(a'_1, a_2, a_3, a'_4; \sigma_a)$"With the membership function ": $f_{\tilde{A}}$ and non-

Membership function $g_{\tilde{A}}$" defined through

$$\text{"}f_{\tilde{A}} = \begin{cases} 0, x < a_1 \\ \frac{x-a_1}{a_2-a_1}\omega_a,\ a_1 \leq x \leq a_2 \\ \omega_a, a_2 \leq x \leq a_3 \\ \frac{x-a_4}{a_3-a_4}\omega_a, a_3 \leq x \leq a_4 \\ 0, a_4 < x\text{"} \end{cases} \quad \text{and "}g_{\tilde{A}} = \begin{cases} 0, x < a'_1 \\ \frac{x-a_2}{a'_1-b_2}\sigma_a, a'_1 \leq x \leq a_2 \\ \sigma_a, a_2 \leq x \leq a_3 \\ \frac{x-a_3}{a'_4-a_3}\sigma_a, a_3 \leq x \leq a'_4 \\ 0. a'_4 < x\text{"} \end{cases}$$

Where "ω_a and σ_a correspond" towards a high range of the contribution and also a low level of the non-contribution and "$0 \leq \omega_a \leq 1, 0 \leq \sigma_a \leq 1, 0 \leq \omega_a + \sigma_a \leq 1$".

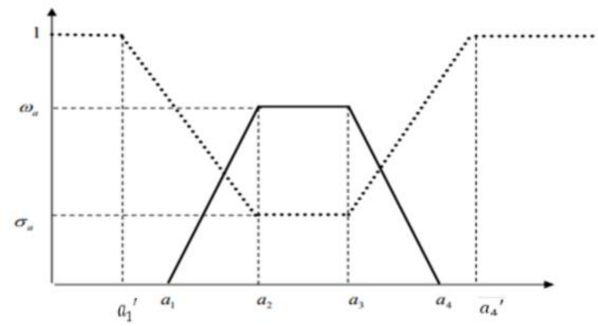

Figure 1: Generalized ITFN

Arithmetic Operations:

Let$\widetilde{A} = \langle (a_1, a_2, a_3, a_4; \omega_a)\ (a'_1, a_2, a_3, a'_4; \sigma_a) \rangle$ *and* $\tilde{B} = \langle (b_1, b_2, b_3, b_4; \omega_b)\ (b'_1, b_2, b_3, b'_4; \sigma_b) \rangle$ be Generalized trapezoidal "Intuitionistic Fuzzy Number"s. Then the arithmetic operations are

(i) Addition:

$$"\widetilde{A} \oplus \tilde{B} = \begin{Bmatrix} (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; min(\ \omega_a, \omega_b)) \\ (a'_1 + b'_1, a_2 + b_2, a_3 + b_3, a'_4 + b'_4; max(\ \sigma_a, \sigma_b)) \end{Bmatrix}"$$

(ii) Subtraction:

$$"\widetilde{A} \Theta \tilde{B} = \begin{Bmatrix} (a_1 - b_4, a_2 - b_2, a_3 - b_3, a_4 - b_1; min(\ \omega_a, \omega_b)) \\ (a'_1 - b'_4, a_2 - b_2, a_3 - b_3, a'_4 - b'_1; max(\ \sigma_a, \sigma_b)) \end{Bmatrix}"$$

(iii) Scalar Multiplication:

$$"k \otimes \tilde{A} = \begin{Bmatrix} (ka_1, ka_2, ka_3, ka_4; \omega_a)(ka'_1, ka_2, ka_3, ka'_4; \sigma_a)\ if\ k \geq 0 \\ (ka_4, ka_3, ka_2, ka_1; \omega_a)(ka'_4, ka_3, ka_2, ka'_1; \sigma_a)\ if\ k < 0 \end{Bmatrix}"$$

(iv) Multiplication:

$$"\widetilde{A} \otimes \tilde{B} = \begin{Bmatrix} (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; min(\ \omega_a, \omega_b))) \\ (a'_1 b'_1, a_2 b_2, a_3 b_3, a'_4 b'_4; max(\ \sigma_a, \sigma_b))) \end{Bmatrix}"$$

3. Mathematical Formulation of Transportation Model using Linear Programming

Mathematically, the fuzzy of transportation issues in the Table 1 stated as follows:

Minimize "$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$,"

subject to "$\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1,2, \ldots, m$"

$$"\sum_{i=1}^{m} x_{ij} \geq b_j, j = 1,2, \ldots, n"$$

and the non-negative restrictions, "x_{ij} (i=1,2,...,m; j = 1,2,...,n)" are vectors.

Table 1: Transportation problem

Destination Sources	D_1	D_2	…	D_n	Supply a_i
S_1	c_{11}	c_{12}	…	c_{1n}	a_1
S_2	c_{21}	c_{22}	…	c_{2n}	a_2
. . .	. . .	. . .	. . .	. . .	. . .
S_m	c_{m1}	c_{m2}	…	c_{mn}	a_m
Demand b_j	b_1	b_2	…	b_n	$\sum_i a_i$=$\sum_j b_j$

where, a_i : Quantity of sources of materials availability at Source(S_i ,i =1,2,...,m)

b_j: Quantity of sources of material required at destination (D_j, j =1,2,...,n)

c_{ij}: Unit cost of transformation from sources S_i to destination D_j.

4. Earth Mover's Distance (EMD)

In this section, we explain the calculation of EMD between two generalized intuitionistic trapezoidal fuzzy numbers and its application in a transportation problem.

4.1 Earth Mover's Distance between Two GITFN [12]

EMD between two GITFN's $\tilde{A}$ and $\tilde{B}$ can be calculated as follows :

1. Compute membership function and non-membership function for both $\tilde{A}$ and $\tilde{B}$.
2. Compute cumulative distribution functions for both $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ and $\nu_{\tilde{B}}$

i.e., "$F_{\mu_{\tilde{A}}}(x)$" and "$F_{\nu_{\tilde{A}}}(x)$"

"$F_{\mu_{\tilde{B}}}(x)$" and "$F_{\nu_{\tilde{B}}}(x)$"

3. Compute EMD for both A and B

$$"EMD_\mu(\tilde{A},\tilde{B}) = \int_{-\infty}^{\infty} |\, F_{\mu_{\tilde{A}}}(x) - F_{\mu_{\tilde{B}}}(x)|\, dx"$$

$$"EMD_\nu(\tilde{A},\tilde{B}) = \int_{-\infty}^{\infty} |\, F_{\nu_{\tilde{A}}}(x) - F_{\nu_{\tilde{B}}}(x)|\, dx"$$

4. Total EMD is calculated as "EMD ($\tilde{A}$,$\tilde{B}$) = $\frac{EMD_\mu(\tilde{A},\tilde{B})+EMD_\nu(\tilde{A},\tilde{B})}{2}$"

In the context of transportation problem, EMD can be employed to measure the dissimilarity between supply and demand distributions, ensuring minimum transportation. EMD is particularly useful when dealing with fuzzy numbers, as it can effectively handle the inherent uncertainty and variability.

4.2 Modified Earth Mover's Distance (EMD) Calculation for GITFNs in Transportation Problems

Consider "Generalized Intuitionistic Trapezoidal Fuzzy Number" (GITFN) represented by"$A = (a_1, a_2, a_3, a_4; \omega_a)(a'_1, a_2, a_3, a'_4; \sigma_a)$" with membership function f_A and non-membership function g_A (as defined in Definition 2.5).

Case (i) : Compute the cumulative distribution functions for both membership functionF_{f_A} and non-membership

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function$F_{g_A}$and calculate for each range using the formula " $\int_{-\infty}^{\infty} |F_{f_A}(x) - F_{g_A}(x)|^2 \ dx$"and then calculate the EMD using the formula EMD(A) = " $\int_{-\infty}^{\infty} |F_{f_A}(x) - F_{g_A}(x)|^2 \ dx$" for each cell in the transportation problem.

Case (ii) : Compute the cumulative distribution functions for both of the membership functionF_{f_A} and non-membership function$F_{g_A}$and calculate for each range using the formula " $\int_{-\infty}^{\infty} | F_{f_A}(x) - F_{g_A}(x)| \, dx$" and then calculate the EMD using the formula EMD(A) = " $\int_{-\infty}^{\infty} | F_{f_A}(x) - F_{g_A}(x)| \, dx \times \sigma_a \times \frac{a_1+a_2+a_3+a_4}{4}$" for each cell in the transportation problem.

Comparison of EMD for Each Cell in the Transportation Problem

To compare the EMD values of two Generalized Intuitionistic Trapezoidal Fuzzy Numbers (GITFNs), GITFN1 and GITFN2, compute their respective EMD values.

The comparison are as follows:

(i) If "EMD(GITFN1) - EMD(GITFN2) > 0", then GITFN1 is considered greater than GITFN2.

(ii) If "EMD(GITFN1) - EMD(GITFN2) < 0", then GITFN1 is considered less than GITFN2.

(iii) If "EMD(GITFN1) - EMD(GITFN2) = 0", then GITFN1 and GITFN2 are considered equal.

In the context of a transportation problem, the EMD values can be used to rank different transportation options or routes. A lower EMD value indicates a more reliable or efficient transportation option.

4.3 Initial Basic Feasible Solutions

"North West Corner Rule":

Step 1: Northwest corner have given to a method because variable have determines with the help of north-west "(i.e. top left corner)".

Step 2: Select extreme northwest side of respective table and also allocating the maximum possible value.

Step 3: Subtract the value from the row & column. If any one of the values either availability or demand is satisfied cross (strike) it and move to next cell.

Step 4: Move to right-hand side and make the second allocation to the extreme northwest cell and this process continues until the availability and demand values are 0.

Least Cost Method:

Step 1: Select cell having least value and allocate how much possible.

Step 2: Subtract the selected value from the availability and demand.

Step 3: If either the row or column allocation is '0' strike it off and continue the process through availability and also the value of demand 0.

"Vogel's Approximation Method":

Step-1: Take difference for first lowest cost, second lowest cost for all the rows and columns. These differences are called as penalties.

Step-2: Select a high penalty (any row, column). Check lowest amount of cost in row and allocate maximum level of possible units and remove that row and column.

Step-3: Repeat steps 1 and 2 until all allocations (assignments) have been made.

4.4 Optimal Solution

"Modified Distribution (MODI) Method"

Step-1: Find the initial Solution of basic feasible through using the methods like "North-west corner", "Least cost method", "Vogel's approximation method".

Assign variables u_i's to rows, v_j's to columns $|u_i + v_j = c_j"|$ cells.

Step-2: Solve the equations in step-1 to find "u_i'sv_j's (u_i =v_j=0 initially)".

Step-3: For non-basic cells find "$d_{ij} = c_{ij} - u_i + v_j$"

Step-4: If $d_{ij} \leq 0$, then basic Solution feasiblly is also optimal and at least "$d_{ij} = 0$", then alternate existence of optimal solution.

Step-5: If at least one $d_{ij} > 0$, then basic feasible Solution is not optimal. Now select largest positive d_{ij} and to maintain total basic "(m + n - 1)", one of existing variables must be removed.

Step-6: Start from the cell of entering basic variable and move horizontally and vertically, take turns only at a basic cell and return to the starting cell to get a closed loop. This loop will have corners as basic cells.

Step-7: Let θ be the maximum units that can be allocated at the entering variable cell. To maintain row and column totals, subtract θ and add θ from the row edges and column edges respectively. Find the minimum value of θ which is minimum of allocations made at the corners of the loop from which have been θ subtracted. This can be new feasible solution.

Step-8: Test the new feasible Solution obtained in step-7 using step-2, 3, and 4. If it is not optimal, then repeat steps 5, 6, 7 until optimality is achieved.

5. Proposed the Transportation Problem Solution through Earth Mover's Distance for Ranking Generalized Intuitionistic "Trapezoidal Fuzzy Numbers"

Efficient transportation of fuzzy solution of generalised intuitionistic transportation problem is obtained by propose process. Proposed process are bellow:

Step 1: Formulate the Generalized "Intuitionistic Transportation Problem"

Generalized "intuitionistic transportation problem" of m sources and n, where each source i has an availability (supply) of "$\tilde{a}_i$ (i = 1, 2, ..., m)" and each destination j has a demand of "$\tilde{b}_j$ (j = 1, 2, ..., n)". If the problem of transportation is balanced ($\sum_i a_i$=$\sum_j b_j$), proceed to Step 2. Introduce a dummy rows with zero number of "intuitionistic fuzzy costs" towards creating a balancing transportation problem.

Step 2: Convert towards "Linear Programming Problem:"

Convert intuitionistic problem for transportation to the issues of linear programme:

Minimizes "$z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij},$"

subject towards "$\sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{a}_i, i = 1,2,\ldots,m$"

$$"\sum_{i=1}^{m} \tilde{x}_{ij} \geq \tilde{b}_j, j = 1,2,\ldots,n"$$

and the "non-negative restrictions", "$\tilde{x}_{ij}$ (i=1,2,...,m; j = 1,2,...,n)" are vectors.

Where $\tilde{c}_{ij}$ ("i=1,2,...,m; j=1,2,...n") represents fuzzy costs of transportation expressed as generalizing "intuitionistic trapezoidal fuzz"y number.

Step 3: Apply EMD as a Ranking Function

Convert linear programming through the Step number 2 with in a new transportation problem using ranking function**EMD,** facilitatedby the "Earth Mover's Distance" (EMD) in section 4**:**

Minimize "$z = \sum_{i=1}^{m} \sum_{j=1}^{n} \boldsymbol{EMD}(\tilde{c}_{ij})\, \tilde{x}_{ij},$"

subject to "$\sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{a}_i, i = 1,2,\dots,m$"

$$"\sum_{i=1}^{m} \tilde{x}_{ij} \geq \tilde{b}_j\,, j = 1,2,\dots,n"$$

and the non-negative restrictions, "$\tilde{x}_{ij}$(i=1,2,...,m; j = 1,2,...,n) ≥ 0"

For each intuitionistic trapezoidal fuzzy number in the cell of a transportation issues, need to identify intervals where the membership function changes. We can take the midpoints of each range and the associated membership values. For each discrete point (x), the cumulative distribution function (CDF) is the sum of all previous membership values up to that point.

Step 4 : Convert to Crisp Numbers

Apply the EMD to convert generalized intuitionistic fuzzy numbers to the crisp numbers.

Step 5: "Solve Linear Programming Problem"

Solving problem of linear programming Step 4 through linear programming techniquesto obtain the values"$\tilde{x}_{ij}$ (i = 1,2,,,,,m ; j=1,2,...,n)". Non- zero values $\tilde{x}_{ij}$represents allocations, while zero values indicate no allocation. Also, obtain the minimum total cost.

Step 6: Obtain Minimum Cost in Generalized Intuitionistic Fuzzy Form

Using the allocations obtained in Step 5, calculate the minimum cost using arithmetic operations in terms of generalized intuitionistic fuzzy numbers.

6. Numerical Example

Generalizing the ITFN transportation issues, reflects by "Agarwal et al.[19]" (Table number 2) to address proposed phenomenon. The issue derives total three different aspects "$S_1, S_2, and\ S_3$" and three destinations "$D_1, D_2, and\ D_3$".

Step 1: Formulate the "Intuitionistic Transportation Problem" (Table 2)

Table 2: The generalization of Intuitionistic for the "Trapezoidal Fuzzy Numbers Transportation Problem"

Supply Demand	D_1	D_2	D_3	Supply s_i
S_1	("2,4,8,15;0.6") ("1,4,8,18;0.3")	("3,5,7,12;0.5") ("1,5,7,15;0.3")	("2,5,9,16;0.7") ("1,5,9,18;0,3")	25
S_2	("2,5,8,10;0.6") ("1,5,8,12;0.2")	("4,8,10,13;0.4") ("3,8,10,15,0.3")	("4,8,10.13;0.4") ("3,8,10,15;0.3")	30
S_3	("2,7,11,15;0.5") ("1,7,11,18;0.3")	("5,9,12,16;0.7") ("3,9,12,19;0.2")	("4,6,8,10;0.6") ("3,6,8,12;0.3")	40
Demand d_j	35	45	15	**95**

Step 2: Convert to Linear Programming Problem

Minimize "$\tilde{z}$ = $\tilde{c}_{11}\otimes\tilde{x}_{11}\oplus\tilde{c}_{12}\otimes\tilde{x}_{12}\oplus\tilde{c}_{13}\otimes\tilde{x}_{13}\oplus\tilde{c}_{21}\otimes\tilde{x}_{21}\oplus\tilde{c}_{22}\otimes\tilde{x}_{22}\oplus\tilde{c}_{23}\otimes\tilde{x}_{23}\oplus\tilde{c}_{31}\otimes\tilde{x}_{31}\oplus\tilde{c}_{32}\otimes\tilde{x}_{32}\oplus\tilde{c}_{33}\otimes\tilde{x}_{33}$"

subject to

"$\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\leq a_1$

$\tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23}\leq a_2$

$\tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33}\leq a_3$

$\tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31}\geq b_1$

$\tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32}\geq b_2$

$\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33}\geq b_3$"

Using Table 2,

Minimize "z = (2,4,8,15;0.6)(1,4,8,18;0.3)$\otimes\tilde{x}_{11}\oplus$(3,5,7,12;0.5)(1,5,7,15;0.3)$\otimes\tilde{x}_{12}\oplus$(2,5,9,16;0.7)(1,5,9,18; 0,3)$\otimes\tilde{x}_{13}\oplus$(2,5,8,10;0.6)(1,5,8,12;0.2)$\otimes\tilde{x}_{21}\oplus$(4,8,10,13;0.4)(3,8,10,15,0.3)$\otimes\tilde{x}_{22}\oplus$(4,8,10.13;0.4)(3,8,10,15;0.3)$\otimes$ $\tilde{x}_{23}\oplus$(2,7,11,15;0.5)(1,7,11,18;0.3)$\otimes\tilde{x}_{31}\oplus$(5,9,12,16;0.7)(3,9,12,19;0.2)$\otimes\tilde{x}_{32}\oplus$(4,6,8,10;0.6)(3,6,8,12;0.3)$\otimes\tilde{x}_{33}$"

subject to

"$\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\leq 25$

$\tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23}\leq 30$

$\tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33}\leq 40$

$\tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31}\geq 35$

$\tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32}\geq 45$

$\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33}\geq 15$"

Step 3: Apply Ranking Function as EMD

Minimize "z = EMD $(\tilde{c}_{11})\otimes\tilde{x}_{11}\oplus$ EMD $(\tilde{c}_{12})\otimes\tilde{x}_{12}\oplus$EMD $(\tilde{c}_{13})\otimes\tilde{x}_{13}\oplus$EMD $(\tilde{c}_{21})\otimes\tilde{x}_{21}\oplus$EMD$(\tilde{c}_{22})\otimes$ $\tilde{x}_{22}\oplus$EMD$(\tilde{c}_{23})\otimes\tilde{x}_{23}\oplus$EMD $(\tilde{c}_{31})\otimes\tilde{x}_{31}\oplus$EMD$(\tilde{c}_{32})\otimes\tilde{x}_{32}\oplus$EMD$(\tilde{c}_{33})\otimes\tilde{x}_{33}$"

subject to

"$\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\leq a_1$

$\tilde{x}_{21}\oplus\tilde{x}_{22}\oplus\tilde{x}_{23}\leq a_2$

$\tilde{x}_{31}\oplus\tilde{x}_{32}\oplus\tilde{x}_{33}\leq a_3$

$\tilde{x}_{11}\oplus\tilde{x}_{21}\oplus\tilde{x}_{31}\geq b_1$

$\tilde{x}_{12}\oplus\tilde{x}_{22}\oplus\tilde{x}_{32}\geq b_2$

$\tilde{x}_{13}\oplus\tilde{x}_{23}\oplus\tilde{x}_{33}\geq b_3$"

It becomes,

Minimize "z = EMD[(2,4,8,15;0.6)(1,4,8,18;0.3)]$\otimes\tilde{x}_{11}\oplus$EMD[(3,5,7,12; 0.5)(1,5,7,15;0.3)]$\otimes\tilde{x}_{12}\oplus$ EMD [(2,5,9,16;0.7)(1,5,9,18;0,3)]$\otimes\tilde{x}_{13}\oplus$EMD[(2,5,8,10;0.6)(1,5,8,12;0.2)]$\otimes\tilde{x}_{21}\oplus$EMD[(4,8,10,13;0.4)(3,8,10,15,0.3)]$\otimes$ $\tilde{x}_{22}\oplus$EMD[(4,8,10.13;0.4)(3,8,10,15;0.3)]$\otimes\tilde{x}_{23}\oplus$[(2,7,11,15;0.5)(1,7,11,18;0.3)]$\otimes\tilde{x}_{31}\oplus$[(5,9,12,16;0.7)(3,9,12,19;0 .2)]$\otimes\tilde{x}_{32}\oplus$EMD[(4,6,8,10;0.6)(3,6,8,12;0.3)]$\otimes\tilde{x}_{33}$"

subject to

"$\tilde{x}_{11}\oplus\tilde{x}_{12}\oplus\tilde{x}_{13}\leq 25$

$\tilde{x}_{21} \oplus \tilde{x}_{22} \oplus \tilde{x}_{23} \leq 30$

$\tilde{x}_{31} \oplus \tilde{x}_{32} \oplus \tilde{x}_{33} \leq 40$

$\tilde{x}_{11} \oplus \tilde{x}_{21} \oplus \tilde{x}_{31} \geq 35$

$\tilde{x}_{12} \oplus \tilde{x}_{22} \oplus \tilde{x}_{32} \geq 45$

$\tilde{x}_{13} \oplus \tilde{x}_{23} \oplus \tilde{x}_{33} \geq 15$"

Step 4: Convert to Crisp Numbers Using EMD

Minimize "$z = 4.86\, x_{11} + 3.97\, x_{12} + 5.46\, x_{13} + 3.14\, x_{21} + 2.99\, x_{22} + 6.06\, x_{23} + 4.3\, x_{31} + 5.43\, x_{32} + 2.61\, x_{33}$"

subject to

"$x_{11} + x_{12} + x_{13} \leq 25$

$x_{21} + x_{22} + x_{23} \leq 30$

$x_{31} + x_{32} + x_{33} \leq 40$

$x_{11} + x_{21} + x_{31} \geq 35$

$x_{12} + x_{22} + x_{32} \geq 45$

$x_{13} + x_{23} + x_{33} \geq 15$"

Step 5: Solve "Linear Programming Problem"

Solving linear program for step 4, we get "$x_{11} = 0$" , "$x_{12} =$ **25**" , "$x_{13} = 0$" , "$x_{21} =$ **30**", "$x_{22} = 0$", "$x_{23} = 0$", "$x_{31} =$ **5**", "$x_{32} =$ **20**", "$x_{33} =$ **15**".

"m+n-1 = 3 +3-1 = 5" allocations, "$x_{12} = 25$" ,$x_{21} = 30$" , "$x_{31} = 5$" , "$x_{32} = 20$", "$x_{33} = 15$".

Step 6: Obtain Minimum Cost in Intuitionistic Fuzzy Form

Minimum Total Cost =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes x_{ij} = \tilde{c}_{12} \otimes x_{12} \oplus \tilde{c}_{21} \otimes x_{21} \oplus \tilde{c}_{31} \otimes x_{31} \oplus \tilde{c}_{32} \otimes x_{32} \oplus \tilde{c}_{33} \otimes x_{33}$$

$= (3,5,7,12;0.5)(1,5,7,15;0.3) \otimes \mathbf{25} \oplus (2,5,8,10;0.6)(1,5,8,12;0.2) \otimes \mathbf{30} \oplus (2,7,11,15;0.5)(1,7,11,18;0.3) \otimes \mathbf{5} \oplus (5,9,12,16;0.7)(3,9,12,19;0.2) \otimes \mathbf{20} \oplus (4,6,8,10;0.6)(3,6,8,12;0.3) \otimes \mathbf{15}$

$= (305,580,830,1145;0.5)\ (165,580,830,1385;0.3)$

7. Comparatively Aspect

The fuzzy optimal costs are derives through used method proposed and also it includes through "Indira and Shankar" [18] also identical. An comparative study present within Table 3.

Table 3: "Comparative Table"

Example	Procedure of Ranking	"Fuzzy Transportation Method"	Fuzzy Optimal Cost
"Agarwal et al.[19]"	"Pardha Saradhi et al. [20]"	"Indira and Shankar [18]"	(305,580,830,1145;0.5) (165,580,830,1385;0.3)
Agarwal et al.[19]	EMD method-1	Proposed Method	(305,580,830,1145;0.5)

			(165,580,830,1385;0.3)
Agarwal et al.[19]	EMD method-2	Proposed Method	(305,580,830,1145;0.5) (165,580,830,1385;0.3)

8. Discussion and Results

The process through through "Generalized Intuitionistic Trapezoidal Fuzzy Numbers" (GITFNs) and Modified "Earth Mover's Distance" (EMD) (Case (i) and Case (ii))were applied in the transportation issues. The comparative approaches have conducted for evaluating the costs of transportation through using proposed process and the existing methodology. The results evaluated the identification results of the methods. The respective findings also focuses to demonstrating that influences and also validity have proposed through "GITFN-based optimization" process to solve the issue of transportation.

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