International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 18s, 2025
https://theaspd.com/index.php

Topological Characteristics Of Rough Finite Automata

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Abstract
This paper aims to explore the attributes of a rough finite state automaton, includ- ing separatedness, connectedness, and retrievability. It explores the interconnections between these properties, providing insights into their relationships. Additionally, this work introduces the concept of the product of two rough finite state automata, adding a novel dimension to our understanding of these computational constructs.

INTRODUCTION

In the realm of classical computation, mathematical models involving automata have held significant importance within theoretical computer science. Broadly, an automaton can be defined as a triad ($\mathcal{Q}$, $\mathcal{X}$, δ), where $\mathcal{Q}$ represents the set of states, $\mathcal{X}$ stands for the set of input symbols, and $\delta : \mathcal{Q} \times \mathcal{X} \to \mathcal{Q}$ denotes the transition relation. In an innovative exploration, Bavel [2, 3] outlined the core of source and successor operators for the state set. In addition, Bavel proposed fascinating ideas and intriguing concepts, including subautomata, separability, retrievability, and homomorphism [4]. Importantly, as elucidated by Malik and Mordeson [6], the successor operator serves a crucial role as an upper approximation operator within approximation spaces and finds utility in topology, and (fuzzy) abstract algebra. After Bavel's research, Shukla and Srivastava [12] discerned that the source operator functions as a closure operator, leading to a broader topology for $\mathcal{Q}$. It is addition- ally asserted [2] that an investigation into these principles of automata contributes organically to an enhanced understanding of automata structure and their practical applications. These concepts have evolved from a quest to comprehend system behavior in different environments [5] and have played a substantial role in shaping the foundational aspects of computer science. On the account of fuzzy automata [14], fuzzy finite state machines were introduced, and Malik and Mordeson initiated the algebraic study of such machines [7, 8] and further studied in [10].
Pawlak's rough set theory [9], analogous to fuzzy set theory, presents a math- ematical framework for managing imperfect knowledge. It has swiftly gained recognition among mathematicians and computer scientists for its utility in mod- eling various aspects of artificial intelligence and cognitive sciences, especially in the domains of knowledge acquisition, decision analysis, and expert systems Subsequently, following the advent of rough set theory, Basu introduced [1] the concept of a rough finite state (semi)automaton and extended this concept by de- vising a recognizer capable of accepting imprecise statements. The idea of source and successor was broadened to encompass rough automata [11], leading to an exploration of properties such as separatedness, connectedness, and retrievability.

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These investigations facilitated the characterization of the topology inherent to rough automata [13].

Preliminaries

In this section, we recall rough automata and the concepts of source and successors related to the state set of the rough automata.

Definition 2.1 *A rough finite state automaton (RFSA) is defined as a four-tuple* $(Q, R, \mathcal{X}, \delta)$, *where* Q *is the finite set of states,* R *is an equivalence relation on* Q, *and* $\mathcal{X}$ *is the set of input symbols. The map* $\delta : Q \times \mathcal{X} \longrightarrow \boldsymbol{D} \times \boldsymbol{D}$ *is called the rough transition map, where* $\boldsymbol{D}$ *denotes the class of all definable sets in the approximation space* (Q, R) *such that for all* $q \in Q$ *and* $x \in \mathcal{X}$, $\delta(q, x) = (\underline{\delta(q, x)}, \overline{\delta(q, x)})$.

Definition 2.2 *For any equivalence class* $[q] \in Q/R$ *and* $x \in \mathcal{X}$, *the transition map* δ *is defined as*

$$\delta([q], x) = (\underline{\delta([q], x)}, \overline{\delta([q], x)}), \quad \text{where}$$

$$\underline{\delta([q], x)} = \cup\{\underline{\delta(p, x)} \mid p \in [q]\}, \ \overline{\delta([q], x)} = \cup\{\overline{\delta(p, x)} \mid p \in [q]\}.$$

Definition 2.3 *Let* $(Q, R, \mathcal{X}, \delta)$ *be a RFSA. Then for a definable set* D *in* (Q, R), *the rough transition* δ *can be extended to* $\delta^D : \boldsymbol{D} \times \mathcal{X} \longrightarrow \boldsymbol{D} \times \boldsymbol{D}$, *called the block transition map such that for* $D' \in \boldsymbol{D}$ *and* $x \in \mathcal{X}$

$$\delta^D(D', x) = (\underline{\delta^D(D', x)}, \overline{\delta^D(D', x)}), \quad \text{where}$$

$$\underline{\delta^D(D', x)} = \cup\{\underline{\delta(q, x)} \mid q \in [p] \subseteq D'\}, \ \overline{\delta^D(D', x)} = \cup\{\overline{\delta(q, x)} \mid q \in [p] \subseteq D'\}.$$

Definition 2.4 *Let* $(Q, R, \mathcal{X}, \delta)$ *be a RFSA. The transition map* δ *can be extended to* $\delta : Q \times \mathcal{X}^* \longrightarrow \boldsymbol{D} \times \boldsymbol{D}$ *such that for* $q \in Q$, $w \in \mathcal{X}^*$, *and* $x \in \mathcal{X}$

$$\delta(q, wx) = (\underline{\delta(q, wx)}, \overline{\delta(q, wx)}), \quad \text{where}$$

$$\underline{\delta(q, wx)} = \underline{\delta^D}(\underline{\delta(q, w)}, x), \quad \overline{\delta(q, wx)} = \overline{\delta^D}(\overline{\delta(q, w)}, x).$$

Definition 2.5 [1] Let $(Q, R, \mathcal{X}, \delta)$ be a RFSA and $p, q \in Q$. Then q is called a **definite successor** (resp. **possible successor**) of p if $\exists\, w \in \mathcal{X}^*$ such that $q \in \underline{\delta(p, w)}$ or $[q] \subseteq \underline{\delta^D([p], w)}$ (resp. $q \in \overline{\delta(p, w)}$ or $[q] \subseteq \overline{\delta^D([p], w)}$).

Definition 2.6 [1] Let $(Q, R, \mathcal{X}, \delta)$ be a RFSA and $T \subseteq Q$. The **definite successor** and **possible successor** of T are respectively defined as the sets

$DS(T) = \bigcup\{DS(q) : q \in T\}$ and

$PS(T) = \bigcup\{PS(q) : q \in T\}$,

where $DS(q) = \{p : p \in \underline{\delta(q, w)}$, for some $w \in \mathcal{X}^*\}$ and $PS(q) = \{p : p \in \overline{\delta(q, w)}$, for some $w \in \mathcal{X}^*\}$.

Analogous to the above concept of a definite (possible) successor, we now introduce the notion of a

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definite (possible) source.

Definition 2.7 *Let* (Q, R, X, δ) *be a RFSA and* $p, q \in Q$. *Then* q *is called a* ***definite source*** *(resp.* ***possible source****) of* p *if* $\exists\ w \in X^*$ *such that* $p \in \underline{\delta(q, w)}$ *or* $[p] \subseteq \delta^D([q], w)$ *(resp.* $p \in \overline{\delta(q, w)}$ *or* $[p] \subseteq \overline{\delta^D}([q], w)$*).*

Definition 2.8 *Let* (Q, R, X, δ) *be a RFSA and* $T \subseteq Q$. *The* ***definite source*** *and* ***possible source*** *of* T *are respectively defined as the sets*
$DS'(T) = \bigcup\{DS'(q) : q \in T\}$ *and*
$PS'(T) = \bigcup\{PS'(q) : q \in T\}$,
where $DS'(q) = \{p : q \in \underline{\delta(p, w)}$*, for some* $w \in X^*\}$ *and* $PS'(q) = \{p : q \in \overline{\delta(p, w)}$*, for some* $w \in X^*\}$.

Definition 2.9 *An RFSA* (Q', R', X, δ') *is called a definite rough sub automa- ton (resp. possible rough sub automaton) of an RFSA* (Q, R, X, δ) *if* $Q' \subseteq Q$, $DS(Q') = Q'$, $R' = R|_Q$ *and* $\delta' = \delta|_{Q' \times X}$ *(resp.* $PS(Q') = Q'$, $R' = R|_Q$ *and* $\delta' = \delta|_{Q' \times X}$*).*

Swapping inputs in an automaton is a distinctive and advantageous feature. Many have explored abelian automata, and their findings can be enhanced by the insights in the following theorems about rough automata's structure.

Definition 2.10 *A RFSA* $M = (Q, R, X, \delta)$ *is called Abelian if and only if*

$(\underline{\delta(q, xy)}, \overline{\delta(q, xy)}) = (\underline{\delta(q, yx)}, \overline{\delta(q, yx)})$,
$\forall q \in Q$ *and* $\forall x, y \in X$.

Theorem 2.1 *The sets of states of disjoint definite rough sub automata have the disjoint definite sources in an Abelian RFSA.*

Proof: Let $M = (Q, R, X, \delta)$ be an Abelian RFSA and $N1$, $N2$ are two definite rough sub automata of M such that $N1 \cap N2 = \emptyset$. Now, for any $q \in DS'(N1) \cap DS'(N2)$, $q \in DS'(p)$, for some $p \in N1$ and $q \in DS'(r)$, for some $r \in N2$. Then there exists $x, y \in X^*$ such that $p \in \underline{\delta(q, x)}$ and $r \in \underline{\delta(q, y)}$, or that, $p \in \underline{\delta(q, xy)}$ and $r \in \underline{\delta(q, yx)}$. But since M is Abelian, $\underline{\delta(q, xy)} = \underline{\delta(q, yx)}$, which contradicts
the disjointness of $N1$ and $N2$.

Theorem 2.2 *The sets of states of disjoint possible rough sub automata have disjoint possible sources in an Ableian RFSA.*

Proof: The proof is similar to that of theorem 2.1.

Lemma 2.1 *Let* $N = (Q', R', X, \delta')$ *be the definite rough sub automaton of an RFSA* $M = (Q, R, X, \delta)$ *and* $T \subseteq Q'$. *Then*
$DS'(T) = \{p \in Q' : DS(p) \cap T \neq \emptyset\}$
(resp. $PS'(T) = \{p \in Q' : PS(p) \cap T \neq \emptyset$

Proof: Follows from the definition of definite (possible) source.

Lemma 2.2 *Let* $N = (Q', R', X, \delta')$ *be the definite rough sub automaton of an* $RFSA$ $M = (Q, R, X, \delta)$ *and* C *be a collection of subsets of* Q'. *Then for* $A1, A2 \in C$,

(i) (*a*) $DS'(A1) - DS'(A2) \subseteq DS'(A1 - A2)$;
(b) $PS'(A1) - PS'(A2) \subseteq PS'(A1 - A2)$;
(ii) (*a*) $Q_N - DS'(A_1) \subseteq DS'(Q_N - A_1)$;
(b) $Q_N - PS'(A_1) \subseteq PS'(Q_N - A_1)$;

Proof: (i) (a) Let $q \in DS'(A1) - DS'(A2)$. Then $q \in DS'(A1)$ and $q \notin DS'(A2)$, i.e., $q \in Q_N$, $DS(q) \cap A_1 \neq \phi$ and $DS(q) \cap A_2 = \phi$, or that, $\exists p \in DS(q)$ and $p \in A_1$, but $p \notin A_2$. Thus $q \in Q_N$, $p \in DS(q)$ and $p \in (A_1 - A_2)$. Hence $q \in Q_N$ and $DS(q) \cap (A1 - A2) \neq \phi$, showing that $q \in DS'(A1 - A2)$. (b) The second part follows similarly.
(ii) (a) Let $q \in Q_N - DS'(A_1)$. Then $q \in Q_N$ and $q \notin DS'(A_1)$, i.e., $q \in Q_N$, $DS, (q) \cap A_1 = \phi$, or that, $\exists p \in DS'(q)$ but $p \notin A_1$. Thus $q \in Q_N$, and $DS'(q) \cap (Q_N - A_1) \neq \phi$, showing that $q \in DS'(Q_N - A_1)$. (b) The second part follows similarly.

Lemma 2.3 *Let N be a definite rough subautomaton of M and let $A \subseteq Q_N$. Then*
$DS(A) = \{q \in Q : DS'_N(q) \cap A \neq \emptyset\} = \{q \in Q_N : DS'_N(q) \cap A \neq \emptyset\}$.
Proof: Since $A \subseteq Q_N$, $Q_N \cap A = A$. Then, $DS'_N(q) \cap A = DS'(q) \cap Q_N \cap A = DS'(q) \cap A$. Now, let $q \in DS(A)$. Then $\exists\, x \in X$ such that $q \in \underline{\delta(p, x)}$ for some $p \in A$, i.e., $p \in A$ and $p \in DS'(q)$, whereby $p \in DS'(q) \cap A$. Thus $DS'(q) \cap A \neq \phi$, showing the first equality. Now, since N is closed, $q \notin Q_N \implies DS'_N(q) = \emptyset$.
This shows the second equality.
Lemma 2.4 *Let N be a possible rough subautomaton of M and let $A \subseteq Q_N$. Then*
$PS_N(A) = \{q \in Q, : PS'_N(q) \cap A \neq \emptyset\} = \{q \in Q_N : PS'(q) \cap A \neq \emptyset\}$.
Proof: The proof is similar to that of lemma 2.3.

The capability to substitute an expression that involves the definite(possible) source with one concerning the definite(possible) successor set, and vice versa, is ex- tremely important. The following theorem and its associated corollaries provide the foundation for such substitutions.
Theorem 2.3 *Let N be a definite rough sub automaton of M, $A1 \subseteq Q$, and $A_2 \subseteq Q_N$. Then $DS'_N(A_2) \cap A_1 = \emptyset \iff A_2 \cap DS(Q_N \cap A_1) = \emptyset$.*
Proof: Let $q \in DS'_N(A2) \cap A1$. Then $q \in DS'_N(A2)$ and $q \in A1$. That is,
$q \in Q_N$, $DS(q) \cap A_2 \neq \phi$ and $q \in A_1$, or that $q \in Q_N \cap A_1$ and $DS(q) \cap A_2 \neq \phi$, whereby $DS(q) \subseteq DS(Q_N \cap A_1)$. Thus $A_2 \cap DS(Q_N \cap A_1) \neq \phi$.
Conversely, Let $q \in A_2 \cap DS(Q_N \cap A_1)$, by lemma 2.3, $DS'_N(q) \cap A_1 \neq \phi$. Hence
$DS'_N(A2) \cap A1 \neq \phi$.
Theorem 2.4 *Let N be a possible rough subautomaton of M, $A1 \in Q$, and $A2 \in Q_N$. Then $PS'(A_2) \cap A_1 = \emptyset \implies A_2 \cap PS(Q_N \cap A_1) = \emptyset$.*
Proof: The proof is similar to that of theorem 2.3.

Corollary 2.1 *If $A_2 \subseteq Q_N$, then*
$DS'(A1) \cap A2 = \emptyset \iff A1 \cap DS(A2) = \emptyset$.
Corollary 2.2 *If $A_2 \subseteq Q_N$, then*

$PS'(A1) \cap A2 = \emptyset \iff A1 \cap PS(A2) = \emptyset$.

Corollary 2.3 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* $A1, A2 \subseteq Q$. *Then*

$DS'(A1) \cap A2 = \emptyset \iff A1 \cap DS(A2) = \emptyset$.

Corollary 2.4 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* $A1, A2 \subseteq Q$. *Then*

$PS'(A1) \cap A2 = \emptyset \iff A1 \cap PS(A2) = \emptyset$.

Corollary 2.5 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* $A1, A2 \subseteq Q$. *Then*

(i) (*a*) $A1 \cap DS(DS'(A2)) = \phi \iff DS(DS'(A1)) \cap A2 = \phi$;

(b) $A1 \cap PS(PS'(A2)) = \phi \iff PS(PS'(A1)) \cap A2 = \phi$;

(ii) (*a*) $A1 \cap DS'(DS(A2)) = \phi \iff DS'(DS(A1)) \cap A2 = \phi$;

(b) $A1 \cap PS'(PS(A2)) = \phi \iff PS'(PS(A1)) \cap A2 = \phi$;

Proof: The proof of (i) and (ii) immediately follows from the two applications in the opposite ways of corollary 3.3 and 3.4.

Remark 2.1 *The two sets* $DS(DS'(A1))$ *and* $DS'(DS(A1))$ *(resp.* $PS(PS'(A1))$ *and* $PS'(PS(A1))$ *mentioned in corollary 3.5 is not necessarily comparable by inclusion, it can be easily verified. However, when* $A1$ *is the set of states of a rough sub automaton, we have the following results:*

Lemma 2.5 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and* N *is a definite rough subau- tomaton of* M,$N1$ *is also a definite rough proper subautomaton of* M *and definite rough subautomaton of* N. *Then*

$DS'(DS(A1)) \subseteq DS(DS'(A1))$

Proof: The proof is immediate follows from the fact that $DS(A1) = A1$ (The set of $DS(DS'(A1))$, *for* $A1 \neq \emptyset$)

Lemma 2.6 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and* N *is a possible rough subau- tomaton of* M,$N1$ *is also a definite rough proper subautomaton of* M *and possible rough subautomaton of* N. *Then*

$PS'(PS(A1)) \subseteq PS(PS'(A1))$

Proof: The proof is the same as above.

Definition 2.11 *An RFSA* (Q', R', X, δ') *is called* ***definite rough subautomaton*** *(resp.* ***possible rough subautomaton****) of an RFSA* (Q, R, X, δ) *if* $Q' \subseteq Q$, $DS(Q') = Q'$, $R' = R/_{Q'}$ *and* $\delta' = \delta/_{Q'\times X}$ *(resp.* $Q' \subseteq Q$, $PS(Q') = Q'$, $R' = R/_{Q'}$ *and* $\delta' = \delta/_{Q'\times X}$*).*

Definition 2.12 *A definite rough subautomaton (resp. possible rough subautoma- ton)* (Q', R', X, δ') *of* (Q, R, X, δ) *is called* ***definite separated subautomaton*** *(resp.* ***possible separated subautomaton****) if* $DS(Q/Q') \cap Q' = \phi$ *(resp.* $PS(Q/Q') \cap Q' = \phi$*).*

Theorem 2.5 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* $Q' \subseteq Q$. *Then the follow- ing are equivalent:-*

(i) (*a*) *The 4 tuple* (Q', R', X, δ') *of* M *is a definite rough sub automaton of* M;

(b) *The 4 tuple* (Q', R', X, δ') *of* M *is a possible separated rough sub automaton of* M;

(ii) (a) $DS'(Q - Q') = Q - Q'$;
(b) $PS'(Q - Q') = Q - Q'$;
(iii) (a) $\forall q \in Q, (DS'(q) \cap Q' \neq \emptyset \Rightarrow q \in Q')$;
(b) $\forall q \in Q, (PS'(q) \cap Q' \neq \emptyset \Rightarrow q \in Q')$;
Proof: (i) $\Leftrightarrow$ (ii) $DS'(Q - Q') = Q - Q' \Leftrightarrow$ then it is enough to show that $DS'(Q - Q') \subseteq Q - Q'$ Let $q \in Q and q \in Q - Q'$, then $q \in Q'$ whereby $q \notin DS'(Q - Q') \Leftrightarrow DS'(Q - Q') \cap Q' = \phi \Leftrightarrow (Q - Q') \cap DS'(Q') = \phi \Rightarrow DS'(Q') \subseteq Q' \Leftrightarrow DS'(Q') = Q' \Leftrightarrow$ the 4 tuple (Q', R', X, δ') of M be a defi- nite rough sub automaton of M .
(ii) $\Leftrightarrow$ (iii) $[\forall q \in Q, (DS'(q) \cap Q' \neq \phi \Rightarrow q \in Q')]$ Let $q \in DS'(Q')$ then $\exists x \in X$ such that $q \in \underline{\delta(p, x)}$ for $p \in Q'$ and $p \in DS'(q)$ whereby $p \in DS'(q) \cap Q'$ – $\forall q \in Q, (q \in Q - Q' \Rightarrow DS'(q) \cap Q' = \phi)] \Leftrightarrow DS'(Q - Q') \subseteq Q - Q' \Leftrightarrow DS'(Q - Q') = Q - Q'$.

The Pure Source

Definition 3.1 *For an RFSA (Q, R, X, δ), the block transition map δ^D : $\boldsymbol{D} \times X \rightarrow \boldsymbol{A}$ can be extended to a map $\delta^D : \boldsymbol{D} \times X^+ \rightarrow \boldsymbol{A}$ as follows: $\forall D' \in \boldsymbol{D}$ and $\forall x \in X^+$, Where $X^+ = (X^* - \epsilon)$*

$$\delta^{D^+}(D', x) = (\underline{\delta^{D^+}}(D', x), \overline{\delta+D(D', x)}), \text{ where}$$
$$\underline{\delta^{D}}(D', x) = \bigcup\{\underline{\delta+(q, x)} : q \in B \subseteq D', B \in Q/R\} \text{ and}$$
$$\overline{\delta^D}(D', x) = \bigcup\{\overline{\delta+(q, x)} : q \in B \subseteq \overline{D'}, B \in Q/R\}.$$

Definition 3.2 *Let (Q, R, X, δ) be an RFSA and $p, q \in Q$. q is called a* ***definite pure successor*** *(resp.* ***possible pure successor****) of p if $\exists\, x \in X^+$ such that $q \in$ ~~$\underline{\delta+(p, x)}$ or~~ $[q] \subseteq \underline{\delta^{D^+}}([p], x)$ (resp. $q \in \overline{\delta+(p, x)}$ or $[q] \subseteq \overline{\delta^{D^+}}([p], x)$).*

Definition 3.3 [1] Let (Q, R, X, δ) be an RFSA and $T \subseteq Q$. The **definite pure successor** and **possible pure successor** of T are respectively defined as the sets
$DS+(T) = \bigcup\{DS+(q) : q \in T\}$ and
$PS+(T) = \bigcup\{PS+(q) : q \in T\}$,

where $DS+(q) = \{p : p \in \underline{\delta+(q, x)}$, for some $x \in X^+\}$ and $PS+(q) = \{p : p \in \overline{\delta+(q, x)}$, for some $x \in X^+\}$.
Analogous to the above concept of definite (possible) pure successor, we now introduce the notion of a definite (possible) pure source.

Definition 3.4 *Let (Q, R, X, δ) be an RFSA and $p, q \in Q$. q is called a* ***definite pure source*** *(resp.* ***possible pure source****) of p if $\exists x \in X^+$ such that $p \in \underline{\delta+(q, x)}$ or $[p] \subseteq \delta^D([q], x)$ (resp. $p \in \overline{\delta+(q, x)}$ or $[p] \subseteq \delta^D([q], x)$).*

Definition 3.5 *Let (Q, R, X, δ) be an RFSA and $T \subseteq Q$. The* ***definite pure source*** *and* ***possible pure source*** *of T are respectively defined as the sets*

$DS'_+(T) = \bigcup\{DS'_+(q) : q \in T\}$ *and*

$PS'_+(T) = \bigcup\{PS'_+(q) : q \in T\}$,

where $DS'_+(q) = \{p : q \in \underline{\delta+(p, x)}$, *for some* $x \in X^+\}$ $PS'_+(q) = \{p : q \in \overline{\delta+(p, x)}$, *for some* $x \in X^+\}$.

OR

Definition 3.6 *Let $N = (Q', R', X, \delta')$ be the definite rough subautomata of an RFSA $M = (Q, R, X, \delta)$ and $A1 \subseteq Q'_N$. Then the set of definite(possible) pure successor of $A2 \subseteq Q$ is define as*

$DS+(A2) = \{\underline{\delta(q, x)} : q \in A2, x \in X^+\}$ $\underline{DS+(\emptyset)} = \emptyset$

$PS+(A2) = \{\delta(q, x) : q \in A2, x \in X^+\}$ $PS+(\emptyset) = \emptyset$

Definition 3.7 *Let $N = (Q', R', X, \delta')$ be the definite rough subautomata of an RFSA $M = (Q, R, X, \delta)$ and $A_1 \subseteq Q_N$. Then the set of definite(possible) pure source of $A1 \subseteq N$ is define as*

$DS'_{+N}(A_1) = \{p \in Q_N : DS_+(q) \cap A_1 \neq \emptyset\}$

$PS'_{+N}(A_1) = \{p \in Q_N : PS_+(q) \cap A_1 \neq \emptyset\}$

Lemma 3.1 *Let $M = (Q, R, X, \delta)$ be an RFSA and $N = (Q', R', X, \delta')$ be a def- inite (possible) rough subautomaton of M and $A1, A2 \subseteq Q'_N$. Then the following holds:*

(i) (*a*) $DS+(A1) = \bigcup\{q \in Q : DS'_+(q) \cap A1 \neq \emptyset\}$;

$= \bigcup\{q \in Q'_N : DS'_+(q) \cap A1 \neq \emptyset\}$;

(b) $PS+(A1) = \bigcup\{q \in Q : DS'_+(q) \cap A1 \neq \emptyset\}$;

$= \bigcup\{q \in Q_N : PS'_+(q) \cap A_1 \neq \emptyset\}$;

(ii) (*a*) *If* $A1 \subseteq A2 \Rightarrow DS'_{+N}(A1) \subseteq DS'_{+N}(A2)$;

(b) *If* $A1 \subseteq A2 \Rightarrow PS'_{+N}(A1) \subseteq PS'_{+N}(A2)$;

(iii) (*a*) $DS'_{+N}(A_1) = Q_N \cap DS'_+(A_1)$;

(b) $PS'_{+N}(A_1) = Q_N \cap PS'_+(A_1)$;

(iv) $(a)\ DS'_{N}(A1) = DS'_{+N}(A1) \cup A1;$

(b) $PS'_{N}(A1) = PS'_{+}(A1) \cup A1;$

(v) $(a)\ DS'_{+N}(DS'_{N}(A1)) \subseteq DS'_{N}(A1);$

(b) $PS'_{+N}(PS'_{N}(A1)) \subseteq PS'_{+}(A1);$

(vi) $(a)\ DS'_{N}(DS'_{+N}(A1)) = DS'_{+N}(DS'_{N}(A1)) = DS'_{+N}(A1);$

(b) $PS'_{N}(PS'_{+N}(A1)) = PS'_{+N}(PS'_{N}(A1)) = PS'_{+N}(A1);$

(vii) $(a)\ DS'_{+N}(A1 \cup A2) = DS'_{+N}(A1) \cup DS'_{+N}(A2);$

(b) $PS'_{+N}(A1 \cup A2) = PS'_{+N}(A1) \cup PS'_{+N}(A2);$

(viii) $(a)\ DS'_{+N}(A1 \cap A2) \subseteq DS'_{+N}(A1) \cap DS'_{+N}(A2);$

(b) $PS'_{+N}(A1 \cap A2) \subseteq PS'_{+N}(A1) \cap PS'_{+N}(A2);$

(ix) $(a)\ DS'_{+N}(A1) - DS'_{+N}(A2) \subseteq DS'_{+N}(A1 - A2);$

(b) $PS'_{+N}(A1) - PS'_{+N}(A2) \subseteq PS'_{+N}(A1 - A2);$

(x) $(a)\ \forall U \subseteq Q,\ DS_{+N}(U) \cap A1 = \emptyset \Leftrightarrow U \cap DS'_{+N}(A1) = \emptyset;$

(b) $\forall U \subseteq Q,\ PS_{+N}(U) \cap A1 = \emptyset \Leftrightarrow U \cap PS'_{+N}(A1) = \emptyset;$

The proof of the above lemma is straightforward and simple.

Lemma 3.2 *Let* M = (Q, *R*, X, *δ*) *be an RFSA and* N1, N2 *are two definite rough subautomaton, in which* N1 *is definite rough subautomaton of* M *whereas* N2 *is also a definite rough subautomaton of* N1 *and a definite proper rough subautomaton of* M, A ⊆ Q_{N2} . *Then*

(i) $A \subseteq DS'_{+N_1}(A)$;

(ii) $DS'_{+N1}(A) = DS'_{N1}(A) = DS'_{N1}(DS_{N1}(A)) = DS'_{N1}(DS_{+N1}(A));$

Proof:(i) Since $DS_{+}(q) \neq \emptyset, \forall q \in Q$ and as $p \in A = Q_N$, $DS_{+}(p) \subseteq DS(p) \subseteq DS(A) = A$, for some $p \in A$. Now we have $DS'_{+}(p) \cap A \neq \emptyset$ Now let $q \in DS'_{+}(A)$ then $\exists x \in X$ such that $q \in \underline{\delta(p, x)}$ for some $p \in A$ and $p \in DS'_{+}(q)$ that is $p \in DS'_{+}(q)$, $p \in A$ whereby $p \in DS'_{+}(q) \cap A\ \forall p \in A$. Hence $A \subseteq DS'_{+}(A)$ by definition.

(ii) As we have $DS'_{+N1}(A) \subseteq DS'_{N1}(A) \subseteq DS'_{n1}(DS_{+}(A)) \subseteq DS'_{N1}(DS(A))$.

Then it is sufficient to prove that $DS'_{N1}(DS(A)) \subseteq DS'(A)$. Let $q \in DS'(DS(A))$ then $q \in DS'(p)$ for some $p \in DS(A)$ Now, $p \in DS(A) \Rightarrow p \in DS(r)$ for some $r \in A$. Thus q is a definite source of p and p is a definite successor of r. i.e $q \in DS(r) \subseteq DS'_{+}(A)$, and also $DS'_{+}(A) \subseteq DS'(DS(A))$.

Hence $DS'(DS(A)) = DS'(A)$. Since $A = Q_{N1}$, $DS(A) = A$ and hence

$DS'(DS(A)) = DS'(A)$ by above lemma, $DS'_{+}(A) = DS'_{+}(A) \cup A = DS'(A)$.

Thus the second part.

Lemma 3.3 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and* $N1$, $N2$ *are two possible rough subautomaton, in which* $N1$ *is possible rough subautomaton of* M *whereas* $N2$ *is also a possible rough subautomaton of* $N1$ *and a possible proper rough subautomaton of* M, $A \subseteq Q_{N2}$ *. Then*

(i) $A \subseteq PS'_{+N_1}(A)$;

(ii) $PS'_{+N1}(A) = PS'_{N1}(A) = PS'_{N1}(PS_{N1N1}(A)) = PS'_{N1}(PS_{+}(A))$;

$Q_{N2} \subseteq PS_{+}(PS'_{+N_1}(Q_{N1}))$.

Proof: The proof is similar to theorem 3.1.

Corollary 3.1 *An RFSA* M *is definite reflexive iff* $\forall Q' \subseteq Q$, $DS+(Q') = DS(Q')$, *iff* $\forall N$ *is definite rough subautomaton of* M, $\forall Q' \subseteq Q_N$, $DS'_{+}(Q') = DS'(Q)$. **Corollary 3.2** *An RFSA* M *is possible reflexive iff* $\forall Q' \subseteq Q$, $PS+(Q') = PS(Q')$,
iff $\forall N$ *is possible rough subautomaton of* M, $\forall Q' \subseteq Q_N$, $PS'(Q') = PS'_{+}(Q')$.

Proof: The proof is similar to that of lemma 4.2.

Definition 3.8 *Let* $M = (Q, R, X, \delta)$ *be a definite (resp. possible) reflexive if and only if* $\forall q \in Q$, $\exists x \in X^*$, *such that* $x \neq \epsilon$ *and* $\underline{\delta(q, x)} = [q]$ *(resp.* $\delta(q, x) = [q]$*).*

Theorem 3.1 *Let* $M = (Q, R, X, \delta)$ *be an RFSA. Then the following are equiva- lent.*

(i) M *is definite reflexive*

(ii) $\forall q \in Q$, $q \in DS'(q)$.

(iii) *for all* N_1 *is a definite rough subaotumaton of* M, $\forall Q' \subseteq_1 Q_N$, $Q' \subseteq DS'_{+}(Q')$.

(iv) *for all* N_1 *is a definite rough subaotumaton of* M, $\forall Q' \subseteq_1 Q_N$, $Q' \subseteq DS+(DS'_{+}(Q'))$.

(iii) $N1$, $N2$ *are two definite rough subautomaton, in which* $N1$ *is definite rough subautomaton of* M *whereas* $N2$ *is also a definite rough subautomaton of* $N1$ *and a definite proper rough subautomaton of* M, *then this implies that*
$Q_{N2} \subseteq DS_{+}(DS'_{+N_1}(Q_{N1}))$.

Proof: (i)⇒ (ii) it follows immediate from definition 4.7.

(ii)⇒ (iii): Let N_1 be a definite rough subautomaton of M $Q' \subseteq Q_N$ and $q \in Q'$ since $q \in DS'_{+}(q)$ by lemma and thus $q \in DS'_{+N1}(Q')$.Thus by prop-[1] erties $q \in DS'\ Q' \subseteq DS'_{+}(Q')_{N_1}(Q')$, i.e for some $q \in Q' \exists x \in X$ such that $q \in \underline{\delta+(q, x)}$. Hence (iii)$\Rightarrow$ (iv): Let N_1 be a definite rough subautomaton of M, $Q' \subseteq Q_N$ and $q \in Q'$ with $Q' = \{q\}$ in (iii), we have $q \in DS'_{+N_1}(q) \subseteq DS'_{+N_1}(Q') \subseteq \underline{\delta+(q, x)}$, for some $q \in Q', \exists x \in X$. Implying $DS'_{+N_1}(q) \cap DS'_{+N_1}(Q') \neq \emptyset$. Now let $q \in DS+_{N_1}(Q')$. Then $\exists x \in X$ such that $q \in \underline{\delta+(q, x)}$ for $q \in Q'$, that is $q \in Q'$ and $q \in DS+_{N_1}(Q')$ where by $q \in DS'_{+N_1}(q) \cap DS'_{+N_1}(Q')$ hence by lemma 4.1(x), $q \in DS+(DS'_{+N_1}(Q'))$.

(iv) $\Rightarrow$ (v): It is also follows immediate;

(v) $\Rightarrow$ (i): Let $q \in Q$. Then by (v) $DS(q) \subseteq DS+(DS+(DS(q)))$ and thus $q \in DS+(DS'(DS(q)))$. Hence $\exists p \in DS'(DS(q))$ and $x \in X$ such that $x \neq \epsilon$ and $\underline{\delta+(p, x)} = q$ But then $p \in DS(q)$ and therefore $\exists y \in X$ such that $\underline{\delta+(q, y} = p$. Hence $\underline{\delta+(q, yx)} = q$. Since x not $\in \epsilon$, M is definite reflexive.

Theorem 3.2 *Let $M = (Q, R, X, \delta)$ be an RFSA. Then the following are equiva- lent.*

(i) *M is possible reflexive.*

(ii) *$\forall q \in Q, q \in PS'_+(q)$.*

(iii) *for all N_1 is a possible rough subaotumaton of M, $\forall Q' \subseteq Q_{N_1}$, $Q' \subseteq PS'_+(Q')$.*

(iv) *for all N_1 is a possible rough subaotumaton of M, $\forall Q' \subseteq Q_{N_1}$, $Q' \subseteq PS+(PS'_+(Q'))$.*

(*iii*)$N1$, $N2$ *are two possible rough subautomaton, in which $N1$ is possible rough subautomaton of M whereas $N2$ is also a possible rough subautomaton of $N1$ and a possible proper rough subautomaton of M, then this implies that.*

Source and Primaries

Theorem 4.1 *Let $M = (Q, R, X, \delta)$ be an RFSA with $\emptyset \neq A \subseteq Q$. Then $< DS(A) >$ is the union of all definite primaries of M in which at least one element of A is a state.*

Proof: Let us suppose U be the union of all definite primaries of M, in which at least one element of A is a state and let any such definite primary be $< q >$. Then there exists $p \in A$ such that $p \in DS(q)$ i.e., $p \in \underline{\delta(q, x)}$. Hence $q \in DS'(p) \subseteq DS(A)$. This implies that, $DS(q) \subseteq DS(DS'(A))$. Consequently $< q >$ is a definite rough subset of $< DS'(A) >$. Hence U is a definite rough subset of $< DS'(A) >$.———(1)

Conversely: Let $r \in DS(DS'(A))$. Then there exists $p \in A$ such that $r \in DS(DS'(p))$ and hence $DS'(r) \cap DS'(p) \neq \emptyset$. Let $m \in DS'(r) \cap DS'(p)$. Then $r \in DS'(m)$ and $p \in DS'(m)$. By the primary decomposition theorem. There exists a definite primary $< n >$ of M in which m is a state. But then $m \in DS(n)$ and thus $r \in DS(n) and p \in DS(q)$ and hence r is a state of a definite primary $< n >$ in which p is also a state. Consequently, r is a state of U and thus $DS'(A)$ is a definite subset of U.——————(2)

By (1) and (2), we have $< DS(A) >= U$.

Theorem 4.2 *Let $M = (Q, R, X, \delta)$ be an RFSA with $\emptyset \neq \subseteq Q$. Then $< PS(A) >$ is the union of all possible primaries of M in which at least one element of A is a state.*

Proof: The proof is similar to that of theorem 5.1.

Lemma 4.1 *Let* $M = (Q, R, X, \delta)$ *be a singly generated RFSA, and let* $q \in RgenM$*. Then* $DS'(q) = RgenM$ *(resp.* $PS'(q) = RgenM$*.*

Proof: $r \in DS'(q) \Leftrightarrow q \in DS(r) \Leftrightarrow r \in RgenM$. (resp. $r \in PS'(q) \Leftrightarrow q \in PS(r) \Leftrightarrow R \in RgenM$).

Definition 4.1 *Let* $M = (Q, R, X, \delta)$ *be an RFSA, and let* $A \subseteq Q$*. Then* A *is definite genetic if and only if f* $DS'(A) \subseteq DS(A)$*(resp. possible genetic if and only if* $PS'(A) \subseteq PS(A)$*).*

Lemma 4.2 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* A *be a nonempty definite genetic subset of* Q*. Then* $< A >$ *is the union of those definite primaries of* M*, in which at least one element of* A *is a state.*

Proof: Since A is definite genetic, so $DS(DS'(A)) \subseteq DS(DS(A)) = DS(A)$. But $A \subseteq DS(A) \Rightarrow DS(A) \subseteq DS(DS'(A))$ and hence $DS(DS'(A)) = DS(A)$. Hence this lemma automatically follows from theorem 4.1.

Lemma 4.3 *Let* $M = (Q, R, X, \delta)$ *be an RFSA and let* A *be a nonempty possible genetic subset of* Q*. Then* $< A >$ *is the union of those possible primaries of* M*, in which at least one element of* A *is a state.*

Proof: The proof is similar to that of lemma 4.2.

Lemma 4.4 *Every definite primary of a RFSA* M *has a definite genetic generating set.*

Proof: Let $< q >$ be a definite primary of an RFSA M. Then by 5.1. $DS'_{<q>}(q) = RgenM < q >$ But $DS'_{<q>}(q) = DS'(q)$ since $< q >$ is maximal singly generated, hence $DS'(q) = Rgen < q >$. Now $Rgen < q > \subseteq DS(q)$ since $DS(q) = Q_{<q>}$ and hence $DS'(q) \subseteq DS(q)$. Consequently, q is a definite genetic generating set of $< q >$.

Lemma 4.5 *Every possible primary of a RFSA* M *has a possible genetic generat- ing set.*

Proof: The proof is similar to that of lemma 4.4.

Lemma 4.6 *A nonempty rough subautomaton* $N1$ *of an RFSA* M *has a definite genetic generating set if and only if* $N1$ *is the union of primaries of* M*.*

Proof: Let $N1 = \bigcup_{i=1}^{j} P_i$, where P_i is a definite primary of M for each $i \in 1, \ldots, j$. Then every P_i has a definite generating set D_i by lemma 5.4. Now $DS'(\bigcup_{i=1}^{j} D_i) = \bigcup_{i=1}^{j} DS'(D_i) \subseteq \bigcup_{i=1}^{j} DS(D_i) = DS(\bigcup_{i=1}^{j} D_i)$. And hence $\bigcup_{i=1}^{j} D_i$ is definite genetic. Also $Q_{N1} = \bigcup_{i=1}^{j} Q_{P_i} = \bigcup_{i=1}^{j} DS(D_i) = DS(\bigcup_{i=1}^{j} D_i)$ and hence $\bigcup_{i=1}^{j} D_i$ generates $N1$. The converse follows immediately by lemma 4.2.

Lemma 4.7 *A nonempty rough subautomaton $N1$ of an RFSA M has a possible genetic generating set if and only if $N1$ is the union of primaries of M.*
Proof: The proof is similar to that of lemma 4.6.
Theorem 4.3 *A nonoempty rough subautomaton N of an RFSA M is a definite primary of M if and only if N is a minimal nonempty rough subautomaton of M, which has a definite genetic generating set.*
Proof: Let us assume N be a definite primary of M. Then N has a definite genetic generating set by lemma 4.4 and no proper rough sub automaton of N has a definite genetic generating set by lemma 4.6. Thus N is minimal with respect to this property.
Conversly: Let N be a minimal nonempty rough subautomaton of M, which has a definite genetic generating set. Then we say that N is the union of definite primaries of M by lemma 4.6. Now, if N consists of more than one definite primary of M, every such definite primary is a definite proper rough subautomaton of N, which has a definite genetic generating set. Thus we have a contradicting definite minimality of N. Hence N is a definite primary of M.
Theorem 4.4 *A nonoempty rough subautomaton N of an RFSA M is a possible primary of M if and only if N is a minimal nonempty rough subautomaton of M, which has a possible genetic generating set.*
Proof: The proof is similar to that of theorem 4.3.

Source and connectivity
In both automata and fuzzy automata, the connected automata and separated sub automata are introduced in [4, 8]. In this section, we introduced analogous notions for RFSA and discussed the interrelationship between them.
Definition 5.1 *Let $N = (Q', R', X, \delta')$ be a definite rough sub automaton (resp. possible rough subautomaton) of M is called the definite separated subautomaton (resp. possible separated subautomaton) if $DS(Q/Q') \cap Q' = \emptyset$(resp. $PS(Q/Q') \cap Q' = \emptyset$).*

Definition 5.2 *Let $M = (Q, R, X, \delta)$ be a nonempty RFSA is called definite con- nected (resp' possible connected) if and only if it has no definite separated proper rough subautomaton (resp. possible separated proper rough subautomaton).*
From the definition point of view, the definite separatedness (resp. possible separatedness) proper rough sub automaton N-depends on whether N can be reached from outside. Reachability from outside, however, is a concept that is better addressed by the definite source(resp. possible source) than by the definite successor (resp. possible successor) operator, as is marked by the simplicity of the following statement.
Theorem 5.1 *Let $M = (Q, R, X, \delta)$ be an RFSA and let $< \emptyset > \neq N$ and N is a definite rough subautomaton of M. Then N is definite separated if and only if $DS'(Q_N) = Q_N$.*
Proof: $DS(Q/Q_N) \cap Q_N = \emptyset \Leftrightarrow (Q/Q_N) \cap DS(Q_N) = \emptyset \Leftrightarrow DS'(Q_N) \subseteq Q_N \Leftrightarrow DS'(Q_N) = Q_N$.
Theorem 5.2 *Let $M = (Q, R, X, \delta)$ be an RFSA and let $< \emptyset > \neq N$ and N is a possible rough subautomaton of M. Then N is possible separated if and only if $PS'(Q_N) = Q_N$.*
Proof: The proof is similar to that of the above theorem.
Corollary 5.1 *Under the condition of above theorem N is definite (possible) sepa- rated iff $(Q/Q_N) \cap DS'(Q_N) = \emptyset$. (resp. $(Q/Q_N) \cap PS'(Q_N) = \emptyset$.*
Proof: The proof immediately follows from the above theorem.
Corollary 5.2 *Let $M = (Q, R, X, \delta)$ be an RFSA and let $\emptyset = A \subseteq Q$. Then*

(i) $DS(DS'(A)) = DS'(A) \Rightarrow (DS'(A), R, X, \delta)$ *is a definite separated rough subautomaton of* M.

(ii) $DS'(DS(A)) = DS(A) \Rightarrow < A >$ *is definite separated.*

Proof: (i) $DS(DS'(A)) = DS'(A). \Rightarrow DS'(A)$.is a set of states of a definite rough subautomaton of M. Thus by properties $DS'(DS'(A)) = DS'(A)$ and hence by theorem 5.1 $(DS'(A), R, X, \delta)$ is definite separated.

(ii) Proof follows immediately from Theorem 5.1.

Definition 5.3 *A nonempty RFSA* $M = (Q, R, X, \delta)$ *is said to be definite (possible) discrete if and only if* $\forall q \in Q$, $DS(q) = q$ *. (resp.*$\forall q \in Q$, $PS(q) = q$*).*

Theorem 5.3 *Let* $M = (Q, R, X, \delta)$ *be an nonempty RFSA, then*

(i) M *is definite retrievable if and only if* $DS'(Q_N) = Q_N$ *for all* N *is a definite rough subautomaton of* M.

(ii) M *is definite discrete if and only if* $DS'(A) = A, \forall A \subseteq Q$.

Proof: (i) A nonempty RFSA $M = (Q, R, X, \delta)$, is definite retrievable if and only if each nonempty definite rough subautomaton of M is definite separated. Thus part (i) follows immediately from theorem 5.1.

(ii) $DS(q) = q \forall q \in Q \Leftrightarrow q = DS'(q), \forall q \in Q$ since $DS'(A) = A, \forall A \subseteq Q \Leftrightarrow$ $DS'(q) = q, \forall q \in Q$. it follows the second part of the theorem.

Theorem 5.4 *Let* $M = (Q, R, X, \delta)$ *be an nonempty RFSA, then*

(i) M *is possible retrievable if and only if* $PS'(Q_N) = Q_N$ *for all* N *is a possible rough subautomaton of* M.

(ii) M *is possible discrete if and only if* $PS'(A) = A, \forall A \subseteq Q$.

Proof: The proof is similar to that of theorem 5.3.

Theorem 5.5 *Let* $< \emptyset > \neq N$ *and* N *is a definite rough subautomaton of M.Then* N *is definitely strongly connected if and only if* $DS'_N(A) = Q_N$ *for every nonempty* $A \subseteq Q_N$.

Proof: For all $q, r \in Q_N; q \in DS(r) \Leftrightarrow \forall q, r \in Q_N, r \in DS'_N(q) \Leftrightarrow q \in Q_N, Q_N \subseteq DS'_N(q). \Leftrightarrow \forall$, *nonempty*$A \subseteq Q_N, Q_N \subseteq DS'_N(A)$.

Theorem 5.6 *Let* $< \emptyset > \neq N$ *and* N *is a possible rough subautomaton of M.Then* N *is possibly strongly connected if and only if* $PS'_N(A) = Q_N$ *for every nonempty* $A \subseteq Q_N$.

Proof: The proof is similar to that of theorem 5.5.

It is well known that every nonempty RFSA has a definite (possible) strongly connected rough subautomaton. The importance of the role played by the definite (possible) connected rough subautomaton of an RFSA is seen with the aid of the definite source(possible source) as is shown in the following theorem.

Theorem 5.7 *The set of states of a nonempty RFSA* $M = (Q, R, X, \delta)$ *is the union of the definite sources of the sets of states of the definite strongly connected rough subautomaton of* M.

Proof: Let $M = (Q, R, X, \delta)$ be an RFSA, and let $q \in Q$. Then $< q >$ has a definite strongly connected rough subautomaton N. Since $Q_N \subseteq DS(q)$, $DS(q) \cap Q_N \neq \emptyset \Rightarrow q \in DS'(Q_N)$. Since q is an arbitrary state of M. Hence the theorem.

Theorem 5.8 *The set of states of a nonempty RFSA* M *is the union of the possible sources of the sets of states of the possible strongly connected rough subautomaton of* M.

Proof: The proof is similar to that of theorem 5.7.

The Shape of Abelian RFSA

In Theorem 3.1, we have shown that the abelian rough automaton has the property that disjoint definite (possible) rough subautomata have disjoint definite (possible) sources. Now the following theorem provides alternate means of discussing this same property.

Theorem 6.1 *Let $M = (Q, R, X, \delta)$ be an nonempty RFSA. Then the following are equivalent.*

(iii) *The sets of states of disjoint definite rough subautomaton have disjoint definite sources.*

(iv) *every definite primary of M has exactly one definite strongly connected rough subautomaton.*

(v) *every block of M has exactly one definite strongly connected rough subau- tomaton.*

(vi) *the number of blocks of M equals to the number of definite strongly connected rough subautomaton of M.*

Proof: (i) ⇒ (ii) Let $N_1, \ldots N_k$ be all the different definite strongly conected rough subautomaton of M. Then by theorem 6.6 $Q = \bigcup^{k} DS'(Q_N)$. Since different definite strongly, (i) implies that $DS'(Q_N) \cap DS'(Q_N) = \emptyset$ if $i \neq j$. But then $Q_{Ni} \cap DS(DS'(Q_N)) = \emptyset$ and thus by theorem 5.1 the definite primaries con- taining Nj are disjoint fron Ni. Consequently, since every definite primary has at least one definite strongly connected rough subautomaton and as Ni is an arbitrary definite strongly connected rough subautomaton of M, every definite primary of M has exactly one definite strongly connected rough subautomaton.

(ii) ⇒ (i): Let us assume (ii) and let $N, N1$ are definite rough subautomaton of M, and $N \cap N_1 =< \emptyset >$. Suppose $DS'(Q_N) \cap DS'(Q_N)) = \emptyset$; then $Q_N \cap DS(DS'(Q_N)) \neq \emptyset$ thus by theorem 5.1 N has satates in the definite primaries of M containing N_1. Let $q \in Q_N \cap Ds(DS'(Q_N))$. Then there exists a definite primary $< r >$ of M such that $q \in DS(r)$ and $p \in DS(r)$ for some $p \in Q_{N_1}$. Now $< q >$ and $< p >$ each have a definite strongly connected rough subautomaton and since both $< p >$ and $< q >$ are in the definite primary $< r >$, so are their definite strongly connected rough subautomaton in $< r >$. However by hypothesis, $< r >$ has only one definite strongly connected rough subautomaton, and thus $< q > \cap < p > \neq \emptyset$. But $< q >$ is a definite rough subautomaton of N and $< p >$ is a definite rough subautomaton of $N1$ and hena $N \cap N1 = \emptyset$. This is contrary to the hypothesis. Thus $Ds'(Q_N) \cap Ds'(Q_N) = \emptyset$.

(ii) ⇒ (iii): Let N be a block of M, let $N1$ and $N2$ be definite strongly connected rough subautomaton of N and assume (ii). Since N is definitely connected, there exists a finite sequence of primaries $P1, \quad Pm$ of A such that $N1$ is a definite rough subautomaton of $P1,D$ is also a definite rough subautomaton of Pm, and $Pi \cap Pi+1 \neq< \emptyset >, \forall\, i \in 1, m-1$. Then $P1 \cap P2$ has a definite strongly connected rough subautomaton $N3$. But since $N3$ is also a definite rough subautomaton of $P1$ and $P1$ in definite rough subautomaton of $N1$, (ii) implies that $N3 = N1$. Similarly, $Pi \cap Pi+1, \forall i = 1, \quad m-1$, has a definite strongly connected rough subautomaton,
ich must be $N1$. Thus $N1$ and $N2$ are definite rough subautomaton of Pm and hence by hypothesis $N1 = N2$. Consequently, every block of M has exactly one definite strongly connected rough subautomaton, since every block of M is the union of definite primaries of M and M is the union of its blocks.

(iii) ⇒ (ii): Suppose every block of M has exactly one definite strongly con- nected

rough subautomaton. Since a definite primary of M must have at least one definite strongly connected rough subautomaton and since no definite primary can be in more than one block, thus every definite primary of M has exactly one definite strongly connected rough subautomaton.
(iii) $\Rightarrow$ (iv): Since every block of any nonempty RFSA has at least one definite strongly connected rough sub automaton, and thud different blocks of M are disjoint, thus it follows the equivalence of (iii) and (iv).

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