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Natural Convection Heat Transfer Enhancement Of Power Law Fluid In A Cylindrical Gap Using Internal Flat Tube

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Abstract— *These A numerical study conducted on the steady state laminar natural convection in the annular space created by inserting internal flat tube concentrically into an external cylinder filled with power-law fluid. The internal flat tube receives hot temperature T_h from heat source while the external cylinder is cooled at temerature T_c. The governing equations for the power-law fluid are solved numerically with the ANSYS-CFX package based on the finite volume technique.*
The effects of the relevant parameters such as Rayleigh number ($10^3 \leq Ra \leq 10^5$), Prandtl number ($10 \leq Pr \leq 10^3$), power law index ($0.6 \leq n \leq 1.4$) and internal flat tube inclination angle ($0° \leq \phi \leq 90°$) on the thermal performance of the channel are presented and discussed, three cases are studied for various values of Rayleigh and Prandtl numbers and power law index in the ranges considered earlier corresponding to inclination angles $\phi=0°$, $\phi=45°$ and $\phi=90°$ respectively. The results are interpreted in form of isotherms, velocity vectors and velocity profiles; besides, the average Nusselt number is determined.The obtained results indicated to an increase in thermal fields disturbances for increasing Ra especially for pseudoplastic fluids and this refers to the large cooling effect in the annular space; Additionally, the magnitude of the velocity component is found to be greater for pseudoplastic fluids and smaller for dilatant fluids when compared to Newtonian fluids ; Also, increasing the inclination angle leads to an improvement in the heat transfer rate for the same set of the other parameters ; Finally, the influences of the precedent parameters on the heat transfer rate are represented and explained in detail.
Keywords— *Flat tube, Inclination angle, Laminar flow, Natural convection, non-Newtonian fluid,* ***Steady state****.*

I. INTRODUCTION

Analyzing the natural convection flows of non-Newtonian fluids which induced by thermal buoyancy resulting from the differences of temperature between the heated internal cylinder and the cold external one in concentric cylindrical annulus caught a considerable regard in the past two decades. This geometry may be encountered in several engineering applications and industries like solar collectors, tubular heat exchangers, cooling of electronic components, thermal energy storage, pulp paper, food processing and polymer engineering..... ; Therefore, it was an important matter finding a method to enhance their efficiency. The natural convection of non-Newtonian fluid in tall vertical rectangular cavity subjected to a horizontal temperature gradient was studied numerically by M. Naïmi et al. [1], they showed the flow and temperature fields' solutions, and the heat transfer rate as functions of the governing parameters. The multiple steady state solutions for natural convection of power-law fluids in a shallow rectangular cavity heated from all sides and in a tilted rectangular slot heated from the vertical side walls was done by M. Lamsaadi et al. [2] and M. Naïmi et al. [3] respectively, they have proven the multiplicity of the solutions by existence of natural and anti-natural flows when Rayleigh exceeds a critical value. M. Naïmi et al. [4], [5] examined the effects of the non-Newtonian behaviour on the heat transfer characteristics and the onset of buoyancy driven flow in a shallow rectangular horizontal cavity uniformly heated from below in the first time while it was heated uniformly from the side walls in the second time. R.Y. Sakr et al. [6] presented experimental and numerical studies on the natural convection heat transfer in elliptic annuli, the average and local Nusselt numbers over the elliptic cylinders at different orientation angles were obtained and discussed.
O. Turan et al. [7]-[11] performed many benchmark studies on the laminar natural convection of power law fluids and the Newtonians in square and rectangular enclosures with differentially heated side walls and even heated from below, they proved that the increasing in Rayleigh number strengthens the convective transport in the numerical domain. I. Pop et al. [12], L. Khezzar et al. [13] investigated the natural convection in of power-law fluid in squared eccentric duct and inclined cavities, respectively, the

effects of the various parameters such as aspect ratio, Rayleigh and Prandtl numbers on the heat and fluid flow are presented. M.H. Matin and W.A. Khan [14] made a survey on the influences of the different parameters on the natural convection of non-Newtonian fluid in a cylindrical gap, they proved that pseudoplastic fluids are better for cooling purposes while dilatant fluids are better for insulating purposes.The natural convection in cold squared enclosure contains two hot cylinders where one is circular and the other one is elliptical [15], [16], and in the case of four elliptical cylinders [17] was studied numerically, the authors presented the thermal structure and focused on the effects of aspect ratio and the inclination angle of the elliptical cylinder to the passage from the steady to unsteady regime. S. Yigit et al. [18] investigated the laminar free convection of power-law fluid in a square enclosure partially heated from below, they found out that the increase in heat source length increases the Nusselt number. Y.G. Park et al. [19] made a survey of the natural convection of moving internal cylinder in a square enclosure filled with non-Newtonian fluid differentially heated from the horizontal side walls, they've proven that the positioning of the cylinder near the heated walls leads to an unsteady regime. S. Yigit et al. [20] investigated the laminar mixed convection in cylindrical enclosures with a cold rotating top cover and a stationary heated bottom wall whereas with a heated rotating top cover was done by O. Turan et al. [21], they discovered that the mean Nusselt values exhibits a non-monotonic trend at low Reynolds numbers.E. Abu-Nada and Ali. J. Chamkha [22] conducted a numerical study on the natural convection heat transfer of CuO-EG-water nanofluid, they presented the isotherm and streamline contours as well as local and average Nusselt numbers. M. Sheikholeslami et al. [23] carried a numerical investigation on the natural convection heat transfer effects of Cu-water nanofluid in an enclosure between an outer cylinder and inner elliptical one. M. Ghalambaz et al. [24] studied the effects of the key parameters such as the volume fraction of nanoparticules, Rayleigh number … on the conjugate natural convection of Ag-Mgo/water hybrid nanofluid in a squared cavity. Influences of Rayleigh number, volume fraction of Al_2O_3/water nanofluid, aspect ratio on the natural convection heat transfer between an eccentric inner cylinder and horizontal elliptic outer one was done by A.A. Mohammed et al. [25], it was found that the highest heat transfer obtained when the eccentricity and the aspect ratio are equal to 0.2.M. Bayareh et al. [26] investigated the natural convection problem of non-Newtonian fluid in L shaped enclosure in the presence of magnetic field, they revealed that the heat transfer rate for shear thinning fluids is smaller than that of Newtonian and shear thickening fluids except for an angle $\alpha \neq 60°$ the heat transfer rate is similar for all fluids. H. Laidoudi and M. Helmaoui [27] carried a numerical investigation of the effects of aspect ratio and the inner cylinder's inclination angle on the laminar free convection between cold outer cylinder and hot inner elliptical one. H. Laidoudi [28] studied the laminar natural convection heat transfer of two hot triangular equal-sized cylinders confined in cold triangular enclosure, the effects of the governing parameters along with the map of isotherms and streamlines were achieved. Controlling the natural convection of power-law fluid in a cavity with differentially heated side walls where a flexible elastic fin was mounted at its hot wall was given by M. Ghalambaz et al. [29], they proved that the fin deflection is higher for a dilatant fluid when compared with a pseudoplastic and a Newtonian fluid. H. Masuda et al. [30] used the velocity scale to study the Rayleigh Benard convection of pseudoplastic fluid, they proposed a new type of velocity scale allows for an approximate estimation of the actual velocity.
The issues of double-diffusive mixed convection and the Rayleigh-Bénard double-diffusive mixed convection in rectangular cavities where they subjected to uniform heat and mass fluxes from their vertical side walls and from their horizontal side walls as well was introduced by Y. Tizakast et al. [31], [32], they presented the effects of the governing parameters on isotherms, streamlines, stream function, Sherwood number and the average Nusselt number. M. Lamsaadi et al. [33] examined the Rayleigh-Bénard convection of non-Newtonian fluid in rectangular cavity subjected to vertical temperature gradient, the effects of Rayleigh and Prandtl numbers, aspect ratio and external magnetic field on the flow interaction and the heat transfer are discussed.
In spite of the unlimited number of studies performed on free convection between two concentric cylinders but few of them treated the problem between external cylinder and internal flat tube ; Hence, the purpose from this paper is to expand the concepts of natural convection in annular spaces and carrying on our previous work [34], We allow the internal flat tube to be tilted by an inclination angle ϕ to check its effect on the heat transfer enhancement. We're seeking from this study to investigate the

effects of Rayleigh number, power-law index and the inclination angle of the internal flat tube on the flow and temperature fields and the heat transfer rate.

II. MATHEMATICAL MODEL

Fig. 1 (a) stated the form of the current problem in the two-dimensional steady state which is a cross-section of an annular space formed by a flat tube located at the core of a circular cylinder containing power-law fluid. The internal surface of the flat tube receives a constant temperature from heat source T_h whereas the external surface of the circular cylinder is kept isothermal at the temperature T_c (T_h>T_c). It is possible to tilt the internal flat tube relative to the horizontal axis by an orientation angle ϕ. The radii ratio is taken for R_o/R_i=5 and the orientation angle ϕ for 3 values (ϕ=0°, ϕ=45° and ϕ=90°). The flow is laminar and the fluid properties are constant except for the change in density so that the Boussinesq approximation is used, this difference in density results in buoyancy in the power-law fluid that must be studied.

A. Governing Equations

The flow inside the annular space is governed by equations that must be solved during the simulation, these equations correspond with conservation of mass, momentum and energy written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho g \beta (T - T_c) \qquad (3)$$

$$\rho C_p u \frac{\partial T}{\partial x} + \rho C_p v \frac{\partial T}{\partial y} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

B. Non-Newtonian Model

The behavior of the power-law model exhibit a non linear relationship between the stress tensor and the strain rate tensor which it may be written as the following equation :

$$\tau_{ij} = \mu_{ref}\, e_{ij} = K\,(e_{kl}e_{kl}/2)^{(n-1)/2} e_{ij} \qquad (5)$$

As the reference viscosity isn't constant over the characteristic length, it needs to be scaled based on characteristic shear rate $\gamma = u_{char}/L \approx \frac{\alpha}{L} * \frac{1}{L} = (\alpha/L^2)$

The last form of μ_{ref} is : $\mu_{ref} = K\,\gamma^{n-1} = K\,(\alpha/L^2)^{n-1}$ (6)

Where K is the consistency index, n is the power-law index, α is the thermal diffusivity and L is the characteristic length.

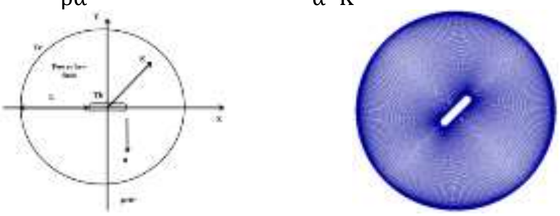
C. Dimensionless Numbers

We define the dimensionless numbers Pr, Ra and Nu by :

$$Pr = \frac{\mu_{ref}\, C_p}{k}, \qquad Ra = \frac{\rho^2 g \beta C_p \Delta T L^3}{\mu_{ref} k^2}, \qquad Nu = \frac{h\,L}{k}$$

After we introduce eq. (6), the definitions of Prandtl and Rayleigh are :

$$Pr = \frac{K L^{2-2n}}{\rho \alpha^{2-n}}, \qquad Ra = \frac{\rho g \beta \Delta T L^{2n+1}}{\alpha^n K}$$



(a) (b)

Fig. 1 Schematic diagram of the present problem (a) and mesh generation (b)

III. NUMERICAL PROCEDURE

The previous equations (1-4) are solved numerically by the Ansys-cfx 16, it is an advanced technology of the computational Fluid Dynamics (CFD) software which used for simulating the fluid flow phenomena, heat transfer and many other related physical and chemical processes especially in aircraft design and manufacturing, design of industrial products and processes and especially in turbomachinery applications.

The ansys cfx uses the finite volume technique as a numerical approach to transform the governing equations into a system of algebric equations after discretizing them into control volumes using a mesh, the SIMPLE algorithm is used to achieve the coupling between pressure and velocity. The second order upwind scheme is used for discritizing the diffusion terms whereas the high resolution scheme is used for the convective term. On the inernal flat tube and the external cylinder, the boundary condition for velocities are null. The laminar model is set to predict the natural convection flow in the computational domain. The grid size is not uniform in the radial component R, rather it is thick around the boundary conditions for a better precition of the heat transfer phenomenon as depicted in Fig. 1 (b). The results are considered after the solution convergence criteria are set to 10^{-8} for continuity and momentum equations and 10^{-6} for the energy equation.

IV. Validation Of The Numerical Procedure

To ensure the numerical results provided here, we chose two benchmark cases of the natural convection in concentric annulus among the wide number of publications in this field to be validated against, this comparison is based on considering the heat transfer rate across the boundary layer to determine the efficiency of the convective heat transfer. The first one is the work of M.H. Matin and W.A. Khan [14] for Newtonian fluid which covered the laminar regime ($Ra \leq 10^5$) and the second one is the experimental work of R.Y. Sakr et al. [6] for Newtonian fluid in the transient regime; for this purpose, the geometry is modified to concentric annulus with elliptic internal cylinder. It is obvious from the comparison results between the present and the prior works (Figs. 2 and 3) that our data are in good agreement with them.

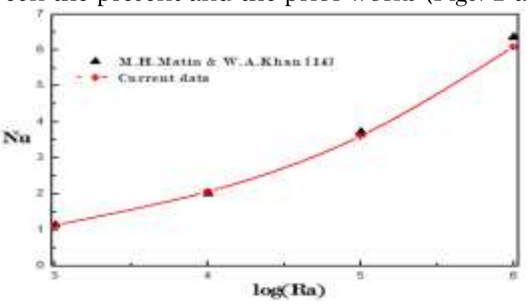


Fig. 2. Comparison of the Nusselt number between the current data and the results of M.H. Matin and W.A. Khan [14] for concentric cylinders when RR=2.6, Pr=0.7 and different Rayleigh numbers.

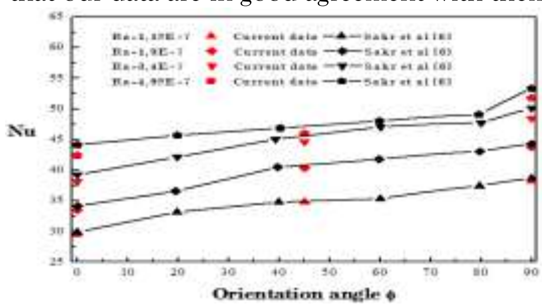


Fig. 3. Comparision between the current data for the Nusselt number and the results of Sakr et al. [6] for concentric cylinders when RR=6.4, Pr=0.71 and different Rayleigh numbers.

V. RESULTS

A. Features of isotherms and velocity vectors

Fig. 4 depicts the isotherms and velocity vectors behaviors for different values of flow indices n and inclination angles ϕ at fixed Prandtl and Rayleigh numbers. Both fluid flow and thermal fields are affected by the changing in flow index n, for isotherms, it is observed that there is a dense region on the crown of the internal flat tube and it diminishes as n grows, this reflects to the fact that the thermal gradient and then the heat transfer is larger in pseudoplastic fluid than in dilatant fluid ; Moreover, the fluid circulation is more rapid for pseudoplastic fluids than dilatant fluids which is explained by the convection transport is stronger for these fluids ; Finally, the fluid circulation intensity for dilatant fluids is concentrated around the internal flat tube so that it uneasily rises upwards in the numerical domain which let us deduce that the conduction is the dominant mechanism of heat transfer in these fluids.Fig. 5 displays the evolution of isotherms and velocity vectors under the effects of Rayleigh numbers $Ra=10^3$, 10^4 and 10^5 respectively. Both the isotherms and velocity vectors are symmetrical about the Y axis except the case of inclined flat tube ($\phi=45°$) where the position of the flat tube is not perpendicular to gravity force. For small values of Rayleigh numbers ($Ra=10^3$), the fluid circulation intensity is very low and it is located around the internal flat tube due to the weak effects of buoyancy force herein, as Rayleigh number values increase ($Ra=10^4$) the fluid circulation inside the numerical domain rise up and gradually increased so that the flow intensity becomes stronger which is a sign for increment the effects of convection, these effects undoubtedly grow for higher Rayleigh numbers ($Ra=10^5$) where it is manifested in the important flow intensity. Isotherm patterns show the same behavior as velocity vectors in axle to the y axis, it is observed that there is a thermal plume region on the upper part of the numerical domain and it extends at higher values of Rayleigh number which can help explaining the large heat transfer here owing to an increase in buoyancy force, the mode of heat transfer changes from conduction to convection dominant for higher values of Rayleigh number.

B. Velocity Profiles*To throw the light on the effects of the inclination angle* ϕ and the flow index n on the velocity, the velocity profiles in the cylindrical gap along the x and y coordinates are drawn and will be discussed for *Pr = 100 and Ra =* 10^5. Notice that these lines pass through where the thermal region is located.

The velocity patterns over the x and y coordinates are represented in Fig. 6. It is obviously visible that the velocity magnitude for pseudoplastic fluids is more important than both Newtonian and dilatant fluids for all cases, this is because of the shear thinning property which decreases the shear strain rate and then the fluid resistance as the shear stress applied augments ; Furthermore, the velocity patterns over the x coordinate are lower than that over the y coordinate and this is due to the fact that the thermal region is concentrated on top of the internal flat tube whereas the horizontal line passes only through its extensions from the lower side ; Finally, the velocity magnitude for the geometry (ϕ = 90°) is bigger than the geometry (ϕ = 0°) and this is clearly visible in pseudoplastic fluids as the inclination angle enhances the fluid movement and then the velocity is higher.

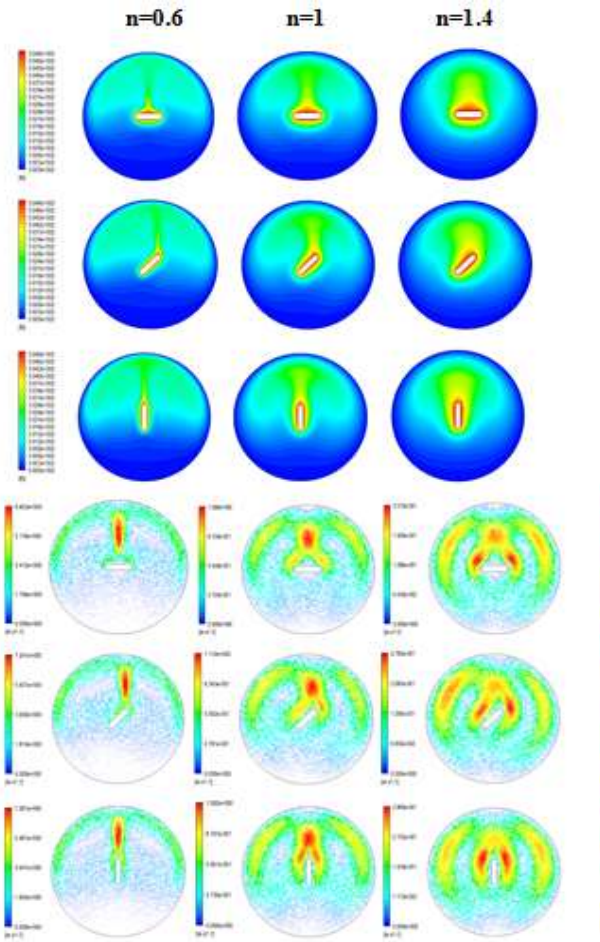


Fig. 4 Isotherms (up) and velocity vectors (down) at Pr=100, Ra=10^4 and various values of power law index n and inclination angle (ϕ=0°) first row, (ϕ=45°) second row, (ϕ=90) third row.

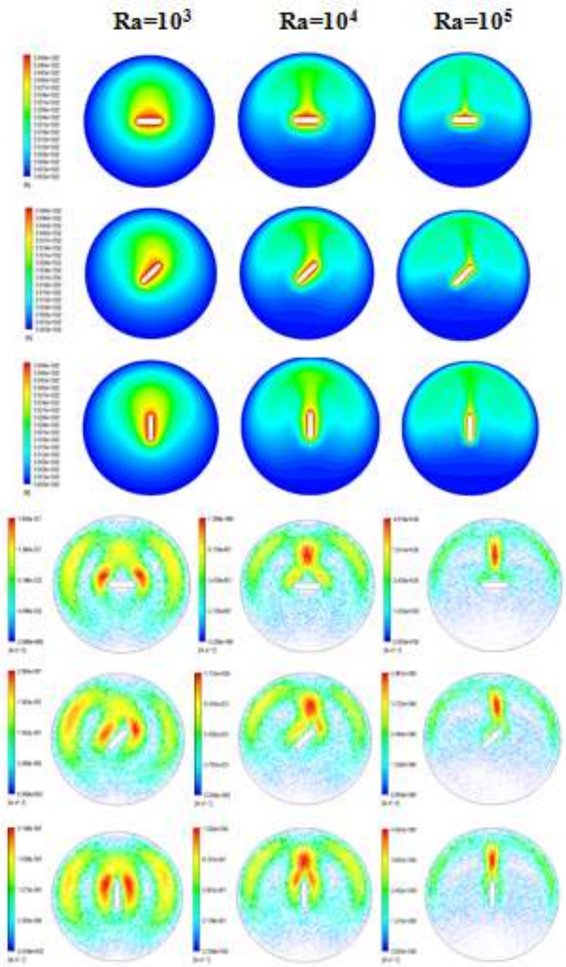


Fig. 5 Isotherms (up) and velocity vectors (down) at Pr=100, n=1 and various values of Rayleigh number Ra and inclination angle (ϕ=0°) first row, (ϕ=45°) second row, (ϕ=90) third row.

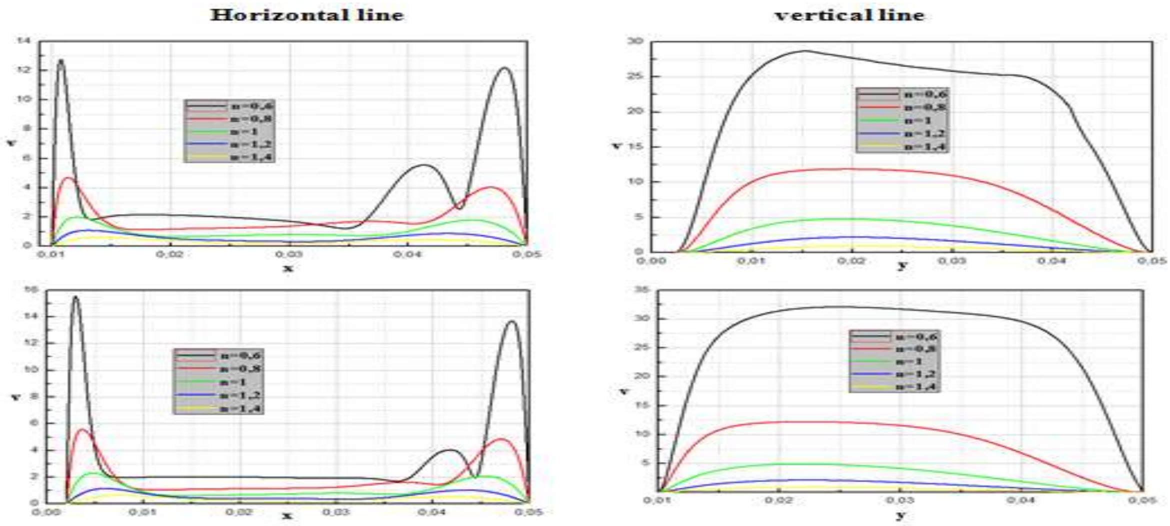


Fig. 6 Velocity profiles along both x and y coordinates for Pr = 100, Ra = 10^5 and ϕ = 0° (first row), ϕ = 90° (second row).

C. Nusselt Number

The changes of the average Nusselt number in the cylindrical gap with respect to the different parameters for ϕ = 0°, ϕ = 45° and ϕ = 90° are represented in Fig. 7. It is easily seen that there is an augmentation in the Nu number when n decreases for a given value of Ra, this means that the heat transfer rate minimizes gradually until settling toward the conduction regime for big values of n (n=1.4) and this is due to the shear thickening property which forced the fluid's particules between them to create a big resistance to the flow at high shear stresses. Also, the Nusselt number is small for low Ra numbers (Ra=10^3) which simply indicates that the mechanism of heat transfer is by conduction, the Nu number starts to grow when Ra increases (Ra=10^4) so the strength of buoyancy effects begins to appear especially for pseudoplastic fluids and less for Newtonians, this means that the heat transfer occurs mainly by convection and it lasts for higher Ra numbers (Ra=10^5) where it becomes more remarkable to include dilatant fluids as well. The effects of Pr number on the heat transfer are negligible except for pseudoplastic fluids at high Ra numbers. An interesting point that can be observable is that the increase in the inclination angle ϕ causes an augmentation in the Nu number, this increment is particularly evident in pseudoplastic fluids at high Ra number values rather than the others due to their characteristics which exhibites lower viscosity and then better flow; Therefore, it is to be concluded that we contributed in the heat transfer enhancement by increasing the inclination angle.

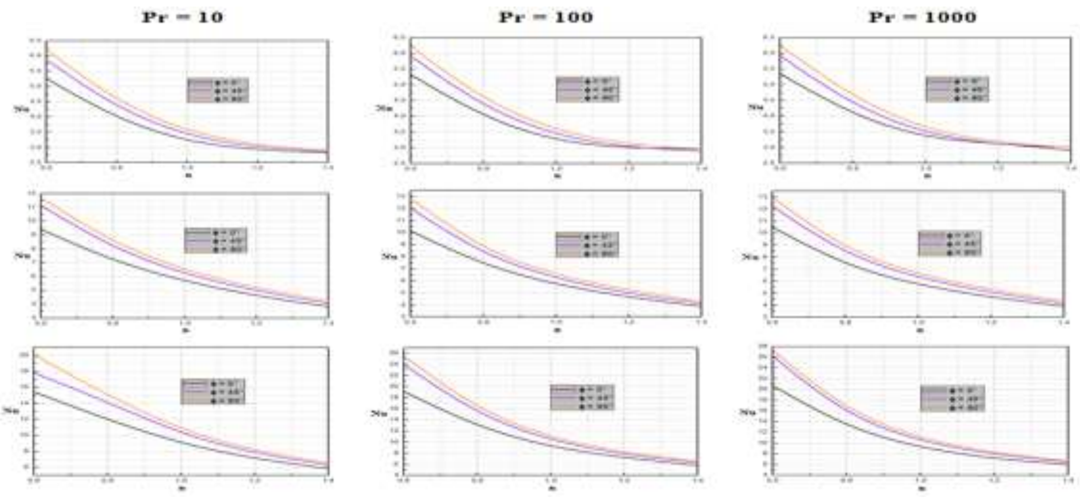


Fig. 7. Average Nusselt number versus power-law ndex n for different parameters of Pr, ϕ and Ra = 10^3 (first row), Ra = 10^4 (second row), Ra = 10^5 (third row).

VI. CONCLUSIONS

The laminar natural convection of non-Newtonian power-law fluid in cross sectioned outer cylinder and inclined internal flat tube was presented numerically. The effects of various power-law indices, inclination angles and Rayleigh and Prandtl numbers on the flow and thermal fields are analysed and interpreted. We come to the following findings :

1/ the fluid movement in pseudoplastic fluids is faster than Newtonian and dilatant fluids, this is apparent at high Rayleigh number values where the fluid recirculation is big and then higher swirl flow due to the gradient in density in the cylindrical gap.

2/ The average Nusselt number is proportional to the Rayleigh number, this last one drives the buoyancy force.

3/ the Prandtl number effects on the heat transfer are insignificant except when combined with high Rayleigh values for pseudoplastic fluids , this refers to the large gradual thinning of momentum and thermal boundary layers in these fluids.

4/ the thermal and velocity fields are symmetric about the y axis for inclination angles $\phi = 0°$ and $\phi = 90°$, this last one corresponds with the maximum heat transfer and accordingly the best for cooling purposes.

NOMECLATURE

C_p : Specific heat capacity, J $kg^{-1}K^{-1}$
e_{ij} : Rate of strain tensor, s^{-1}
g : Gravity acceleration, m s^{-2}
h : Heat transfer coefficient, W m^{-2} K^{-1}
k : Thermal conductivity, W m^{-1} K^{-1}
K : Consistency index of the power-law
L : Characteristic Length, m
n : Flow index
Nu_{ave} : Average Nusselt number
P : Pressure
Pr : Prandtl number
Ra : Rayleigh number
R : Radius, m
T : Temperature
T_h : Internal flat tube temperature, °K
T_c : external cylinder temperature, °K
u, v : Radial and tangential velocities, m s^{-1}
x, y : Cartesian coordinates
x_i : Coordinate in the ith direction, m
Greek symbols
α : Thermal diffusivity, m^2 s^{-1}
β : Volume coefficient of expansion, K^{-1}
γ : rate of strain tensor, s^{-1}
τ_{ij} : Stress tensor, Pa
ΔT : Difference between hot and cold temperatures, $T_h - T_c$
θ : Dimensionless temperature
μ : Dynamic viscosity, N s m^{-2}
μ_{ref} : Reference viscosity
ρ : Density, Kg m^{-3}
ϕ : Orientation angle, °

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