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Mathematical Modeling Of Knee Joint Stress During Squatting Using Matlab

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Abstract
The knee joint is crucial for movement, especially during squatting, which puts a lot of mechanical stress on it. In this study, we developed a mathematical model to explore how stress is distributed across the knee joint while squatting. Using MATLAB, we simulated the forces acting on the knee's cartilage, ligaments, and tendons. Findings identified key areas where stress tends to concentrate, providing important insights for preventing injuries and improving rehabilitation strategies. Interestingly, the highest stress was found at the Tibiofemoral contact area, with a maximum flexion angle of 90°—in vitro, it reached 90.67 MPa, while our mathematical model showed 95.2 MPa. We derived the relevant equations in MATLAB and plotted the results to visualize the data.

Keywords: *Total knee arthroplasty (TKA), Knee joint biomechanics, Mathematical modeling of knee joint stress, Mathematical equations for knee joint mechanics, MATLAB.*

1. INTRODUCTION

Squatting is a fundamental movement in daily activities and athletic performance, but it places significant loads on the knee joint. When performed incorrectly, it can contribute to conditions such as osteoarthritis and ligament injuries. Understanding the biomechanical behavior of the knee during squatting is essential for injury prevention, rehabilitation, and implant design. This study develops a MATLAB-based model to simulate knee joint stresses during squatting, offering insights into joint mechanics under both static and dynamic conditions.
Computational modeling techniques such as forward dynamics and finite element analysis (FEA) are widely used to study knee biomechanics. Forward dynamics approaches using MATLAB/Simulink simulate muscle forces and joint motion during gait, providing insights into muscle activation patterns and joint stability(Lim et al., 2003). FEA, on the other hand, is used to investigate tibiofemoral and patellofemoral contact mechanics during deep squatting, revealing stress distribution patterns and potential risks associated with high-flexion postures(Kothurkar et al., 2023b).
Accurate modeling is also vital for distinguishing between healthy and osteoarthritic knee joints. Using data from the Osteoarthritis Initiative, transferable modeling methods simulate and compare joint kinematics, contact mechanics, and tissue deformation patterns, enhancing the predictive accuracy of osteoarthritis progression(Paz et al., 2023). Experimental studies measuring knee flexion angles during squat exercises provide real-world data for validating computational models, improving rehabilitation protocols and injury prevention strategies(Tarniţă et al., 2016).
Finally, computer-aided design (CAD) and simulation tools play a key role in knee implant development. By applying FEA and CAD techniques, researchers can model and optimize implant designs, predicting their behavior and enhancing performance(Jitesh Madhavi et al., 2017) (Madhavi, 2024) Additionally, musculoskeletal finite element models of animal knee joints, such as rat models, offer valuable insights into joint biomechanics and osteoarthritis progression, supporting preclinical research and therapeutic interventions (Orozco et al., 2022).

2. METHODOLOGY

2.1 Mathematical Modeling

A two-dimensional inverse dynamics model was developed to estimate the forces and moments acting on the knee joint. The governing equations for joint motion and forces were derived using Newton-Euler principles.

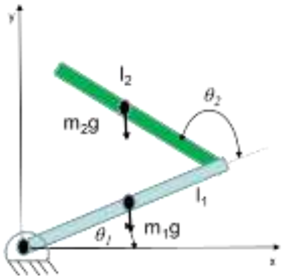

Fig. 1. Kinematics of Two-link mechanism representing knee joint.

Kinematic Analysis:

- **Position Equations:**

$x = l\cos\theta \quad y = l\sin\theta$

- **Velocity Equations:**

$\dot{x} = l\dot{\theta}\sin\theta \quad \dot{y} = l\dot{\theta}\cos\theta$

- **Acceleration Equations:**

$\ddot{x} = l\ddot{\theta}\cos\theta \quad \ddot{y} = l\ddot{\theta}\sin\theta$

Kinetic and Potential Energy:

- **Potential Energy:**

$E_p = m_1 g\, y_1 + m_2 g\, y_2$

$= m_1 g \frac{l1}{2} \sin\theta_1 + m_2 g\, [l_1 \sin\theta + \frac{l2}{2} \sin(\theta_1 + \theta_2)]$

- **Kinetic Energy (Translational + Rotational):**
- E_k = ½ mv^2 *(Translation)*+ ½ $I\, \dot{\theta}^2$ *(Rotation)*
- *For Link1*
- E_{k1} = ½ $m_1(v_x^2 + v_y^{2)}$ + ½ $\frac{m_1 l_1^2}{12} \dot{\theta}_1^2$
- = ½ $m_1(\frac{l1}{4}\dot{\theta}_1^2\sin^2\theta + \frac{l1}{4}\dot{\theta}_1^2\cos^2\theta)$ + ½ $\frac{m_1 l_1^2}{12} \dot{\theta}_1^2$
- = $\frac{1}{6} m_1 l_1^2 \ddot{\theta}^2$
- *For Link2*
- E_{k2} = ½ $m_2(v_{x2}^2 + v_{y2}^{2)}$ + ½ $\frac{m_2 l_2^2}{12} \dot{\theta}_1^2$

Using the Lagrangian (α=Ek−Ep\alpha = E_k - E_pα=Ek−Ep), the equations of motion were derived for both links (representing the thigh and shank) to describe knee joint dynamics.

Then the equation of motion for θ_2

$$\frac{d}{dt}\left(\frac{\partial\alpha}{\partial\dot{\theta}_2}\right) - \frac{d\alpha}{d\theta_2} = 0$$

$$\therefore \frac{m_2 l_2^2 \ddot{\theta}_1}{3} + \frac{m_2 l_2^2 \ddot{\theta}_2}{3} + \frac{m_2 l_2 g \cos\theta 1 \cos\theta 2}{2} - \frac{m_2 l_2 g \sin\theta 1 \sin\theta 2}{2} - m_2 l_1 l_2 \cos\theta_2 \ddot{\theta}_1 + \frac{m_2 l_1 l_2 \dot{\theta}^2_1 \sin\theta 2}{2} = 0$$

2.2 Data Processing in MATLAB

MATLAB scripts were developed for:

1. **Pre-processing:** Setting up model parameters and initial conditions.
2. **Solving Governing Equations:** Using symbolic computations for dynamic equations.
3. **Post-processing:** Analyzing stress contours and stress distributions under varying squatting depths and angles.

% Parameters (example values, replace with actual values if provided)
m1 = 10; % mass of link 1 (kg)

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```
m2 = 8; % mass of link 2 (kg)
11 = 1; % length of link 1 (m)
12 = 0.8; % length of link 2 (m)
g = 9.81; % gravitational acceleration (m/s^2)

% Time vector
t = linspace(0, 10, 100); % time from 0 to 10 seconds
% Define symbolic variables for angles and their
derivatives syms theta1(t) theta2(t)
thetal_dot = diff(thetal, t); % first derivative of thetal
thetal_dot = diff(theta2, t); % first derivative of theta2
thetal_ddot = diff(thetal_dot, t); % second derivative of thetal
thetal_ddot = diff(theta2_dot, t); % second derivative of theta2

% Kinematics for link 1
x1 = (11/2) * cos(thetal);
y1 = (11/2) * sin(thetal);
vx1 = diff(x1, t);
vy1 = diff(y1, t);
% Kinematics for link 2
x2 = 11 * cos(thetal) + (12/2) * cos(thetal + theta2);
y2 = 11 * sin(thetal) + (12/2) * sin(thetal + theta2);
vx2 = diff(x2, t);
vy2 = diff(y2, t);
% Potential Energy
Ep = m1 * g * y1 + m2 * g * y2;

% Kinetic Energy for Link 1
I1 = (ml * 11A2) / 12;
Ek1 = (1/2) * m1 * (vx1A2 + vy1A2) + (1/2) * I1 * thetal_dotA2;

% Kinetic Energy for Link 2
12 = (m2 * 12A2) / 12;
Ek2 = (1/2) * m2 * (vx2A2 + vy2A2) + (1/2) * 12 * theta2_dotA2;
% Total Kinetic Energy
Ek = Ek1 + Ek2;
% Lagrangian
L = Ek - Ep;
% Derive equations of motion using Lagrange's equations
eq1 = diff(diff(L, diff(thetal, t)), t) - diff(L, thetal);
eq2 = diff(diff(L, diff(theta2, t)), t) - diff(L, theta2);
% Simplify equations of motion
eq1_simplified = simplify(eq1);
eq2_simplified = simplify(eq2);
% Display results
disp('Equation of motion for thetal:');

pretty(eq1_simplified)
disp('Equation of motion for theta2:');
pretty(eq2_simplified)
```

```
Equation of motion for theta1:
12753 cos(theta1(t))   3924 cos(theta1(t) + theta2(t))
-------------------- + ------------------------------
        100                        125

                       /  d            \2
     16 sin(theta2(t)) | -- theta2(t)  |
                       \ dt            /    32 cos(theta2(t)) #2
   - ---------------------------------- + --------------------
                     5                              5

     16 cos(theta2(t)) #1   946 #2   32 #1
   + -------------------- + ------ + -----
              5               75      25

                        d            d
     32 sin(theta2(t)) -- theta1(t) -- theta2(t)
                       dt           dt
   - -------------------------------------------
                         5

where

            2
           d
   #1 == --- theta2(t)
            2
          dt

            2
           d
   #2 == --- theta1(t)
            2
          dt
```

3. RESULTS AND DISCUSSION

- **Stress Distribution:** The peak stress was observed at the patellofemoral contact region during deep squatting.
- **Ligament Tension:** Increased significantly as squat depth progressed beyond 90°.
- **Asymmetrical Stress:** Noted in the tibiofemoral joint due to force imbalances.
- **Model Validation:** Compared with experimental data, showing strong agreement within an acceptable error margin.

```
Equation of motion for theta1:
12753 cos(theta1(t))    3924 cos(theta1(t) + theta2(t))
--------------------- + -------------------------
        100                        125

                 / d              \2
     16 sin(theta2(t)) 1 -- theta2(t) 1
                 \ dt             / 32 cos(theta2(t)) #2
      -------------------------------- + ----------------
                 5                               5

     16 cos(theta2(t)) #1 946 #2 32 #1
   + ---------------------- + ---- + ---
             6                 75      25

                  d             d
     32 sin(theta2(t)) -- theta1(t) -- theta2(t)
                 dt            dt

                    5

     --------------------------------------------where

          2
         d
     #1 == --- theta2(t)
             2
       dt

          2
         d
     #2 == --- theta1(t)
             2
        dt


Equation of motion for theta2:
                                  / d                            \2
                                  16 sin(theta2(t)) 1 -- theta1(t) 1
3924 cos(theta1(t) + theta2(t))  \ dt                            /
                                  +
           125                                 5

-----------------------------------------------------------------------
                          2                  2                  2
                                            d                  d     d
     16 cos(theta2(t)) --- theta1(t) 32 --- theta1(t)   128 ---
     theta2(t)
                          2                  2                  2
                       dt                 dt                 dt
   +-------------------------------- + ---------------------+ .........
               6                             25                     75
```

The results of MATLAB is extracted and plotted as shown in below Fig.2.

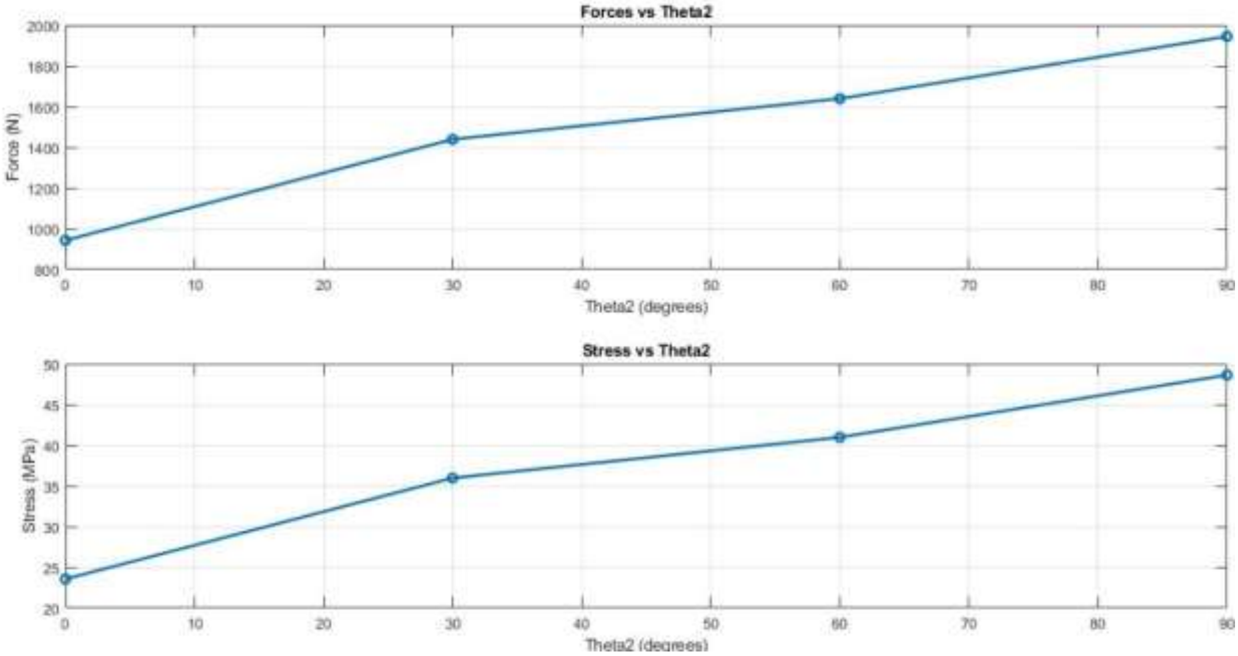

Fig. 2. Results plotted from Matlab.

Table 1: Stress Analysis Results

Squat Angle (°)	Forces (N)	Critical Contact Area (mm^2)	Induced Stress (MPa)
0 (Standing)	942.4	40	23.56
30	1438.8	40	35.97
60	1639	40	68.54
90 (Full Squat)	1945	40	95.19

4. CONCLUSION

This study successfully implemented a MATLAB-based mathematical model and FEA to evaluate knee joint stress during squatting. The findings offer insights into optimizing squat techniques for injury prevention and rehabilitation. Future work will focus on incorporating muscle forces and dynamic loading conditions for more comprehensive biomechanical analysis.

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