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Fully Fuzzy Linear Programming Resolution Through Ranking Function Methodology

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Abstract—*This study introduces a novel and efficient approach for solving Fully Fuzzy Linear Programming Problems (FFLPP) by leveraging Triangular Fuzzy Numbers (TFN). The proposed method simplifies FFLPP by converting it into an equivalent Crisp Linear Programming Problem (LPP) through a systematic defuzzification process. This transformation ensures computational efficiency while maintaining solution accuracy. Comparative evaluations against existing techniques highlight the advantages of the proposed methodology. Numerical examples are included to validate the approach and demonstrate its effectiveness in practical applications.*
Keywords *– Linear Programming with Fuzzy Constraints, Triangular, Fuzzy Number Coefficients, Fully Fuzzy Optimization, Fuzzy Simplex Algorithm*

INTRODUCTION

Linear programming serves as a powerful analytical tool in operations research for solving practical optimization problems. However, conventional linear programming approaches often face limitations when dealing with uncertain or imprecise parameters. To overcome this challenge, fuzzy set theory has been increasingly adopted as an effective framework for decision-making under uncertainty. The integration of fuzzy set theory with Fuzzy Linear Programming Problems (FLPP) enables practitioners to model and solve optimization problems where coefficients and constraints contain inherent ambiguity.
The foundations of fuzzy linear programming were established through significant contributions from various researchers. Zimmermann [6] pioneered the formulation of FLP models, while Tanaka et al. [1] expanded the theoretical framework based on Bellman and Zadeh's [2] fundamental concepts. Zimmermann [6] further demonstrated how FLPP models could be converted into equivalent crisp linear programming problems (LPP), showing that fuzzy decisions emerge from the intersection of goals and constraints. These developments have spurred extensive research in FLPP applications for constrained optimization problems with uncertain data.Over the past thirty years, duality theory in FLPP has attracted considerable attention. Numerous studies have explored fuzzy dual problems, with Verdegay [9] employing parametric linear programming to establishconditions under which fuzzy primal and dual problems yield identical solutions.Ebrahimnejad [3] proposed a simplified method for addressing FLP problems, utilizing a coefficient matrix with real numbers and representing objective coefficients and right-hand side values with symmetric trapezoidal fuzzy numbers (STFN). The study showed that solving a corresponding crisp LP problem could provide an accurate solution to the initial FLP problem.
The present study proposes a novel FLPP solution method where triangular fuzzy numbers (TFN) represent all parameters - including objective function coefficients, constraint coefficients, and right-hand side values. We establish that proper defuzzification of TFN parameters can efficiently yield optimal FLPP solutions.This paper is structured as follows: Section 2 reviews essential concepts of fuzzy arithmetic operations. Section 3 presents the formal problem formulation for Fuzzy Linear Programming, extending the works of [4] and [3]. Section 4 details our proposed methodology and compares its performance with existing approaches. Finally, Section 5 concludes the study and suggests potential directions for future research.

PRELIMINARIES

In fuzzy set theory, elements are characterized by their degree of belonging through a membership function that assigns each element in the domain a value between 0 and 1, where 0 indicates complete non-membership, 1 represents full membership, and intermediate values denote partial belonging - this continuous spectrum enables the representation of vague or uncertain information by allowing elements to simultaneously belong to multiple sets with varying degrees of membership, unlike classical set theory's binary classification, making it particularly useful for modeling real-world scenarios where boundaries between categories are often imprecise or overlapping.

A. Definition:

A fuzzy number $\tilde{B}$ constitutes a special class of fuzzy sets characterized by its membership function $\mu_{\tilde{B}}(t)$: $\mathbb{R} \rightarrow [0,1]$ that must satisfy three fundamental mathematical properties:

(1) **Continuity Condition:** The membership function $\mu_{\tilde{B}}(t)$ exhibits piecewise continuity over its domain, ensuring well-defined transitions between membership grades.

(2) **Convexity Property:** For any $\alpha \in (0,1]$, the α-cut $\tilde{B}\_\alpha = \{t \mid \mu_{\tilde{B}}(t) \geq \alpha\}$ forms a convex subset of $\mathbb{R}$, mathematically expressed as: $\forall t_1, t_2 \in \mathbb{R}$, $\forall \lambda \in [0,1]$: $\mu_{\tilde{B}}(t)(\lambda t_1 + (1-\lambda) t_2) \geq \min(\mu_{\tilde{B}}(t_1), \mu_{\tilde{B}}(t_2))$

(3) **Normality Condition:** At least one component is present $t_0 \in \mathbb{R}$ such that sup $\mu_{\tilde{B}}(t) = 1$, guaranteeing maximal membership attainment.

B. Definition: The α-cut of a fuzzy set A, denoted as A_a, is mathematically expressed as: $A_a = \{t \in X \mid \mu_a(t) \geq \alpha\}$ where X represents the universal set and $\mu_a(t)$ is the membership function. This concept is alternatively termed as a level set.

Strong α-cut:

A strict version, called the strong α-cut, is defined by: $A_a^+ = \{t \in X \mid \mu_a(t) > \alpha\}$

C.Classical (Crisp) Sets:

A set A within a universal collection X is considered a crisp set if it can be precisely defined through a binary membership function $\chi_A(t)$: X → {0,1}. This characteristic function operates as follows:

$\chi_A(t)$:) = 1 indicates complete inclusion of element x in set A

$\chi_A(t)$: = 0 signifies absolute exclusion of x from set A

This binary classification creates a sharp, unambiguous boundary between members and non-members of the set.

D. Fuzzy Sets:

A collection Ã is classified as a fuzzy subset of a universal set X when it is characterized by a membership function $\mu_{\tilde{A}}(t)$: X → [0,1]. This function assigns each element t∈ X a membership grade where:

1) $\mu_{\tilde{A}}(t) = 1$ denotes full membership
2) $\mu_{\tilde{A}}(t) = 0$ indicates complete non-membership
3) Any value between 0 and 1 represents a partial membership degree

E. A triangular fuzzy number, denoted as $\tilde{A} = (a_1, a_2, a_3)$, is formally defined by its piecewise linear membership function $\mu_{\tilde{A}}(x)$ that satisfies the following conditions:

$$\mu_A(x) = \begin{cases} 0, & x > a_3, x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2 \\ 1, x = a_2 & \\ \frac{x-a_3}{a_2-a_3}, & a_2 < x \leq a_3 \end{cases} \quad (1)$$

F: Definition: If $\widetilde{A}$ is a fuzzy number, the robust ranking index is defined by

$$R(\widetilde{A}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha \quad (2)$$

Where $(a_\alpha^L, a_\alpha^U) = \{(b-a)\alpha + a, d - (d-c)\alpha\}$ is the $\alpha -$ cut of the fuzzy numberA.

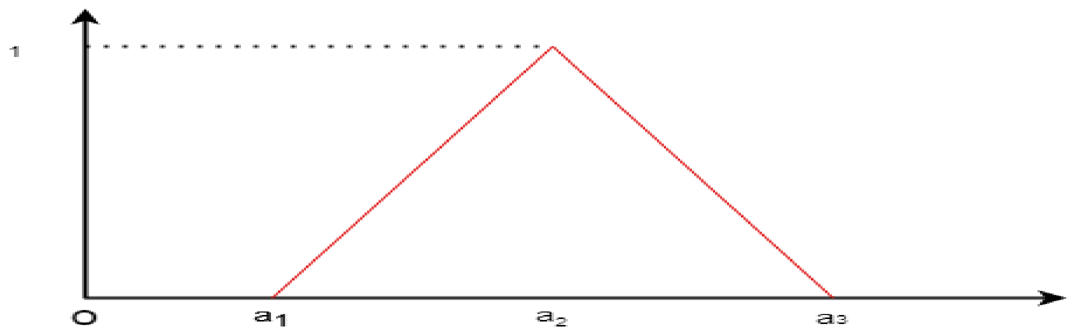

Fig. Triangular fuzzy number (TFN)

I. FLP APPROACHES BY BY RESEARCHERS

In [4],authors presented a unique FLP solution strategy, utilizing real-number coefficient matrices alongside symmetric trapezoidal fuzzy numbers for additional parameters. The model is defined by the following formulation

$S \approx [\tilde{P} * \tilde{t}]$ (3)

Subject to: $A\tilde{t} \preccurlyeq \tilde{B}$

$\tilde{t} \succcurlyeq \tilde{0}$

Where:

- $\tilde{P} \in F(R)^n$, $\tilde{B} \in F(R)^n$ are fuzzy parameters
- $A \in R^{m\times n}$ is the crisp coefficient matrix
- $\tilde{t} \in F(R)^n$ is the fuzzy decision variable

Fuzzy Basic Solution Definition:

A solution vector $\tilde{t} = (\tilde{t}_1, \tilde{t}_2,..., \tilde{t}_k)$ where each element $\tilde{t}_j \approx (-t_j, t_j, \delta_j, \delta_j)$ is a fuzzy basic solution if:

1. $\tilde{t}_j \succcurlyeq \tilde{0}$ and $\delta_j \geq 0$ for all j
2. The non-zero components correspond to linearly independent columns of A

The constraint system expands to:

$a_1\tilde{t}_1 + a_2\tilde{t}_2 + ... + a_k\tilde{t}_k + a_{k+1}(-t_{k+1}, t_{k+1}, \delta_{k+1}, \delta_{k+1}) + ... + a_n(-t_n, t_n, \delta_n, \delta_n) \approx \tilde{B}$

Ebrahimnejad-Tanava Solution Method:

Fuzzy Problem:

Max $\tilde{S} = \sum \tilde{p}_j\tilde{t}$ (4)

Subject to:

$\sum A_{ij}\tilde{t}_j \preccurlyeq \tilde{b}_i$

Ranking Function:

For STFN $\tilde{B} = (\delta^L, \delta^U, h, h)$

$R(\tilde{B}) = \frac{(\delta^L + \delta^U)}{2}$

Rewriting the given equation enables transform the Fuzzy LP problem stated above into the subsequent classical Linear PP. $\tilde{t} \succcurlyeq \tilde{0}$

Crisp Transformation:

The equivalent crisp LPP becomes

Max $\tilde{S}^* = \sum R(\tilde{p}_j)\, \tilde{t}_j^*$ (5)

Subject to:

$\sum A_{ij}\tilde{t}_j \leq R(\tilde{b}_i)$

$\tilde{t} \succcurlyeq \tilde{0}$

This transformed linear program can be solved using standard operation research methods.

THE DEVELOPED FFLP SOLUTION APPROACH

Here we extends the solution methodology developed by Ebrahimnejad and Tavana to address Fully Fuzzy Linear Programming Problems (FFLPP) featuring fuzzy coefficients $\tilde{t}$_j in the constraint equations. We formulate the fuzzy linear programming problem (FLPP) with these fuzzy parameters, building upon previous work while introducing new computational approaches to handle the fuzzy constraint coefficients effectively. Our extended method maintains the theoretical rigor of the original framework while adapting it to solve more complex FFLPP cases where all constraint terms exhibit fuzzy characteristics, ultimately leading to robust solutions for problems with imprecise parameters in both objectives and constraints.

Objective Function: Max $\tilde{S} = \sum_{j=1}^{n} \tilde{p}_j \tilde{t}_j$ (6)

Subject to Constraints: $\sum_{i=1}^{m} A_{ij} \tilde{t}_j \lessapprox \tilde{b}_i$

$\tilde{t} \geq \tilde{0}$

then, convert the above FLPP problem (6) to crisp LP problem as

Max $S^* = \sum_{j=1}^{n} p_j^* t_j^*$(7)

such that $\sum_{i=1}^{m} A_{ij} t_j \leq b_i^*$

$t \geq 0$

where

$$p_j^* = 0.5 \int_0^1 (p_\alpha^L, p_\alpha^U) d\alpha, \; b_i^* = 0.5 \int_0^1 (b_\alpha^L, b_\alpha^U) d\alpha, \; A_{ij} = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha \quad (8)$$

The optimization problem (7) can be effectively solved using the conventional simplex method to obtain crisp optimal values for tj. Our approach for solving the FFLPP involves four systematic steps: First, we perform the necessary multiplication operations on the relevant trapezoidal fuzzy numbers. Next, we employ a ranking function to transform the fuzzy objective function into its crisp equivalent. Subsequently, we convert all fuzzy constraints into crisp formulations. Finally, Through implementation of the standard simplex algorithm, we compute the optimal solution for the transformed crisp linear program

II. WORKHED EXAMPLE

Calculate the optimum result for this Fully Fuzzy Linear Programming example.

$$\text{Max } \tilde{S} \approx (1,3,4)\tilde{t}_1 + (1,2,3)\tilde{t}_2 \quad (9)$$

(9)

Such that

$(0,1,3)\tilde{t}_1 + (2,3,5)\tilde{t}_2 \leq (3,4,6)$

$(1,2,4)\tilde{t}_1 + (0,1,2)\tilde{t}_2 \leq (1,2,5)$ (10)

where$\tilde{t}_1$, $\tilde{t}_2 \geq \tilde{0}$. Following the defuzzification process using the specified formula from Definition (F), we obtain

$$p_1^* = 0.5 \int_0^1 \{(3-1)\alpha + 1 + 4 - (4-3)\alpha\} d\alpha$$

$$= 0.5 \int_0^1 \{2\alpha + 1 + 4 - \alpha\} d\alpha$$

$$= 0.5 \int_0^1 \{5 + \alpha\} d\alpha$$

$$= 2.75$$

$\therefore p_1^* = 2.75$, Similarly we can find the other values as below

$$p_2^* = 2$$

$b_1^* = 4.25$
$b_2^* = 2.5$
Similarly, we can find

$$A_{11} = 1.25, \quad A_{12} = 3.25,$$
$$A_{21} = 2.25, \quad A_{22} = 1$$

By applying the derived values, we can transform the fuzzy linear programming problem (9) into its equivalent crisp formulation as follows

Objective function: Max $S = 2.75t_1 + 2t_2$ (11)

Subject to constraints :
$1.25\, t_1 + 3.25\ t_2 \leq 4.25$
$2.25\, t_1 - \ t_2 \leq 2.5$
$t_1, t_2 \geq 0$
The standard simplex method allows us to formulate problem (11) with the following objective function representation:

Max $S = 2.75t_1 + 2t_2 + 0s_3 + 0s_4$ (12)
Subject to constraints:

$$1.25\, t_1 + 3.25\ t_2 + 0s_3 = 4.25$$
$$2.25\, t_1 - \ t_2 + 0s_4 = 2.5$$
$$t_1, t_2, \ s_3, \ s_4, \geq 0$$

where s_3, s_4 are slack variables.
TABLE I

(B)	p_b	t_b	t_1	t_2	s_3	s_4	(R)
			2.75	2	0	0	
$\mathbf{s_3}$	0	4.25	1.25	3.25	1	0	3.4
s_4	0	2.5	2.25	-1	0	0	1.111
Max			2.75	2	0	0	

The linear programming (LP) formulation in Problem 12 can be effectively addressed through the conventional primal simplex approach. Table I displays the initial simplex tableau, where t_1 enters as the incoming variable while s_2 is identified as the departing variable. Subsequent iterations proceed to Table II, generated through pivot operations, which now identifies t_2 as the entering variable and s_2 as the exiting variable. The algorithm culminates in Table III, representing the final optimal tableau obtained after performing the necessary pivot transformations.
Having established this optimal basis for the crisp LP formulation (Problem 12), we subsequently applied these results to determine the corresponding fuzzy optimal solution for the original fuzzy linear programming (FLP) formulation (Problem 9). This solution methodology demonstrates how crisp optimization techniques can be effectively leveraged to solve fuzzy programming problems while maintaining mathematical rigor.
TABLE II

(B)	p_b	t_b	t_1	t_2	s_3	s_4	(R)
			2.75	2	0	0	
s_4	0	2.86	0	3.81	1	-0.56	0.75
$\mathbf{t_1}$	0	1.11	1	-0.44	0	0.44	-2.5
Max -3.0556			2.75	-1.222	0	1.222	

TABLE III

(B)	p_b	t_b	t_1	t_2	s_3	s_4	(R)
			2.75	2	0	0	
t_2	0	0.75	0	1	0.26	-0.15	
t_1	0	1.45	1	0	0.12	0.38	
Max=5.48			0.75	1.45	0.85	0.75	

Based on the preceding Simplex Table III, we derive the optimal solution $S = 5.48$

CONCLUSION

This study presents an enhanced computational approach for solving Fully Fuzzy Linear Programming Problems (FFLPP) where all parameters - including objective function coefficients, constraint coefficients, and right-hand side values - are represented as Triangular Fuzzy Numbers (TFN). Our developed methodology demonstrates two key advantages: (1) it yields solutions with greater precision compared to existing methods, and (2) it achieves these results with significantly reduced computational time requirements.

The proposed technique specifically addresses FFLPP formulations where:

1. Objective function coefficients employ TFN representation
2. Constraint coefficients utilize TFN notation
3. Right-hand side values are expressed as TFN

These improvements in both solution accuracy and computational efficiency make our method particularly suitable for practical applications requiring fuzzy optimization under uncertainty.

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