International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 13s, 2025
https://theaspd.com/index.php

An Hypothesis Testing for Ranking of Design Evaluation Criteria in the Material Flow Layout Selection Problem in the Automobile Industry

Rahul Kumar Gautam[1],**Sudarsanam S.K.**[2]
[1,2]VIT Business School Chennai, Vellore Institute of Technology Chennai

Abstract
When selecting a material flow layout, the layout's design is assessed using various Cr, including distance, throughput time, shape ratio, adjacency, and safety. To rank these Cr, multiple algorithms are employed. Fuzzy AHP, for instance, is utilized to rank the Cr and determine their respective weights. Within the Analytic Hierarchy Process (AHP), a ratio scale, represented as $(Pi_1, Pi_2..., Pi_t)$, *establishes the priorities of evaluation Cr* $\{Cr_1, Cr_2..., Cr_t\}$ *for decision-making. The ratio* Pi_i/P_{ij} *quantifies the preference of Cr* C_i *over . For group decision-making, experts provide subjective estimates of* P_i/P_j, *which form entries in a pairwise comparison matrix. The AHP algorithm then calculates the weights of these Cr, which in turn establishes their ranking. This article applies a statistical method to validate the ranking of design evaluation Cr. This approach involves hypothesis testing to confirm the AHP algorithm's ranking, relying on comparative ratings from automobile industry experts.*
Keywords: *Hypothesis testing, Non-linear programming model, Rank Order testing, Likelihood-ratio test, Analytic Hierarchy Process, Fuzzy Analytical Hierarchy Process, Multi-criteria Decision Making, F-test.*

INTRODUCTION

Rank ordering of the evaluation criteria for material flow layout selection is a key decision-making problem in the automobile industry. The evaluation criteria for evaluating each of the material flow layout alternatives include both qualitative and quantitative criteria such as Distance, Adjacency, Shape ratio, Throughput time, Cost, Efficiency, Sustainability, Flexibility, Safety, Space Utilization, Quality and Lead time.

In the collection of evaluation criteria $\{Cr_1, Cr_2, Cr_3, Cr_4, Cr_5, Cr_6, Cr_7, Cr_8, Cr_{t-1}, Cr_t\}$ for the material flow layout selection problem, the priority vector of the evaluation criteria is $(Pi_1, Pi_2, ..., Pi_{10})$ and for each i and j, the ratio Pi_i/Pi_j gives the preference of evaluation criteria Cr_i to that of Cr_j.

H_0 : $Pi_1 = Pi_2 = Pi_3 = Pi_4, = Pi_5 = Pi_6 = Pi_7 = Pi_8 = Pi_{t-1} = Pi_t$ has to be tested against the alternate hypothesis

H_a : $Pi_1 \geq Pi_2 \geq Pi_3 \geq Pi_4 \geq Pi_5 \geq Pi_6 \geq Pi_7 \geq Pi_8 \geq Pi_{t-1} \geq Pi_t$

where atleast one inequality holds strictly,For any distinct criteria i and j, an expert provides a subjective estimate, aij, for the ratio Cri/Crj. This estimate, aij, is conceptualized as the true ratio Cri/Crj perturbed by a multiplicative error, eij. This type of multiplicative modelling approach has precedents in the work of Dejong (1984) and Crawford and Williams (1985).

a_{ij} can then be written as

$$a_{ij} = \left(\frac{\pi_i}{\pi_j}\right) * e_{ij} \qquad \text{...........} \qquad (1)$$

Let $a_{i,i+1}^{(k)}$ for k=1, 2..., n be the estimate of π_i / π_{i+1} provided by the k-th expert for k =1, 2..., n.

Then $\left(\frac{1}{a_{i+1,i}^{k}}\right)$ can be taken as an estimate of $\left(\frac{Pi_{i+1}}{Pi_i}\right)$, if $a_{i,\,i+1}$ provides an estimate of $\left(\frac{Pi_i}{Pi_{i+1}}\right)$.

Furthermore, an estimate of $(Pi_1,/Pi_{i+1})$ can be obtained by multiplying other aij values, where i/j. Crawford and Williams (1985) confirmed that the distribution of aij demonstrates reciprocal properties. In the context of the multiplicative model (1), it is appropriate to assume that the log-normal distribution effectively represents the common underlying distribution. Based on this, it is reasonable to consider the eij terms as independent and that they follow a log-normal distribution with a zero mean and a variance of σ^2.

Basak (1990) proposed a statistical technique for performing hypothesis tests to assess the null hypothesis (H0) in comparison to the alternative hypothesis (Ha), although the resulting distribution of the test statistic was found to be quite complex. Section 2 will introduce the simplified hypothesis testing procedure later proposed by Basak (2013).

The detailed computation of the proposed test statistic and its distribution will be elaborated in Section 3. To practically demonstrate this hypothesis testing method, a real-world case study from the automobile sector will be analyzed in Section 4. The concluding observations and suggestions for further research will be presented in Section 5.

Hypothesis testing of hierarchical structures

In the context of material flow layout selection, determining the priority of design evaluation criteria is a crucial step within the multi-criteria decision-making (MCDM) process. Various techniques are available to rank these criteria, among which the Analytic Hierarchy Process (AHP) has been selected for this study.

The foundational work of Saaty (1980) offers a detailed explanation of the AHP methodology, laying out its core principles, conceptual framework, and step-by-step procedures. This work is widely regarded as the cornerstone for understanding how AHP can be effectively applied across different domains, including the design of material flow layouts in manufacturing environments.

Karim et al. (2017) provided significant insights into the practical application of AHP in production and operations management. Their study covered key areas such as supply chain management, facility layout planning, and process improvement. They discussed both the strengths and limitations of AHP, offering guidance on how it can be strategically used for decision-making in manufacturing settings.

Wu et al. (2017) introduced a structured approach for warehouse layout design aimed at enhancing material flow efficiency. Their work identified essential factors like travel distance, processing time, and safety considerations, and proposed an AHP-based decision-making framework to assist in selecting the most suitable layout design.

A case study by Bhattacharya et al. (2017) illustrated the practical use of AHP for selecting material handling equipment within a warehouse, emphasizing its relevance to shop floor layout design decisions.

Additionally, Arunyanart et al. (2018) developed an integrated model that combines Data Envelopment Analysis (DEA) with AHP for evaluating plant layout alternatives based on multiple performance criteria.

Earlier, Chakraborty and Banik (2007) applied a multi-criteria decision-making approach to address facility layout selection. Their study assessed six critical layout evaluation factors—material flow, information flow, equipment flow, maintenance, flexibility, and adjacency—across ten different layout options. They utilized the AHP framework to systematically evaluate and rank these alternatives.

While assigning priority rankings and weights to evaluation criteria typically suffices to address material flow layout selection problems, it is essential to validate the ranking methodology through statistical rigor. By subjecting the ranking process to hypothesis testing, researchers can provide empirical support for the reliability of the criteria rankings. This validation strengthens the argument that such rankings can be confidently adopted by organizations across various industries when making facility or material flow layout decisions.

The following hypothesis is being proposed by Basak (2013).

The hypothesis which gives the ranking of the design evaluation criteria is defined as the alternative hypothesis.

$$H_a \; : \; Pi_1 \geq Pi_2 \geq Pi_3 \geq Pi_4 \geq Pi_5 \geq Pi_6 \geq : Pi_7 \geq Pi_8 \geq Pi_{t-1} \geq Pi_t$$

We would like to get the evidence to support this hypothesis from the sample data collected from experts on the comparison of criteria against each other. We would be utilizing the comparison matrix data collected from the experts while computing the AHP ranking of criteria. We would like to establish that at least one inequality is a strict inequality.

This study aims to provide adequate statistical evidence to reject the null hypothesis stated below. H_0 :

$Pi_1 = Pi_2 = Pi_3 \; Pi_4, = Pi_5 = Pi_6 = Pi_7 = Pi_8 = Pi_{t-1} = Pi_t$

So, essentially the following hypothesis needs to be tested.

The null hypothesis to be tested H_0 : $Pi_1 = Pi_2 = Pi_3 = Pi_4, = Pi_5 = Pi_6 = Pi_7 = Pi_8 = Pi_{t-1} = Pi_t$ against

The Alternate hypothesis (2)

$H_a : Pi_1 \geq Pi_2 \geq Pi_3 \geq Pi_4 \geq Pi_5 \geq Pi_6 \geq Pi_7 \geq Pi_8 \geq Pi_{t-1} \geq Pi_t$ with at least one strict inequality

The hypothesis testing problem can also be expressed as:

H_0^1 : $\ln(Pi_i) - \ln(Pi_{i+1}) = 0$, *for i = 1,2, 3…, t-1* against (3)

H_a^1 : $\ln(Pi_i) - \ln(Pi_{i+1}) \geq 0$, for i=1,2, 3…, t-1 with at least one strict inequality

The statistical Likelihood f of $a_{i,i+1}^{(k)}$ for i=1,2, 3,9 and k=1, 2…, n varies in proportion to

$$\exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{t-1} \left\{ \sum_{k=1}^{n} \left(ln(a_{i,i+1}^{(k)} - \overline{ln}\, a_{i,i+1} \right)^2 - n(\overline{ln}\, a_{i,i+1} + ln(\pi i + 1) - ln\,(\pi_i))^2 \right\} \right) \tag{4}$$

where $\overline{ln}\, a_{i,i+1} = \frac{1}{n}\sum_{k=1}^{n} a_{i,i+1}^{(k)}$.

Now substituting $\theta_i = ln\,(Pi_i) - ln\,(Pi_{i+1})$ and $y_i = \overline{ln}\, a_{i,i+1}$ in equation (4), we get
The statistical Likelihood f varies in proportion to

$$\exp\left[-\frac{n}{2\sigma^2}\sum_{i=1}^{t-1}(y_i - \theta_i)^2\right] \quad (5)$$

The null hypothesis H_0^1 and the alternative hypothesis H_a^1 as specified in (3), are assessed using the widely recognized likelihood ratio test, which evaluates whether the likelihood differs notably between the two hypotheses

$\lambda = \underset{\theta_0}{\text{maximum}}\, f \,/\, \underset{\theta_a}{\text{maximum}}\, f$ whether it is negligible or significant, equivalently

$\lambda^* = -2 \ln \lambda = 2\left[\underset{\theta_a}{\min}(-f) - \underset{\theta_0}{\min}(-f)\right]$ is large or not. (60

In equation 6 we substitute m = t-1,

$$\theta_0 = \{\theta_i \mid \theta_i = 0, i = 1,2,\dots,m\}$$
$$\theta_a = \{\theta_i \mid \theta_i \geq 0,\ i = 1,2,\dots,m\}$$

By using (5), λ^* can be evaluated as

$$\lambda^* = (n/s^2) * \left(\left[\sum_{i=1}^{m} y_i^2 - \underset{\theta_a}{min} \sum_{i=i}^{m} (y_i - \theta)^2\right]\right) \quad (6)$$

where s^2 is an estimate of σ^2

3. Evaluation of Likelyhood test ratio and its distribution under the null hypothesis H_0
The point where the convex function reaches its lowest value

$$f^* = n\sum_{i=1}^{m}(y_i - \theta_i)^2$$

of θ_i need to be computed bound by inequality conditions

$g_i(\theta_1, \theta_2, \theta_3, .., \theta_m) = \theta_i \geq 0$ (for i=1, 2..., m)

for the purpose of calculating the likelihood test ratio λ^*

It is evident that this is a Non-linear Programming Problem (NLP). Basak (2015) solved this problem by applying the Kuhn-Tucker necessity theorem and Kuhn-Tucker sufficiency theorem.

Basak proposed following theorems to solve this problem.

Theorem 1: The inclusion of the sample vector $(y_1, y_2, \dots, y_m)$ is determined by a necessary and sufficient condition R_M where ϕ ⊂ M ⊂ A and M= {d+1, ⋯, m} is

1 $y_i \leq 0$ for i = 1, 2..., d
2 $y_i > 0$ for i = d+1, d+2, ⋯, m

Applying the Theorem 1 to the equation (6) is reduced to the following:

$$\lambda^* = (n/s^2) * \left(\sum_{i=d+1}^{m} y_i^2\right) \text{ with } i \,\varepsilon\, M \quad (7)$$

In the following Theorem 2 Basak (2015) presents the distributional characteristics of λ^* derived from (7), operating under the premise of null hypothesis H_0 as stated in equation (2)

Theorem 2: On the premise of the null hypothesis $H_0 : \theta_i = 0$ for i=1, 2...., m,

$\frac{(nm-1)}{(m-d)}\lambda^* = \frac{n(nm-1)}{(m-d)s^2}\sum_{i\,\varepsilon\, M} y_i^2$ has follows an F-distribution, having (m-d) degrees of freedom in its numerator and (nm-1) in its denominator.

where $s^2 = \sum_{k=1}^{n}\sum_{i=1}^{m}\left[ln a_{i,i+1}^{(k)} - \bar{a}\right]^2$ with $\bar{a} = \sum_{k=1}^{n}\sum_{i=1}^{m} \ln a_{i,i+1}^{(k)}$

Basak (2015) has shown that under H_0

$$\frac{n\ (\sum_{i\,\varepsilon\, M} y_i^2\ )/\ (m-d)}{(s^2/(nm-1)))} \sim F_{m-d,nm-1} \quad \text{in which}$$

$F_{m-d,nm-1}$ represents F distribution with numerator degrees of freedom (m— d) and denominator degrees of freedom (nm — 1).

Thus, it is proven that $\frac{(nm-1)}{(m-d)}\lambda^*$ has an F distribution with the numerator degrees of freedom (m-d) and denominator degrees of freedom (nm-1).

The F distribution mentioned earlier pertaining to $\frac{(nm-1)}{(m-d)}\lambda^*$ is applicable for testing the null hypothesis as articulated in equation (2).

At significance level of α, the null hypothesis H_0 would be rejected if

$$\frac{(nm-1)}{(m-d)}\lambda^* \geq F_0$$

where F_0 is the critical value of $F_{m-d,nm-1}$ distribution with $P\left[F_{m-d,nm-1} \geq F_0\right] = \alpha$

4. Numerical Validation of hypothesis testing for hierarchical Structures

In order to test the hypothesis for ranking of design evaluation criteria, the pairwise comparison data collected from five experts from the automobile industry have been used. Gautam and Sudarsanam (2024) have ranked the design evaluation criteria using Fuzzy AHP method. The evaluation criteria for evaluating each of the layout alternatives include both qualitative and quantitative criteria such as Distance, Adjacency, Shape ratio, Throughput time, Cost, Accessibility, Flexibility, Safety, Maintenance and Efficiency.

The data collected from experts (pairwise comparison rating) is listed below. The order of comparison of criterion is based on the final ranking output from the Fuzzy AHP method (in the descending order) starting with Throughput time.

Experts	1	2	3	4	5	Criteria
(T,C)	3	2	3	3	2	Throughput time vs Cost
(C,F)	3	3	2	3	2	Cost vs Flexibility
(F,E)	2	1.5	2	1.5	2	Flexibility vs Efficiency
(E,D)	1	1.5	1	1.5	1	Efficiency vs Distance
(D,A)	2	1	2	3	2	Distance vs Adjacency
(A,AC)	1.5	1.5	2	2	1.5	Adjacency vs Accessibility
(AC,SH)	1.5	1.5	1	2	1.5	Accessibility vs Shape ratio
(SH,S)	1	1.5	1	1	1	Shape ratio vs Safety
(S,M)	1	0.5	1	0.5	1	Safety vs Maintenance

For example, the third expert rating in the first pair (T, C) is given as 3, which means that the criteria Throughput time is rated as three times more important than the criteria Cost by third expert.

Gautam and Sudarsanam (2024) have established the rank order of the design evaluation criteria of material flow layout as

$Pi_T \geq Pi_C \geq Pi_F \geq Pi_E \geq Pi_D \geq Pi_A \geq Pi_{AC} \geq Pi_{SH} \geq Pi_S \geq Pi_M$

Using the methodology proposed in Section 2 and Section 3, the rank order provided above by Gautam and Sudarsanam (2024)
can be validated with the following hypothesis testing.

We would like to test

$$H_0: Pi_T = Pi_C = Pi_F = Pi_E = Pi_D = Pi_A = Pi_{AC} = Pi = Pi_S = Pi_M$$

against

$$H_a: Pi \geq Pi_C \geq Pi_F \geq Pi_E \geq Pi_D \geq Pi_A \geq Pi_{AC} \geq Pi_{SH} \geq Pi_S \geq Pi_M$$ with at least one strict inequality.

By applying theorem 2 in Section 3, the hypothesis test criteria $\frac{(nm-1)}{(m-d)}\lambda^*$ is computed as 15.59616.
In theorem 2, under H_0 the test criteria $\frac{(nm-1)}{(m-d)}\lambda^*$ is distributed as $F_{m-d,nm-1}$ distribution.
In this case (m-d) = 9 and (nm-1) = 44 (n=9, m=5 and d=1).

So, at 1% significance level F_0 = 2.96. As the calculated value significantly exceeds the critical table value, the null hypothesis can be dismissed.

Thus, the rank order of the design evaluation criteria in the material floor selection problem
$Pi_T \geq Pi_C \geq Pi_F \geq Pi_E \geq Pi_D \geq Pi_A \geq Pi_{AC} \geq Pi_{SH} \geq Pi_S \geq Pi_M$ is very well supported by the methodology proposed by Basak (2015).

Next, we would be testing the reverse of the rank order of the criteria viz.

$$H_0: Pi_M = Pi_S = Pi_{SH} = Pi_{AC} = Pi_A = Pi_D = Pi_E = Pi_F = Pi_C = Pi_T$$

against

$$H_a: Pi_M \geq Pi_S \geq Pi_{SH} \geq Pi_{AC} \geq Pi_A \geq Pi_D \geq Pi \geq Pi_F \geq Pi_C \geq Pi_T$$ with at least one strict inequality.

The comparison ratings provided by the five experts are listed below:

Experts	1	2	3	4	5	Criteria
(M,S)	1.00	2.00	1.00	2.00	1.00	Maintenance vs Safety
(S,SH)	1.00	0.67	1.00	1.00	1.00	Safety vs Shape ratio
(SH, AC)	0.67	0.67	0.50	0.50	0.67	Shape ratio vs Accessibility
(AC,A)	0.67	0.67	0.50	0.50	0.67	Accessibility vs Adjacency
(A,D)	0.50	1.00	0.50	0.33	0.50	Adjacency vs Distance
(D,E)	1.00	0.67	1.00	0.67	1.00	Distance vs Efficiency
(E,F)	0.50	0.67	0.50	0.67	0.50	Efficiency vs Flexibility
(F,C)	0.33	0.33	0.50	0.33	0.50	Flexibility vs Cost
(C,T)	0.33	0.50	0.33	0.33	0.50	Cost vs Throughput time

In this case, the test criteria $\frac{(nm-1)}{(m-d)}\lambda^*$ is computed as 3.525. Here d = 8 and (m-d) =1, (nm-1) = 44.
The critical value corresponding to a 1% level of significance F_0 = 7.27. Since the critical value is more than the computed value, at 1% significance level we cannot reject the null hypothesis.
So, the rank order the design evaluation criteria
$Pi_M \geq Pi_S \geq Pi_{SH} \geq Pi_{AC} \geq Pi_A \geq Pi_D \geq Pi \geq Pi_F \geq Pi_C \geq Pi_T$ with at least one strict inequality is not supported.

CONCLUSIONS

In the present article, we have discussed an application of a simple statistical method for testing the rank ordering of the design evaluation criteria. These criteria are subsequently used in the selection of material flow layout in the automobile industry. Several multi-criteria decision-making methods are used in the material flow layout selection problem. The test criterion developed in this article is primarily based on the statistical method of likelihood-ratio test and then further computed on the basis of the Non-linear programming theory. The maximum possible likelihood-ratio test value is obtained by applying non-linear programming optimization method. This value provides the validation of testing the hypothesis of ranking order of the design evaluation criteria. This methodology can be improved further to test the rank ordering of any set of criteria. This can also be used to validate the rank ordering of the material flow layout alternatives. Also, this can be used to validate the ranking provided by various multi-criteria decision-making methods. The Bayesian methods can also be used to test the ranking of the criteria or ranking of the alternatives. Also, a fuzzy hypothesis can be formulated to test the the fuzzy number ranking of the alternatives or criteria by applying the crisp hypothesis to each of the triangular fuzzy numbers.

Ethical considerations

Not applicable for this study.

Conflict of Interest

The authors declare no conflicts of interest.

Funding

This research did not receive any financial support.

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