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Characteristics Of Heat Transfer For A Fin Under Dehumidification

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Abstract:
A study of the effect of relative humidity on the two dimensional heat transfer characteristics for the two types of annular fins of (constant thickness and variable thickness) have been carried out using finite difference method and grid generation technique of algebraic method to simplify the geometry of fin. The temperature distribution for a fully wet fin and dry fin have been determined at relative humidity of (0, 50, 75 and 100)% also fin efficiency, fin effectiveness and heat transfer have been calculated for a range of relative humidity at (10, 20, 30, 40, 50 and 100)% and length of fin from (0.01 to 0.1)m. The effect of the atmospheric pressure was also studied. The study shows that the fin effectiveness and fin efficiency of a fin in the case of fully wet fin are found to be smaller than those in the dry fin and the temperature gradient at the tip for a dry fin is greater than that for the wet fin
Key words: *Annular fins, relative humidity, heat transfer*

Nomenclature

A	Fin surface area	m^2
cp	Specific heat of moist air	kJ/(kg.°K)
h	Heat transfer coefficient	W/(m^2.°K)
h_{fg}	Latent heat of condensation (h_{fg} = 2430)	kJ/kg
h_m	Mass transfer coefficient	W/m^2.°K
k	Thermal conductivity for the fin (copper material, k=400)	W/m.°K
Le	Lewis number	-------
Lf	Length of fin (ro-ri)	m
M,N	Number of grid in radial and transverse directions	-------
P_{atm}	Atmospheric pressure	Bar
P_s	Pressure at saturated temperature	Bar
P_v	Water vapour pressure	Bar
QF_n	Heat transfer from fin numerically	W
QF_{MAX}	Maximum heat transfer	W
QNF	Heat transfer at the base without fin	W
ri	Inner radius of fin	m
ro	Outer radius of fin	m
R	Ratio of (r/ri)	-------
RH	Relative humidity	%
T_s	Surface temperature of fin	°C
T_b	Base temperature of fin (2°C)	°C
T_f	Fluid temperature (30°C)	°C
w	Thickness of rectangular fin section	m
wi	Inner thickness for the trapezoidal fin section	m
wo	Outer thickness for the trapezoidal fin section	m
ε	Fin effectiveness	-------
η_f	Fin efficiency	-------
θ	Dimensionless local fin surface temperature $(T_f-T_s)/(T_f-T_b)$	-------
ω_f	Humidity ratio of air	kg_v /kg_a
ω_s	Humidity ratio of fin surface	kg_v /kg_a
β	Parameter defined as $h_{fg}/(cp*Le^{2/3})$	°K

INTRODUCTION:

Fins are appendages intimately connected to the primary surface for the augmentation of heat transfer. Finned tube heat exchangers are commonly used for air cooling and dehumidifying. When the fin surface temperature is below the dew point temperature of the air, dehumidification of the air occurs. With dehumidification, fin surface is wetted and simultaneous heat and mass transfers occur. Therefore, the study of the performance and optimization of the wet surface heat exchangers are considerably different from that of dry surface heat exchangers. Among the factors that affect the thermal analysis of such heat exchangers are the thermo-geometric, arterials and psychrometric conditions [4]. Several researchers have investigated the effect of variable thermal properties on the performance of fins. The first extensive analysis was carried out by Rosario and Rahman [1]. They investigated the radial fin assembly under fully wet operating conditions and assumed a fixed sensible to total heat ratio to obtain their numerical solution. Their results showed that there is a strong relationship between the fin efficiency and relative humidity of the incoming air.Wu and Bong [2] provided an analytical solution for the efficiency of a straight fin under both fully wet and partially wet conditions using the temperature and humidity ratio differences as the driving forces for heat and mass transfer. They assumed linear relationship between the humidity ratios of the saturated air on the fin surface. Their result shows that there is not much change of the fin efficiency with the relative humidity.Liang et al. [3] made a comparison of one-dimensional and two-dimensional models for the wet fin efficiency of a plate-fin-tube heat exchanger using the fourth order Runge–Kutta method and a second-order central-difference scheme. It is worth noting that the fin efficiency of a 2-D rectangular plate-fin was approximated by a 1-D equivalent annular circular fin having the same surface area. Their results showed that the wet fin efficiency obtained from the 1-D model agreed with that obtained from the 2-D model.Kundu [4] analyzed analytically the fin performances of longitudinal wet fins of different profiles. In this connection, it can be mentioned that the consideration of linear relationship is an approximate. The incoming air near the fin surface becomes saturated during condensation. Therefore, the specific humidity of air on the fin surface varies with the temperature according to the psychrometric variation which follows a process along the saturation curve on the psychrometric chart. It is well known that the saturation curve on the psychrometric chart is a curvilinear in nature. Therefore, the fin efficiency with linear relationship may be associated with an erroneous prediction.

Kundu and Das [5] developed a generalized analytical technique for longitudinal, and spine fins under dry and fully wet conditions. Their mathematical formulation was based on the assumption of a linear relationship between temperature and specific humidity, while the method of Frobenius power series expansion has been used to solve the governing differential equation.Naphon [6] theoretically investigated the temperature distribution for annular fin of constant thickness under dry-surface, partially wet-surface, and fully wet-surface conditions. The mathematical models based on the conservation equations of energy and mass are developed and solved by the central finite difference method to obtain the temperature distribution along the fin. His results were agreement with other models.Sharqawy and Zubair [7] studied analytically the efficiency and optimization of one dimensional heat transfer of longitudinal fin with combined heat and mass transfer. They introduced to a new modified fin parameter can be calculated without knowing the actual fin-tip condition. Recently, the analysis of annular fins of constant cross-sectional in one dimensional heat flow area with the combined heat and mass transfer mechanisms has been demonstrated by Sharqawy and Zubair [8] numerically.

Kundu and Barman [9] are investigated numerically the temperature distribution and efficiency of on dimensional of an annular fin rectangular profile. They employed a cubic polynomial relationship between specific humidity and temperature for betterment of mass transfer effect on thermal analysis of wet annular fins under dehumidifying conditions. All the above researches are considered with study of the characteristics of heat transfer for annular and longitudinal fins in one dimensional heat transfer, therefore, the purpose of this paper is study numerically the effect of relative humidity on heat transfer characteristics for annular fin of constant thickness and variable thickness (trapezoidal section) in two dimensional heats flow and compared the results with dry air and other researchers.

2. Analysis:

A steady state case is carried out on an annular fin exposed to moving air stream as shown in figure (1).

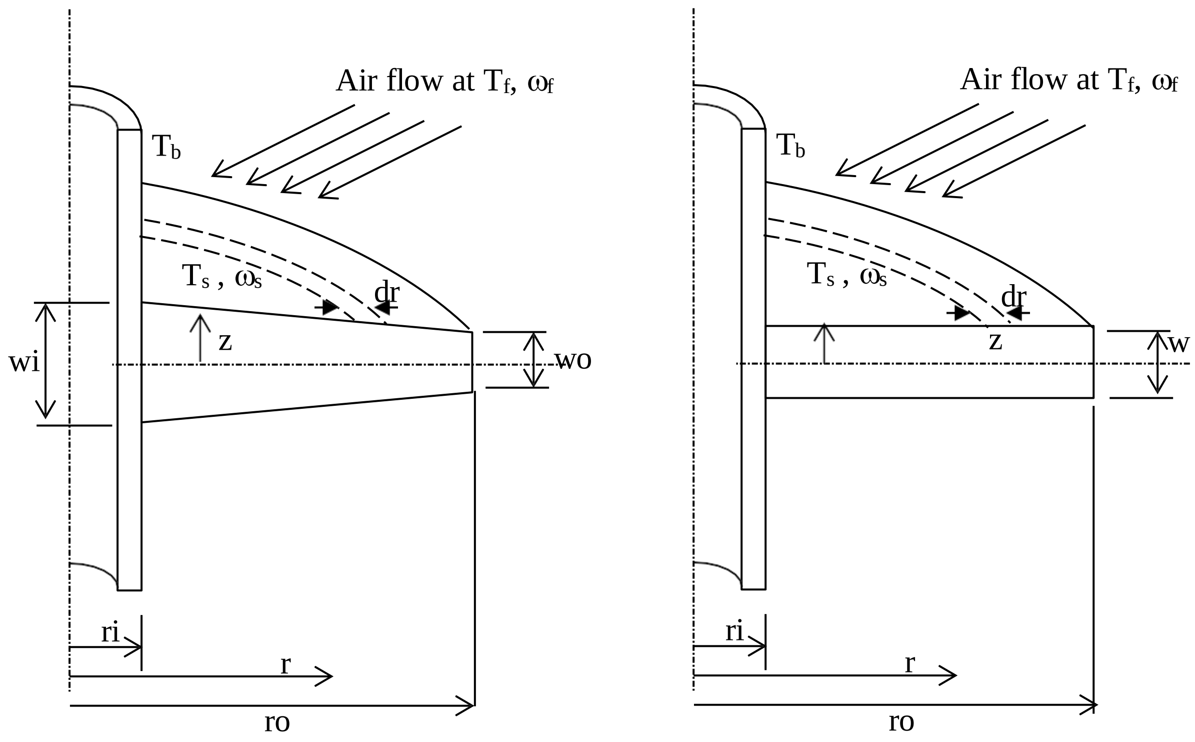

Figure (1) Schematic diagram of fully wet annular fin of rectangular and trapezoidal profiles

This model depending on some assumptions to simplify the solution, these assumptions are:

1- The thermal conductivity of fin material is constant.
2- The pressure, temperature and relative humidity of the surrounding air are constant.
3- The fin base temperature is constant.
4- The effect of pressure drop of air due to air flow is neglected.
5- The heat flow and temperature distributions throughout the fin are independent of time.
6- There is no heat source inside the fin.
7- The conductive heat and mass transfer coefficients are constant.

The fin surface can be classified as fully dry and fully wet. When the temperatures over the entire surface of the fin are higher than the dew point of the surrounding air, then the surface is called fully dry and no humidification occurs. The fin surface will be fully wet when the temperatures over the entire tin surface are lower than the dew point of the air and simultaneous heat and mass transfer takes place [4]. In the present work, the temperature at every point on fin surface is assumed to be below the dew point temperature of the surrounding air.

3. The Numerical Solution:

3.1. For an annular fin of constant thickness:

The fin was divided to the radial and transverse directions of (N and M) divisions. The thickness of each division is (Δr) in the radial direction and (Δz) in the transverse direction. The general equation for the temperature distribution in the two dimensional heat flow case can be shown as:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0.0 \qquad \ldots(1)$$

In the annular fin of constant thickness, the following boundary conditions can be subjected in equation above:

$\frac{\partial T}{\partial r} = 0$ at the tip of fin , $\frac{\partial T}{\partial z} = 0$ at center line of fin and $T=T_b$ at the base of fin, while the boundary condition at the surface of fin is:

$$-k\frac{\partial T}{\partial z} = h(T_f - T_s) + h_{fg}.h_m(\omega_f - \omega_s) \qquad \text{…(2)}$$

Where ω_f is the humidity ratio of air , ω_s is the humidity ratio on the fin surface, h_m is the mass transfer coefficient, h_{fg} is the latent heat of condensation.

The saturated humidity ratio on the surface of fin ω_s can be calculated from the correlation given by [3] :

$$\omega_s = (3.744 + 0.3078 * T_s + 0.0046 * T_s^2 + 0.0004 * T_s^3) \times 10^{-3} \qquad \text{..(3)}$$

when $0^\circ C < T_f < 30^\circ C$

According to Chilton-Colburn analogue [11], the relation between the heat transfer coefficient and mass transfer coefficient is:

$$\frac{h}{h_m} = Cp \times Le^{\frac{2}{3}} \qquad \text{…(4)}$$

Therefore, for the energy balance in the equation (2) yields the following differential equation:

$$-k\frac{\partial T}{\partial z} = h\left[(T_f - T_s) + \frac{h_m}{h} \times h_{fg} \times (\omega_f - \omega_s)\right] \qquad \text{…(5)}$$

$$-k\frac{\partial T}{\partial z} = h\left[(T_f - T_s) + \frac{h_{fg}}{cp \times Le^{\frac{2}{3}}} \times (\omega_f - \omega_s)\right] \qquad \text{…(6)}$$

and then:

$$-k\frac{\partial T}{\partial z} = h\left[(T_f - T_s) + \beta \times (\omega_f - \omega_s)\right] \qquad \text{…(7)}$$

The latent heat of the water evaporation h_{fg} , Lewis number Le and specific heat of air Cp can be assumed are constant, thus β can be considered as a constant [5, 7, 8, 9].

$$\omega_f = 0.622 \times \frac{P_v}{P_{atm}} \qquad \text{…(8)}$$

$$P_v = RH \times P_s \qquad \text{…(9)}$$

Then,

$$\omega_f = 0.622 \times \frac{P_s \times RH}{P_{atm}} \qquad \text{…(10)}$$

When P_v is the water vapour pressure, P_{atm} is the atmospheric pressure, RH is the relative humidity of surrounding air and the P_s is the pressure at the saturated temperature of air.

Then, the second order central difference formulation is used to descritize equation (1) and a second order forward or backward formulas are used for the flux terms. These yields:

$$T(i, j) = aT(i+1, j) + b(T(i, j+1) - T(i, j-1)) + c(T(i-1, j) \qquad \text{…(11)}$$

For the interior points, where:

$a= (2+\gamma) / (4+4\alpha) = (0.5+0.5\gamma) / (1+ \alpha)$ …(12)

$b= \alpha / (2+2 \alpha)$ …(13)

$c= (0.5-0.25\gamma)/ (1+ \alpha)$ …(14)

$\gamma_i= (\Delta r/r_i)$, $\alpha =(\Delta r/\Delta z)^2$ …(15)

For the fin tip, the formula is:

$$T(N, j) = \frac{4T(N-1, j) - T(N-2, j)}{2\Delta r} \qquad \text{…(16)}$$

For the fin surface:

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$$T(i,M) = \left[\frac{1}{2 \in \Delta z + 3}\right]\left(-4T(i,M-1) + T(i,M-2)\right) + \frac{T_f}{1 + \left(\dfrac{3}{2 \in i\Delta z}\right)} \quad \ldots(17)$$

$\in = h\,(1 + \beta(\omega_f - \omega(i,M))$

and for the symmetry line :

$$T(i,1) = \frac{4T(i,2) - T(i,3)}{3} \quad \ldots(18)$$

The above system of equations is solved using the SOR technique and the heat dissipation by the fin is obtained from the summation of the flux over the fin surface, i.e.:

$$QF_n = \sum_{i=1}^{i=N} 4\pi * r(i,M) * \Delta r * h * ((T_f - T(i,M) + \beta * (\omega_f - \omega(i,M))) \quad \ldots(19)$$

$$QF_{MAX} = \sum_{i=1}^{i=N} 4\pi * r(i,M) * \Delta r * h * ((T_f - T(1,1) + \beta * (\omega_f - \omega(1,1))) \quad \ldots(20)$$

$$QNF = \sum_{i=1}^{i=N} 4\pi * ri * w * h * ((T_f - T(1,1)) \quad \ldots(21)$$

The fin efficiency and fin effectiveness are calculated as:

$$\eta_f = \frac{QF_n}{QF_{MAX}}, \varepsilon = \frac{QF_n}{QNF} \quad \ldots(22)$$

3.2. For an annular fin of variable thickness (Trapezoidal section):

Starting from the heat conduction formula in equation (1) using annular fin of trapezoidal cross section, the grid generation technique of algebraic method is used to solve the problem of the irregular mesh on the fin surface. The physical coordinate (r , z) has been converted to calculated coordinate (ξ , η) as shown in figure (2). Assuming each interval between any two nodes $\Delta\xi$ and $\Delta\eta$ are equal 1.0.

Every term of the equation (1) can be written a function of (ξ,η) coordinate as [10]:

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial \xi} * \frac{\partial \xi}{\partial r} + \frac{\partial T}{\partial \eta} * \frac{\partial \eta}{\partial r} \quad \ldots(23)$$

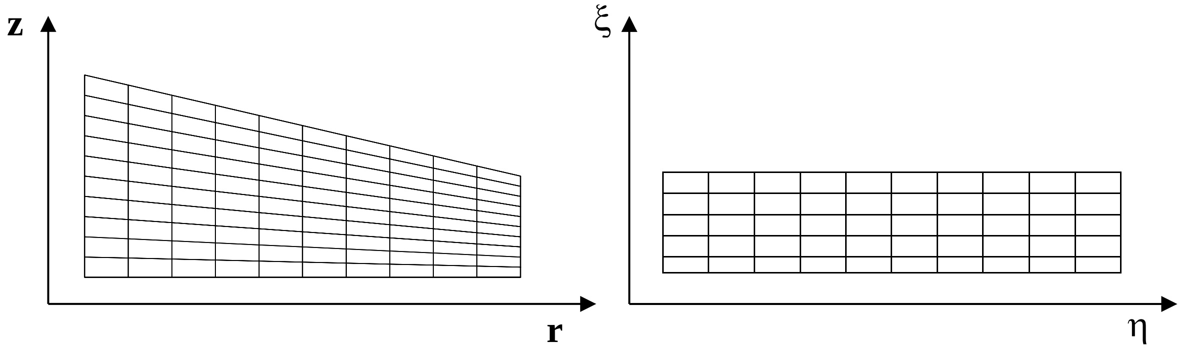

Figure (2) the transformation method to regular grid

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial \xi} * \frac{\partial \xi}{\partial z} + \frac{\partial T}{\partial \eta} * \frac{\partial \eta}{\partial z} \quad \ldots(24)$$

Where: $z = 0.5\left[\frac{(wi - wo) * (ro - r)}{ro - ri} + wo\right]$

$$\frac{\partial^2 T}{\partial r^2} = \left(\frac{\partial T}{\partial \xi} * \frac{\partial^2 \xi}{\partial r^2}\right) + \left(\frac{\partial T}{\partial \eta} * \frac{\partial^2 \eta}{\partial r^2}\right) + \left(\frac{\partial^2 T}{\partial \xi^2}\right)\left(\frac{\partial \xi}{\partial r}\right)^2 + \left(\frac{\partial^2 T}{\partial \eta^2}\right)\left(\frac{\partial \eta}{\partial r}\right)^2 + 2\left(\frac{\partial^2 T}{\partial \eta \partial \xi}\right)\left(\frac{\partial \eta}{\partial r}\right)\left(\frac{\partial \xi}{\partial r}\right)$$

$$\frac{\partial^2 T}{\partial z^2} = \left(\frac{\partial T}{\partial \xi} * \frac{\partial^2 \xi}{\partial z^2}\right) + \left(\frac{\partial T}{\partial \eta} * \frac{\partial^2 \eta}{\partial z^2}\right) + \left(\frac{\partial^2 T}{\partial \xi^2}\right)\left(\frac{\partial \xi}{\partial z}\right)^2 + \left(\frac{\partial^2 T}{\partial \eta^2}\right)\left(\frac{\partial \eta}{\partial z}\right)^2 + 2\left(\frac{\partial^2 T}{\partial \eta \partial \xi}\right)\left(\frac{\partial \eta}{\partial z}\right)\left(\frac{\partial \xi}{\partial z}\right)$$

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The derivatives $\frac{\partial T}{\partial \xi}, \frac{\partial T}{\partial \eta}, \frac{\partial^2 T}{\partial \eta^2}, \frac{\partial^2 T}{\partial \xi^2}$ and $\frac{\partial^2 T}{\partial \xi \partial \eta}$ are represents the central difference for Tylor series respect to η , ξ as:

$$\left(\frac{\partial f}{\partial x}\right) = \frac{f(i+1, j) - f(i-1, j)}{2\Delta x} \quad \text{and} \quad \left(\frac{\partial f}{\partial y}\right) = \frac{f(i, j+1) - f(i, j+1)}{2\Delta y} \qquad \ldots(25)$$

$$\left(\frac{\partial^2 T}{\partial r^2}\right) = \frac{f(i+1, j) - 2f(i, j) + f(i-1, j)}{(\Delta r)^2} \quad \text{and} \quad \left(\frac{\partial^2 T}{\partial z^2}\right) = \frac{f(i, j+1) - 2f(i, j) + f(i, j-1)}{(\Delta z)^2} \qquad \ldots(26)$$

$$\left(\frac{\partial^2 T}{\partial r \partial z}\right) = \frac{f(i+1, j+1) - f(i-1, j+1) - f(i+1, j-1) + f(i-1, j-1)}{4\Delta r \Delta z} \qquad \ldots(27)$$

$$\left(\frac{\partial \xi}{\partial r}\right) = \left(\frac{\partial z}{\partial \eta}\right)\frac{1}{Jac} \qquad \ldots(28)$$

$$\left(\frac{\partial \eta}{\partial r}\right) = -\left(\frac{\partial z}{\partial \xi}\right)\frac{1}{Jac} \qquad \ldots(29)$$

$$\left(\frac{\partial \xi}{\partial z}\right) = -\left(\frac{\partial r}{\partial \eta}\right)\frac{1}{Jac} \qquad \ldots(30)$$

$$\left(\frac{\partial \eta}{\partial z}\right) = \left(\frac{\partial r}{\partial \xi}\right)\frac{1}{Jac} \qquad \ldots(31)$$

Where (Jac) is the Jacobian matrix.

The derivatives $\frac{\partial z}{\partial \xi}, \frac{\partial z}{\partial \eta}, \frac{\partial r}{\partial \xi}$ and $\frac{\partial r}{\partial \eta}$ are represents the central difference Tylor series for temperature respect to η , ξ as equation (28)

$$Jac = \left[\left(\frac{\partial r}{\partial z}\right)\left(\frac{\partial z}{\partial \eta}\right)\right] - \left[\left(\frac{\partial r}{\partial \eta}\right)\left(\frac{\partial z}{\partial \xi}\right)\right] \qquad \ldots(32)$$

By derivative the equations (28 and 29) respect to (r) and (30 and 31) respect to (z), the results will be as the following:

$$\frac{\partial^2 \xi}{\partial r^2} = \frac{-1}{Jac^2}\left[\left(\frac{\partial z}{\partial \eta}\right) Jac_r - \left[\left(\frac{\partial^2 z}{\partial \eta \partial \xi}\right)\left(\frac{\partial z}{\partial \eta}\right) - \left(\frac{\partial^2 z}{\partial \eta^2}\right)\left(\frac{\partial z}{\partial \xi}\right)\right]\right] \qquad \ldots(33)$$

$$\frac{\partial^2 \xi}{\partial z^2} = \frac{1}{Jac^2}\left[\left(\frac{\partial r}{\partial \eta}\right) Jac_z - \left[\left(\frac{\partial^2 r}{\partial \eta^2}\right)\left(\frac{\partial r}{\partial \xi}\right) - \left(\frac{\partial^2 r}{\partial \eta \partial \xi}\right)\left(\frac{\partial r}{\partial \eta}\right)\right]\right] \qquad \ldots(34)$$

$$\frac{\partial^2 \eta}{\partial r^2} = \frac{1}{Jac^2}\left[\left(\frac{\partial z}{\partial \xi}\right) Jac_r - \left[\left(\frac{\partial^2 z}{\partial \xi^2}\right)\left(\frac{\partial z}{\partial \eta}\right) - \left(\frac{\partial^2 z}{\partial \eta \partial \xi}\right)\left(\frac{\partial z}{\partial \xi}\right)\right]\right] \qquad \ldots(35)$$

$$\frac{\partial^2 \eta}{\partial z^2} = \frac{-1}{Jac^2}\left[\left(\frac{\partial r}{\partial \xi}\right)_{i,j} Jac_z - \left[\left(\frac{\partial^2 r}{\partial \eta \partial \xi}\right)\left(\frac{\partial r}{\partial \xi}\right) - \left(\frac{\partial^2 r}{\partial \xi^2}\right)\left(\frac{\partial r}{\partial \eta}\right)\right]\right] \qquad \ldots(36)$$

The derivatives $\frac{\partial^2 r}{\partial \xi^2}, \frac{\partial^2 r}{\partial \eta^2}, \frac{\partial^2 z}{\partial \xi^2}, \frac{\partial^2 z}{\partial \eta^2}, \frac{\partial^2 r}{\partial \xi \partial \eta}$ and $\frac{\partial^2 z}{\partial \xi \partial \eta}$ are represents the central difference for Tylor series in equations (26 and 27) respect to η , ξ .

After simplifying and substituting the equations above, the temperature distribution in the interior node is yield as:

$$T(i,j)=\frac{AT(i-1,j)+BT(i+1,j)+CT(i,j-1)+DT(i,j+1)+E}{A+B+C+D} \quad \ldots(37)$$

When;

$$A=\left(\frac{\partial\xi}{\partial r}\right)^2+\left(\frac{\partial\xi}{\partial z}\right)-0.5*\left(\left(\frac{\partial^2\xi}{\partial r^2}\right)+\left(\frac{\partial^2\xi}{\partial z^2}\right)\right)-\frac{0.5}{r}*\left(\frac{\partial\xi}{\partial r}\right) \quad \ldots(38)$$

$$B=\left(\frac{\partial\eta}{\partial r}\right)^2+\left(\frac{\partial\eta}{\partial z}\right)-0.5*\left(\left(\frac{\partial^2\eta}{\partial r^2}\right)+\left(\frac{\partial^2\eta}{\partial z^2}\right)\right)-\frac{0.5}{r}*\left(\frac{\partial\eta}{\partial r}\right) \quad \ldots(39)$$

$$C=\left(\frac{\partial\eta}{\partial r}\right)^2+\left(\frac{\partial\eta}{\partial z}\right)+0.5\left(\left(\frac{\partial^2\eta}{\partial r^2}\right)+\left(\frac{\partial^2\eta}{\partial z^2}\right)\right)+\frac{0.5}{r}*\left(\frac{\partial\eta}{\partial r}\right) \quad \ldots(40)$$

$$D=\left(\frac{\partial\xi}{\partial r}\right)^2+\left(\frac{\partial\xi}{\partial z}\right)+0.5*\left(\left(\frac{\partial^2\xi}{\partial r^2}\right)+\left(\frac{\partial^2\xi}{\partial z^2}\right)\right)+\frac{0.5}{r}*\left(\frac{\partial\xi}{\partial r}\right)$$

$$E=2\left(\frac{\partial\xi}{\partial r}\cdot\frac{\partial\eta}{\partial r}+\frac{\partial\xi}{\partial z}\cdot\frac{\partial\eta}{\partial z}\right)\left(\frac{\partial^2 T}{\partial\xi\partial\eta}\right) \quad \ldots(41)$$

When 1<i<N and 1<j<M

The boundary condition and the temperature distribution in the center line axis can be given as :

$$\frac{\partial T}{\partial z}=0 \quad \ldots(42)$$

$$\frac{\partial T}{\partial z}=\frac{\partial T}{\partial\xi}\cdot\frac{\partial\xi}{\partial z}+\frac{\partial T}{\partial\eta}\cdot\frac{\partial\eta}{\partial z}=0 \quad \ldots(43)$$

$$T(i,1)=\frac{1}{3}\left[\left(T(i+1,1)-T(i-1,1)*\frac{\partial\xi}{\partial z}+4T(i,2)-T(i,3)*\frac{\partial\eta}{\partial z}\right)\right]\frac{1}{\frac{\partial\eta}{\partial z}} \quad \ldots(44)$$

T_η and r_η are calculated for T,r respect to η as:

$$\frac{\partial T}{\partial x}=\frac{4f(i,j+1)-3f(i,j)-f(i,j+2)}{2\Delta x} \quad \ldots(45)$$

The boundary condition and the temperature distribution at the tip of fin can be written as:

$$\frac{\partial T}{\partial r}=0 \quad \ldots(46)$$

$$\frac{\partial T}{\partial r}=\frac{\partial T}{\partial\xi}\cdot\frac{\partial\xi}{\partial r}+\frac{\partial T}{\partial\eta}\cdot\frac{\partial\eta}{\partial r}=0 \quad \ldots(47)$$

$$T(N,j)=\frac{1}{3}\left[\left(4T(N-1,j)-T(N-2,j)+T(N,j-1)-T(N,j+1)*\frac{\partial\eta}{\partial r}\right)\right]\frac{1}{\frac{\partial\xi}{\partial r}} \quad \ldots(48)$$

T_ξ and z_ξ are calculated for T,z respect to ξ as:

$$\frac{\partial T}{\partial x}=\frac{f(i-2,j)+3f(i,j)-4f(i-1,j)}{2\Delta x} \quad \ldots(49)$$

The boundary condition at the fin surface can be written as:

$$-k\frac{\partial T}{\partial z}=h\left[(T_f-T_s)+\beta\times(\omega_f-\omega_s)\right] \quad \ldots(50)$$

$$T(i,M)=\frac{A1*T(i-1,M)+B1*T(i+1,M)+C1*T(i,M-1)+D1(T_f+\beta(\omega_f-\omega(i,M)))}{E1}$$ …(51)

Where A1,B1 and C1 are represents the (kA/dx) on the surface in each point around T(i,j). C1 represents the heat conductance (hA) between the surface of fin and ambient.

D1=A1+B1+C1+D1 …(52)

The temperature for the corner points can be calculated as:

$$T(N,1)=(T(N-1,1)+T(N,2))/2 \quad \text{and} \quad T(N,M)=(T(N-1,M)+T(N,M-1))/2$$ …(53)

$$QFn=\sum_{i=1}^{i=N} h*4\pi rS((T_f-T(i,j)+\beta*(\omega_f-\omega(i,M))$$ …(54)

Where, S is the distance of interval on the fin surface.

$$QF_{MAX}=\sum_{i=1}^{i=N} 4\pi*r(i,M)S*h*((T_f-T(1,1)+\beta*(\omega_f-\omega(1,1)))$$ …(55)

$$QNF=\sum_{i=1}^{i=N} 4\pi*ri*wi*h*((T_f-T(1,1))$$ …(56)

The fin efficiency and fin effectiveness can be calculated as:

$$\eta_f=\frac{QF_n}{QF_{MAX}},\ \varepsilon=\frac{QF_n}{QNF}$$ …(57)

4. RESULTS AND DISCUSSIONS:

In the figures (3,a-b), the temperature distribution over the fin surface is plotted against the dimensionless radius R= (r/r_i) for relative humidity RH= (50, 75 and 100) % for the two types of fins. For these three values of relative humidity, the fin tip temperature is below the dew point of the air, therefore, the fin is fully wet. It can be seen that at the same location on the fin, the temperature dimensionless $\theta = (T_f-T_s)/(T_f-T_b)$ is smaller for a wet fin than for a dry one. Thus, the surface temperature increases when there is moisture condensation, the higher relative humidity leads to high temperature on the fin surface from fin base to fin tip due to latent heat of vaporization coming into play for a wet fin. Therefore, the temperature gradient at the tip for a dry fin is greater than that for the wet fin. The figure (3,a) shows the comparison with the temperature distribution of one dimensional annular fin rectangular profile by [9].

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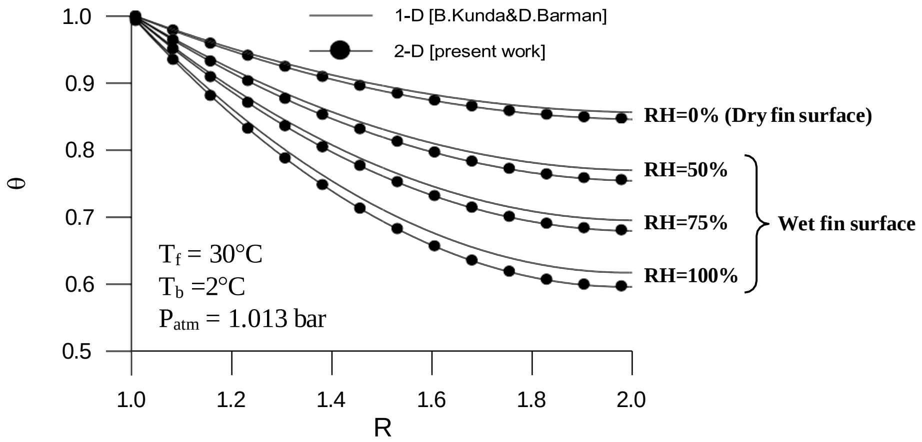


Figure ( ) Comparison the temperature profiles of the dry and wet of two dimensional of annular fin of rectangular section with one dimensional of annular fin of [9]

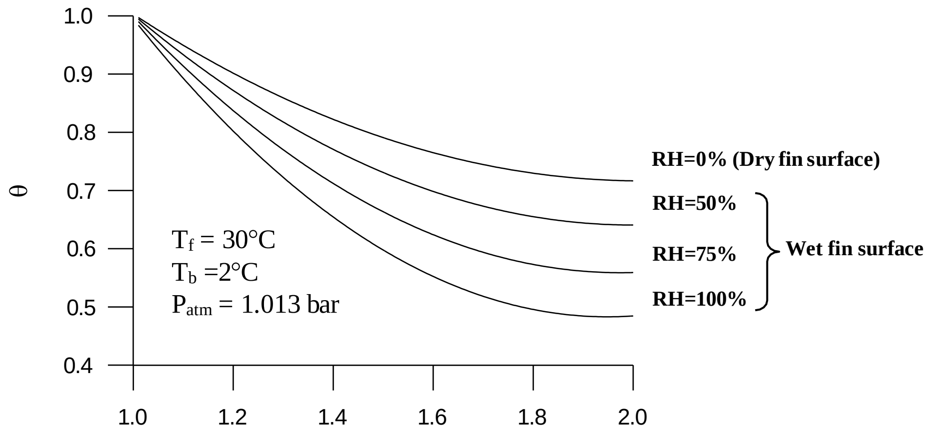
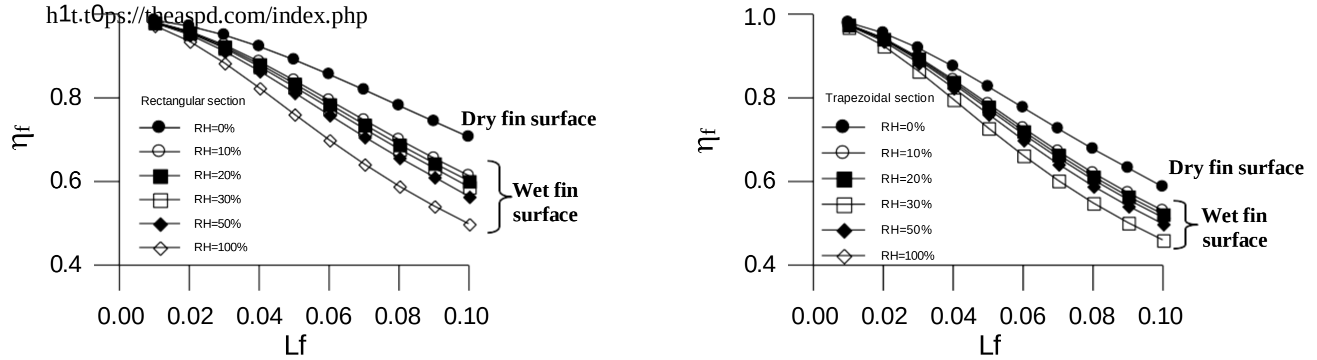
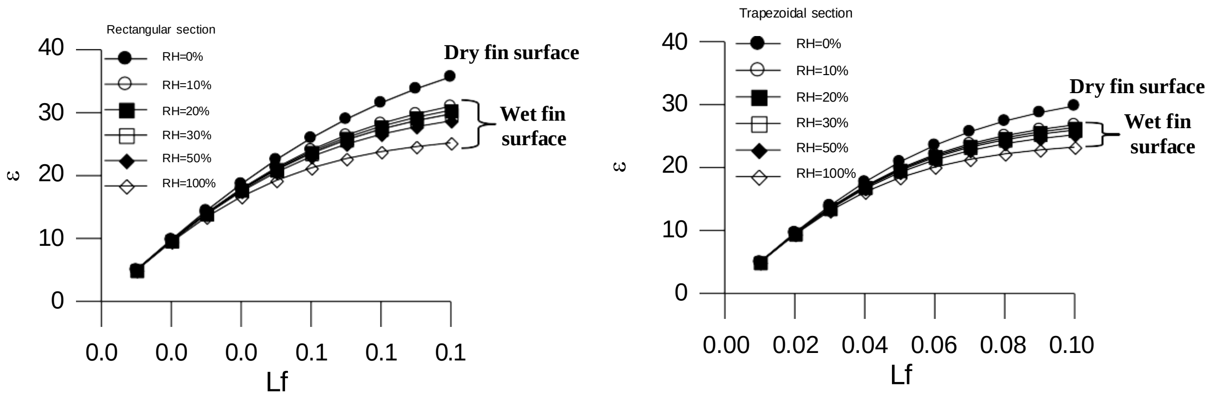


Figure (3,b), the temperature profiles of the dry and wet of annular fin of trapezoidal

Now in figure (4,a,b), the fin efficiency and fin effectiveness estimated by the present model are plotted against the length of fin. The fin efficiency decreases but the fin effectiveness increases with the increases in the length of fin because the increases in length of fin lead to increases in the range of temperature in the fin for which reduces the fin efficiency. However, fin effectiveness increases with increase in length of fin due to increment of actual of heat transfer rate through the fin.



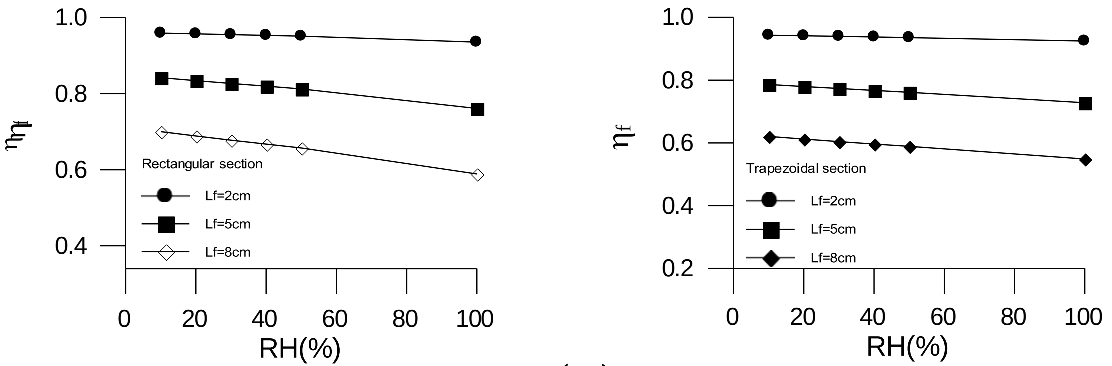
(a)



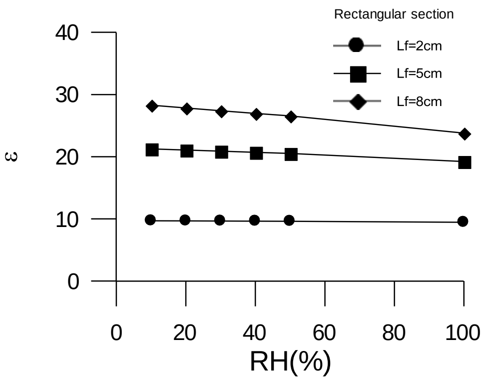
(b)

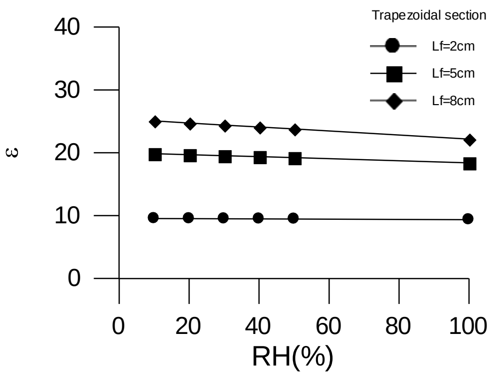
Figure (4,a,b), Variation of fin performance parameters of an annular fin with the length of fin at different length of fin: **(a)** fin efficiency **(b)** fin effectiveness

Also, it's clear in the fig (5,a,b) for the fin efficiency and effectiveness against the relative humidity of air of (10, 20, 30, 40, 50 and 100) % at variable length of fin of (0.02, 0.05 and 0.08) m.



(a)





(b)

Figure (5,a,b), Variation of fin performance parameters of an annular fin with the relative humidity at different length of fin: **(a)** fin efficiency **(b)** fin effectiveness

Figure (6) presents the fin efficiency against the relative humidity at different atmospheric pressures. It's clear that the fin efficiency increases with the increases in the atmospheric pressure. When the atmospheric pressure increase the humidity ratio of air will increase. This makes the driving force of the mass transfer process on the fin surface and then enhancement in the heat transfer due to condensation.

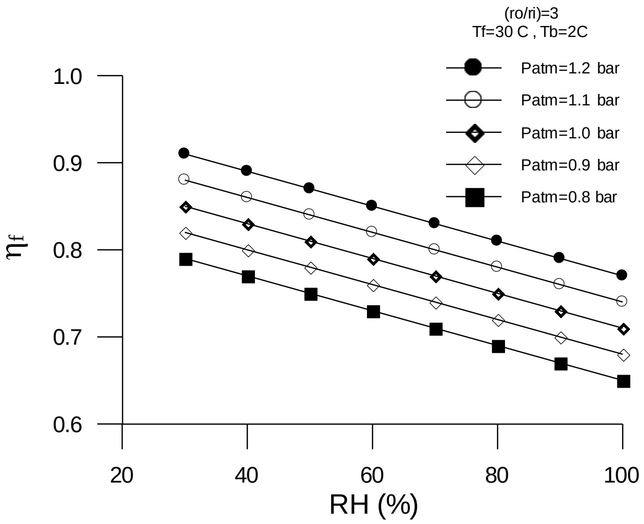


Figure (6), fin efficiency of an annular fin longitudinal section against the relative humidity at different atmospheric pressure

Figure (7) shows the relation between the heat flux against the length of fin at range of relative humidity of (10, 20, 30, 50 and 100) % for the two types of annular fins. At a constant relative humidity, the increases in length of fin lead to increases in the heat flux because the increases in the surface area of fin. When the relative humidity increases, the heat flux also increases because the surface temperature increases when there is moisture condensation, the higher relative humidity leads to high heat transfer on the fin surface along the fin.

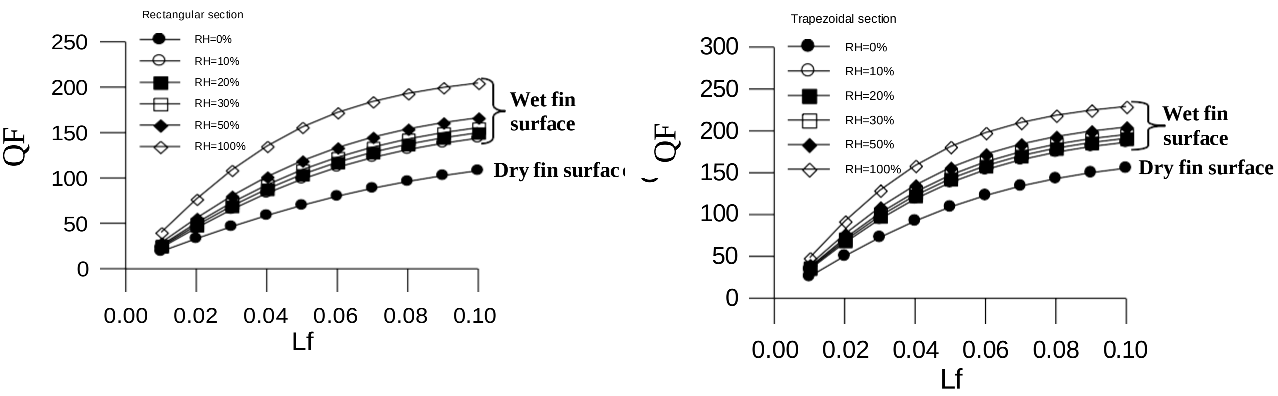


Figure (7), Heat flux against the length of fin at different relative humidity for rectangular and trapezoidal sections

5. CONCLUSIONS:

In this paper, the two dimensional heat equation for annular fins (trapezoidal and longitudinal sections) under different air conditions have been solved numerically using some simplified assumptions. The study included both dry and wet conditions of fin. The following general conclusions can be drawn from this study:

1- The temperature gradient at the tip for a dry fin is greater than that for the wet fin and increases the surface temperature when increasing in the relative humidity.
2- For the fully wet fin and at the same length of fin, the fin efficiency and fin effectiveness are decreases with increase the relative humidity.
3- For the fully wet fin, the fin efficiency depends on the atmospheric pressure, when the atmospheric pressure increases the fin efficiency also increases.
4- For the fully wet fin, the increases in the relative humidity lead to increases in the heat transfer.

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