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Combining Boolean Functions Technique and Pentagonal Fuzzy Numbers to Assess Sugar Plant Reliability

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Abstract. *In this research study, we focused on assessing the reliability of the feeding system within a sugar plant, utilizing a combination of the Boolean function method, Weibull distribution, and pentagonal fuzzy numbers. The sugar plant operates as a complex network of interconnected subsystems, and its overall performance is heavily dependent on the seamless functioning of these components. To analyze this, we developed a mathematical model using a logical matrix, which was further interpreted through the Boolean function approach. The primary objective was to address the uncertainties associated with the lifespan of the plant's components. For this purpose, we opted for the pentagonal fuzzy Weibull lifetime distribution, which is particularly suited to modeling situations with inherent uncertainties in lifespan parameters. Our analysis included deriving equations for fuzzy reliability, which measures the probability that the system performs without failure over a specified period; fuzzy mean time to future, which predicts the expected operational time before failure; and the fuzzy hazard function, which assesses the instantaneous failure rate. Furthermore, we examined these concepts through their α-cut representation, a technique used to handle fuzzy numbers by breaking them down into intervals that represent different levels of certainty. This approach allowed us to provide a more comprehensive and nuanced understanding of the system's dependability, accounting for the ambiguity and variability inherent in real-world operations.*

INTRODUCTION

The analysis of reliability within a sugar plant is a critical process that focuses on evaluating the dependability and performance of various equipment and subsystems involved in sugar production. This analysis encompasses the assessment of failure probabilities, potential downtime, maintenance schedules, and the optimization of operational efficiency to ensure a consistent production output. By identifying potential failure points and implementing preventive strategies, reliability analysis plays a pivotal role in enhancing overall plant productivity and minimizing costly disruptions.

Examining the reliability, long-term availability, and mean time before failure of the butter-oil manufacturing plant can enhance both the production and quality of the butter-oil, as noted by [13]. The methods available for analysing the reliability of k-out-of-n cold standby systems, especially those involving components with age-dependent hazard (failure) rates, are somewhat limited, as discussed by [22]. [21] presents the concept of a pentagonal fuzzy number, along with its characteristics and its application in fuzzy equations. [8] provides a cohesive perspective, specifically a failure-oriented approach to system failure engineering. The concept of failure can be modelled using fuzzy sets and has a broad range of interpretations. [1] introduced a numerical method for conducting reliability analysis of serial processes within the feeding system of a sugar plant. [16] evaluates and analyses the reliability of a paper plant using fuzzy logic techniques. It also determines the fuzzy Weibull distribution and the lifespan of a component. [2] primarily focused on coherent systems and the series arrangement of k-out-of-n standby subsystems, which have component lifetimes that follow an exponential distribution. [7] examined a one-component system that operates under changing conditions over time, with the development of these operating conditions being dictated by a Markov process. The objective of [4] to find the various reliability factor of refinery unit which is a subunit of Vacuum Distillation Unit (VDU). The subunits used in the considered unit are Pump, Desalter, Furnace, atmospheric fractionator and gas separator and these subunits are connected through pipes. The goal of [4] is to determine the various reliability factors of a refinery unit, which serves as a component of the Vacuum Distillation Unit (VDU). The subunits analysed in this study include the Pump, Desalter, Furnace, Atmospheric Fractionator, and Gas Separator, all interconnected through piping systems. An investigation into the reliability of the paper manufacturing system utilizing a hex decagonal fuzzy number and a Boolean function derived from the Weibull distribution as presented in [23]. [20] has explored the evaluation of reliability analysis for a sugar manufacturing plant utilizing Boolean function techniques.

This paper aims to delve into the synergistic application of fuzzy logic and Boolean function techniques specifically within the context of reliability analysis for sugar plant feeding systems. Through a comprehensive review of existing literature, alongside case studies and simulation studies, we aim to demonstrate the effectiveness and practicality of this integrated approach. By harnessing the complementary strengths of fuzzy logic and Boolean functions, we strive to present a robust methodology for assessing, predicting, and optimizing the reliability of feeding systems. The project aids in enhancing the field of industrial reliability engineering by offering a detailed and improved structure to guarantee the ongoing and efficient functioning of sugar manufacturing facilities.

Beginnings

Here we have introduced the fuzzy sets and fuzzy numbers (Pentagonal fuzzy number) and also some fundamental definitions of reliability context.

Definition (Fuzzy set): Let X be a non-empty subset of **R** (Real line). A fuzzy set A in X, denoted by pair (X, μ_A) is illustrated as

A = {(x, μ_A(x)): x $\in$X)} or $A = \left\{ \left(\frac{\mu_A(x)}{x} \right) : x \in X \right\}$ (for finite X), where $\mu_A(x)$ is called the membership function for the fuzzy set, defined by

$\mu_A(x)$: X ϵ [0, 1] such that

$$\mu_A(x) = \begin{cases} 1 & ; \quad x \in X \\ 0 & ; \quad x \notin X \\ (0,1) & ; \quad x \text{ is partly included in } X \end{cases}$$

Definition (Furzy Number): A fuzzy number is a distinct type of fuzzy set, denoted as 'A', which satisfies the following characteristics:

Subset of Real Numbers: The set A $\subset$ R.

Membership Function: The membership function, denoted as $\mu_A(x)$, is piecewise continuous. This means that the function is continuous in segments over its domain.

Normality: There exists an element x_0 in the set A such that the membership function $\mu_A(x_0) = 1$. This indicates that there is at least one element in the set with full membership.

Convexity: For any two elements x_1 and x_2 in the set A and any value λ in the interval [0,1], the following condition is met by the membership function

$$\mu_A(\lambda x_1+(1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

This implies that the membership function is highest at the peak and does not decrease for elements between any two points in the set.

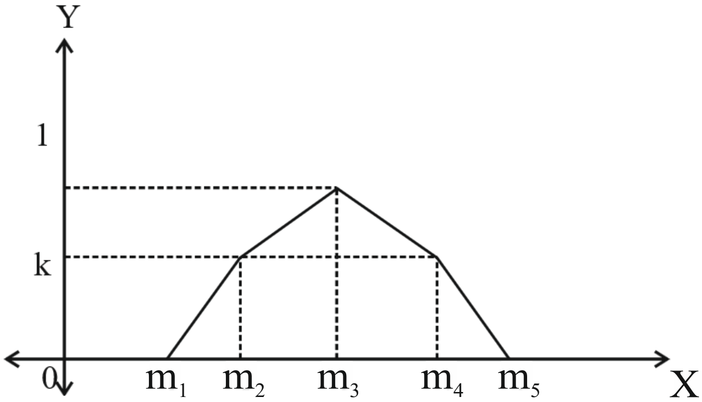
Essentially, a fuzzy number ensures that the membership values are well-defined, continuous, peak at full membership, and maintain a certain smoothness within the set. This makes fuzzy numbers useful in handling uncertain or imprecise data in various mathematical and applied contexts.

Definition [Linear pentagonal fuzzy number with symmetry] [LPFNS] A LPFNS is defined as $\tilde{\theta} = (m_1, m_2, m_3, m_4, m_5; k)$ and its membership function is as follows

$$\xi_{\tilde{\theta}LS}(x) = \begin{cases} 0 & \text{when} \quad x < m_1 \\ k\dfrac{x-m_1}{m_2-m_1} & \text{when} \quad m_1 \leq x \leq m_2 \\ 1-(1-k)\dfrac{x-m_2}{m_3-m_2} & \text{when} \quad m_2 \leq x \leq m_3 \\ 1 & \text{when} \quad x = m_3 \\ 1-(1-k)\dfrac{m_4-x}{m_4-m_3} & \text{when} \quad m_3 \leq x \leq m_4 \\ k\dfrac{m_5-x}{m_5-m_4} & \text{when} \quad m_4 \leq x \leq m_5 \\ 0 & \text{when} \quad x > m_5 \end{cases}$$

Definition [α-cut of linear PFN with symmetry] α-cut of linear PFN with symmetry is defined as:

$$\tilde{\theta}[\alpha] = \{x \in X : \mu_{\tilde{\theta}LS}(x) \geq \alpha\} =$$



$$= \begin{cases} \left[m_1 + \dfrac{\alpha}{k}(m_2 - m_1) \quad , \quad m_5 - \dfrac{\alpha}{k}(m_5 - m_4) \right] & \text{for } \alpha \in [0, k) \\ \left[m_2 + \dfrac{1-\alpha}{1-k}(m_3 - m_2) \quad , \quad m_4 + \dfrac{1-\alpha}{1-k}(m_4 - m_3) \right] & \text{for } \alpha \in [k, 1) \end{cases}$$

Note: In symmetric PEN, the point chosen on the left side matches the point chosen on the right side.

Assumptions

At the start, the whole system is functioning properly.
The failure rates of the components are independent of one another.
Once a component fails, it cannot be repaired.
There is already available data regarding the dependability of each component.
The system is structured to incorporate elements of imprecision or ambiguity.

Characters

θ_1 : Working state of cutting unit.

θ_2 : Cold standby state of cutting unit.

θ_3 : State of crushing unit.

θ_4 : The operational status of the bagasse transportation unit.

θ_5 : The idle condition of the bagasse transport unit.

θ_6 : Working state of heat generating unit.

θ_7 : Stand by state of heat generating unit.

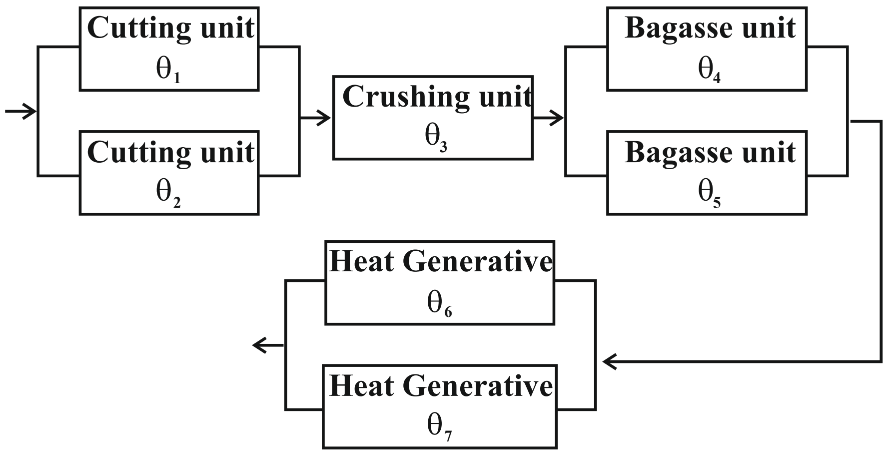
$\theta_i (i = 1, 2, ...6)$: 1, for Excellent condition and 0, for poor condition.

$\tilde{\theta}$: Fuzzy number.

$\tilde{S}(t)$: Fuzzy reliability functions of a module over time t.

$\xi_{\tilde{\theta}}$: Membership functions of a fuzzy number.

$\wedge / V$: Linkage/Separation.



(Diagram illustrating the operational sequence of the sugar plant's feeding system.)

Analysis and Formation of mathematical model of system

The operational capability conditions of the system can be represented using a logical matrix through the application of Boolean function techniques.

$$F(\theta_1, \theta_2, ...\theta_7) = \begin{bmatrix} \theta_1 & \theta_3 & \theta_4 & \theta_6 \\ \theta_1 & \theta_3 & \theta_4 & \theta_7 \\ \theta_1 & \theta_3 & \theta_5 & \theta_6 \\ \theta_1 & \theta_3 & \theta_4 & \theta_7 \\ \theta_2 & \theta_3 & \theta_4 & \theta_6 \\ \theta_2 & \theta_3 & \theta_4 & \theta_7 \\ \theta_2 & \theta_3 & \theta_5 & \theta_6 \\ \theta_2 & \theta_3 & \theta_5 & \theta_7 \end{bmatrix} \qquad ....(1)$$

Result of the Framework

Utilizing the principles of logical algebra, equation (i) can be reformulated or expressed as follows:

$$F(\theta_1, \theta_2, ...\theta_7) = [\theta_3 \wedge f(\theta_1, \theta_2, \theta_4, \theta_5, \theta_6, \theta_7)] \qquad ....(2)$$

Where $$f(\theta_1, \theta_2, ...\theta_7) = \begin{bmatrix} \theta_1 & \theta_4 & \theta_6 \\ \theta_1 & \theta_4 & \theta_7 \\ \theta_1 & \theta_5 & \theta_6 \\ \theta_1 & \theta_5 & \theta_7 \\ \theta_2 & \theta_4 & \theta_6 \\ \theta_2 & \theta_4 & \theta_7 \\ \theta_2 & \theta_5 & \theta_6 \\ \theta_2 & \theta_5 & \theta_7 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \end{bmatrix} \qquad ....(3)$$

$$R_1 = [\theta_1 \ \theta_4 \ \theta_6] \qquad ....(4)$$

$$R_2 = [\theta_1 \ \theta_4 \ \theta_7] \qquad ....(5)$$

$$R_3 = [\theta_1 \ \theta_5 \ \theta_6] \qquad ....(6)$$

$$R_4 = [\theta_1 \ \theta_5 \ \theta_7] \qquad ....(7)$$

$$R_5 = [\theta_2\ \theta_4\ \theta_6] \qquad ....(8)$$

$$R_6 = [\theta_2\ \theta_4\ \theta_7] \qquad ....(9)$$

$$R_7 = [\theta_2\ \theta_5\ \theta_6] \qquad ....(10)$$

$$R_8 = [\theta_2\ \theta_5\ \theta_7] \qquad ....(11)$$

Fuzzy Weibull Distribution:

The Fuzzy Weibull Distribution is an extension of the classic Weibull distribution, which is a continuous probability distribution used extensively in reliability analysis, failure analysis, and life data analysis. The Weibull distribution is characterized by its flexibility in modeling various types of data through its shape parameter. The probability density function of Fuzzy Weibull Distribution is expresses as:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} \qquad ....(12)$$

Where □, □ and □ are represented as scale parameter, shape parameter and location parameter, respectively.

Fuzzy Reliability Function: Fuzzy reliability, based on Lotfi Zadeh's fuzzy set theory, estimates the chance that a unit or system will keep working past a certain time, known as time t. This method uses fuzzy logic to express survival probability, offering a detailed and adaptable measure of the likelihood that the unit will operate beyond the set time. By applying fuzzy set theory, it provides a flexible and thorough understanding of reliability, addressing uncertainties and variations found in real-world situations. If X is the random variable for the lifespan of system modules, and $\tilde{F}_x(t) = \tilde{P}(X \le t)$ is the fuzzy cumulative distribution function (CDF) of X, is used to define the fuzzy reliability function at a specific time-

$$\begin{aligned}\tilde{S}(t) &= \tilde{P}(X > t)\\ &= 1 - \tilde{F}_x(t) \qquad ....(13)\\ &= \{[1 - F_{max}(x)[\alpha], 1 - F_{min}(x)[\alpha]], \mu_{F(x)} = \alpha\}, t > 0\end{aligned}$$

We aim to assess the dependability of a module whose lifespan is characterized by a fuzzy Weibull distribution. To achieve this, we express the parameter $\tilde{\theta}$ as a linear pentagonal fuzzy number that exhibits symmetry, i.e., $\tilde{\theta} = (m_1, m_2, m_3, m_4, m_5; k)$.

The membership function $\xi_{\tilde{\theta}}(x)$ of X is given by

$$\xi_{\tilde{\theta}LS}(x) = \begin{cases} 0 & \text{for } x < m_1 \\ k\dfrac{x-m_1}{m_2-m_1} & \text{for } m_1 \le x \le m_2 \\ 1-(1-k)\dfrac{x-m_2}{m_3-m_2} & \text{for } m_2 \le x \le m_3 \\ 1 & \text{for } x = m_3 \\ 1-(1-k)\dfrac{m_4-x}{m_4-m_3} & \text{for } m_3 \le x \le m_4 \\ k\dfrac{m_5-x}{m_5-m_4} & \text{for } m_4 \le x \le m_5 \\ 0 & \text{for } x > m_5 \end{cases} \qquad ....(14)$$

The □-cuts of linear PFN with symmetry is denoted as

$$\tilde{\theta}[\alpha] = \{x \in X : \xi_{\tilde{\theta}LS}(x) \ge \alpha\}$$

$$= \begin{cases} \left[m_1 + \dfrac{\alpha}{k}(m_2 - m_1) \quad , \quad m_5 - \dfrac{\alpha}{k}(m_5 - m_4)\right] & \text{for } \alpha \in [0,k] \\ \left[m_2 + \dfrac{1-\alpha}{1-k}(m_3 - m_2) \quad , \quad m_4 + \dfrac{1-\alpha}{1-k}(m_4 - m_3)\right] & \text{for } \alpha \in [k,1] \end{cases} \qquad ....(15)$$

Note: The essence of symmetric PFN is that the point chosen on the left side is identical to the point selected on the right side. So, for a component the fuzzy reliability function is

$$\tilde{S}(t)[\alpha]=\left\{\int_0^\infty \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^\beta} dx \mid \theta\in\tilde{\theta}[\alpha]\right\}=\left\{e^{-\left(\frac{x-\delta}{\theta}\right)^\beta} \mid \theta\in\tilde{\theta}[\alpha]\right\} \quad \text{.........(16)}$$

Based on the α-cuts, the fuzzy reliability function is

$$\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{\left\{-\left(\frac{t-\delta}{m_1+\frac{\alpha}{k}(m_2-m_1)}\right)^\beta\right\}},\; e^{\left\{-\left(\frac{t-\delta}{m_5-\frac{\alpha}{k}(m_5-m_4)}\right)^\beta\right\}}\right] \\ \left[e^{\left\{-\left(\frac{t-\delta}{m_2+\frac{1-\alpha}{1-k}(m_3-m_2)}\right)^\beta\right\}},\; e^{\left\{-\left(\frac{t-\delta}{m_4+\frac{1-\alpha}{1-k}(m_4-m_3)}\right)^\beta\right\}}\right]\end{cases} \quad \text{.......(17)}$$

$\tilde{S}(t)[\alpha]$ is a function of α and t, where ($0 \le \alpha \le 1$ and $t > 1$). It is a linear PFN with symmetry for t_0, so the membership of $\tilde{S}(t_0)$ is

$$\xi_{\tilde{\theta}LS}(x)=\begin{cases}\dfrac{x-e^{\left\{-\left(\frac{t_0-\delta}{m_1}\right)^\beta\right\}}}{e^{\left\{-\left(\frac{t_0-\delta}{m_2}\right)^\beta\right\}}-e^{\left\{-\left(\frac{t_0-\delta}{m_1}\right)^\beta\right\}}} & ,\; e^{\left\{-\left(\frac{t_0-\delta}{m_1}\right)^\beta\right\}}\le x< e^{\left\{-\left(\frac{t_0-\delta}{m_2}\right)^\beta\right\}} \\ \dfrac{x-e^{\left\{-\left(\frac{t_0-\delta}{m_2}\right)^\beta\right\}}}{e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}}-e^{\left\{-\left(\frac{t_0-\delta}{m_2}\right)^\beta\right\}}} & ,\; e^{\left\{-\left(\frac{t_0-\delta}{m_2}\right)^\beta\right\}}\le x\le e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}} \\ 1 & ,\; x=e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}} \\ \dfrac{e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}}-x}{e^{\left\{-\left(\frac{t_0-\delta}{m_4}\right)^\beta\right\}}-e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}}} & ,\; e^{\left\{-\left(\frac{t_0-\delta}{m_3}\right)^\beta\right\}}\le x\le e^{\left\{-\left(\frac{t_0-\delta}{m_4}\right)^\beta\right\}} \\ \dfrac{e^{\left\{-\left(\frac{t_0-\delta}{m_5}\right)^\beta\right\}}-x}{e^{\left\{-\left(\frac{t_0-\delta}{m_5}\right)^\beta\right\}}-e^{\left\{-\left(\frac{t_0-\delta}{m_4}\right)^\beta\right\}}} & ,\; e^{\left\{-\left(\frac{t_0-\delta}{m_4}\right)^\beta\right\}}\le x\le e^{\left\{-\left(\frac{t_0-\delta}{m_5}\right)^\beta\right\}}\end{cases} \quad \text{.......(18)}$$

Here, the "Fuzzy Mean Time to Failure" (FMTTF) can be described in simpler terms as an estimate of the average time until a system or component is expected to fail. This estimate incorporates uncertainty and imprecision, typical of real-world scenarios, by using fuzzy logic. Essentially, it provides a more flexible and realistic prediction of failure time by considering a range of possible outcomes rather than a single fixed value.

As Buckley [13] outlines, the FMTTF (Fuzzy Mean Time to Failure) of this fuzzy system is represented as a fuzzy number. The calculation of this can be achieved through the following method:

$$M\tilde{T}TF[\alpha]=\left\{\int_0^\infty x(x)dx \mid \theta\in\tilde{\theta}[\alpha]\right\}=\left\{\int_0^\infty S(t)dx \mid \theta\in\tilde{\theta}[\alpha]\right\} \quad \text{........(19)}$$

When the likelihood of failure is described by a fuzzy Weibull distribution, it implies that the failure times are modeled using a Weibull distribution, which incorporates elements of uncertainty or fuzziness in its parameters or outcomes. This approach is often used to handle situations where precise data is not available, allowing for a more flexible representation of variability and uncertainty in the failure times, so

$$M\tilde{T}TF[\alpha]=\left\{\theta\Gamma(1+\beta^{-1}) \mid \theta\in\tilde{\theta}[\alpha]\right\}$$

$$=\begin{cases}[(m_1+2\alpha(m_2-m_1))\Gamma(1+\beta^{-1})\;,\;(m_5-2\alpha(m_5-m_4))\Gamma(1+\beta^{-1})] \\ [(m_2+2(1-\alpha)(m_3-m_2))\Gamma(1+\beta^{-1})\;,\;(m_4-2(1-\alpha)(m_4-m_3))\Gamma(1+\beta^{-1})]\end{cases} \quad \text{........(20)}$$

Accordingly, to expression (20) defined above, the membership function is derived as follows

$$\xi_{\tilde{S}(t_0)}(x)=\begin{cases} k\left(\dfrac{x-m_1\Gamma(1+\beta^{-1})}{(m_2-m_1)\Gamma(1+\beta^{-1})}\right) & \text{for } \; m_1\Gamma(1+\beta^{-1})\le x\le m_2\Gamma(1+\beta^{-1}) \\ 1-(1-k)\left(\dfrac{x-m_2\Gamma(1+\beta^{-1})}{(m_3-m_2)\Gamma(1+\beta^{-1})}\right) & \text{for } \; m_2\Gamma(1+\beta^{-1})\le x\le m_3\Gamma(1+\beta^{-1}) \\ 1 & \text{for } \; x=m_3\Gamma(1+\beta^{-1}) \\ 1-(1-k)\left(\dfrac{m_4\Gamma(1+\beta^{-1})-x}{(m_4-m_3)\Gamma(1+\beta^{-1})}\right) & \text{for } \; m_3\Gamma(1+\beta^{-1})\le x\le m_4\Gamma(1+\beta^{-1}) \\ k\left(\dfrac{m_5\Gamma(1+\beta^{-1})-x}{(m_5-m_4)\Gamma(1+\beta^{-1})}\right) & \text{for } \; m_4\Gamma(1+\beta^{-1})\le x\le m_5\Gamma(1+\beta^{-1}) \end{cases} \quad \text{.......(21)}$$

Fuzzy Hazard Function

The fuzzy hazard function is a fundamental concept in fuzzy reliability theory, which is a branch of mathematics focused on assessing the reliability of systems and components when uncertainty is present. This function is grounded in the fuzzy probability measure and is typically represented through its α-cuts, which are slices of the fuzzy set at different levels of confidence.

Essentially, the fuzzy hazard function measures the probability of a system or component failing within a brief period, from t to (t + dt), given that it has been functioning without failure up to the time t. This probability is conditional, meaning it takes into account the item's operational history and current state of reliability. Due to this characteristic, the fuzzy hazard function is referred to as the rate at which failures occur instantaneously, as it provides a snapshot of the failure probability at a specific moment in time.

Mathematically, the fuzzy hazard function provides a framework for evaluating the likelihood of an unsuccessful outcome occurring at a specific moment, incorporating the item's past performance and reliability trends. By doing so, it allows engineers and reliability analysts to better understand and predict the behavior of systems under uncertainty, thereby aiding in the design and maintenance of more reliable products and processes.

$$\begin{aligned} \tilde{h}(t)[\alpha] &= \lim_{\Delta t\to 0}\frac{\tilde{P}(t<X<t+\Delta t \mid X>t)}{\Delta t} \\ &= \left\{\lim_{\Delta t\to 0}\frac{S(t)-S(t+\Delta t)}{\Delta t S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\} \qquad \text{........(22)} \\ &= \left\{\frac{-S'(t)}{S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\} \\ &= \left\{\frac{f(t)}{S(t)} \mid \theta\in\tilde{\theta}[\alpha]\right\} \end{aligned}$$

For a Weibull distribution with $\delta = 0$, we have

$$\tilde{h}(t)[\alpha]=\left\{\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \mid \theta\in\tilde{\theta}[\alpha]\right\} \qquad \text{...........(23)}$$

The function $\tilde{h}(t)[\alpha]$ depends on two variables α and t ($0 \le \alpha \le 1$ and $t > 1$).

Quantitative Examination

The lifespan of the component is depicted using a Weibull distribution model, with one parameter being uncertain or imprecise, indicated by a fuzzy value, also the shape parameter is given as δ = 0, and the scale parameter is represented by a fuzzy set as $\tilde{\theta}$ = (1.25, 1.30, 1.35, 1.40, 1.45; 0.5). The α-cuts of the fuzzy reliability function, the fuzzy hazard function, and the fuzzy mean time to failure can be thoroughly explained using their corresponding mathematical formulas and interpretations.

i) **Fuzzy reliability is:**

$$\tilde{\theta}[\alpha]=\begin{cases}[1.25+2\alpha(0.05) \quad , \quad 1.50-2\alpha(0.05)\,] \\ [1.35+2(1-\alpha)(0.05) \quad , \quad 1.45+2(1-\alpha)(0.05)]\end{cases}$$

$$\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{\left\{-\left(\frac{t}{1.25+2\alpha(0.05)}\right)^{\beta}\right\}},e^{\left\{-\left(\frac{t}{1.50-2\alpha(0.05)}\right)^{\beta}\right\}}\right] \\ \left[e^{\left\{-\left(\frac{t}{1.45-(0.1)\alpha)}\right)^{\beta}\right\}},e^{\left\{-\left(\frac{t}{1.55-(0.1)\alpha)}\right)^{\beta}\right\}}\right]\end{cases} \qquad ....(24)$$

$$\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{\left\{-\left(\frac{t-\delta}{1.25+2\alpha(0.05)}\right)^{\beta}\right\}},e^{\left\{-\left(\frac{t-\delta}{1.50-2\alpha(0.05)}\right)^{\beta}\right\}}\right] \\ \left[e^{\left\{-\left(\frac{t-\delta}{1.45-(0.1)\alpha)}\right)^{\beta}\right\}},e^{\left\{-\left(\frac{t-\delta}{1.55-(0.1)\alpha)}\right)^{\beta}\right\}}\right]\end{cases} \qquad ....(25)$$

(1) If t = 0.5

$$\xi_{\tilde{\theta}(0.5)}(x)=\begin{cases}\dfrac{x-e^{\left\{-(0.4)^{\beta}\right\}}}{e^{\left\{-(0.3847)^{\beta}\right\}}-e^{\left\{-(0.4)^{\beta}\right\}}} & , \; e^{\left\{-(0.4)^{\beta}\right\}}\le x< e^{\left\{-(0.3847)^{\beta}\right\}} \\ \dfrac{x-e^{\left\{-(0.3847)^{\beta}\right\}}}{e^{\left\{-(0.3704)^{\beta}\right\}}-e^{\left\{-(0.3847)^{\beta}\right\}}} & , \; e^{\left\{-(0.3847)^{\beta}\right\}}\le x\le e^{\left\{-(0.3704)^{\beta}\right\}} \\ 1 & , \; x=\exp^{\left\{-(0.3704)^{\beta}\right\}} \\ \dfrac{e^{\left\{-(0.3704)^{\beta}\right\}}-x}{e^{\left\{-(0.3571)^{\beta}\right\}}-e^{\left\{-(0.3703)^{\beta}\right\}}} & , \; e^{\left\{-(0.3704)^{\beta}\right\}}\le x\le e^{\left\{-(0.3571)^{\beta}\right\}} \\ \dfrac{e^{\left\{-(0.3448)^{\beta}\right\}}-x}{e^{\left\{-(0.3448)^{\beta}\right\}}-e^{\left\{-(0.3571)^{\beta}\right\}}} & , \; e^{\left\{-(0.3571)^{\beta}\right\}}\le x\le e^{\left\{-(0.3448)^{\beta}\right\}}\end{cases} \qquad ....(26)$$

(2) If α = 0, then reliability is perfect or absolute, and is given by

$$\tilde{S}(t)[0]=\tilde{S}(t)[\alpha]=\begin{cases}\left[e^{\left\{-(0.8)^{\beta}\right\}},e^{\left\{-(0.6667)^{\beta}\right\}}\right] \\ \left[e^{\left\{-(0.6896)^{\beta}\right\}},e^{\left\{-(0.6452)^{\beta}\right\}}\right]\end{cases} \qquad ....(27)$$

Fuzzy hazard function is

$$\tilde{h}(t)[\alpha] = \begin{cases} \dfrac{\beta}{1.50-2\alpha(0.05)}\left(\dfrac{t}{1.50-2\alpha(0.05)}\right)^{\beta-1}, \dfrac{\beta}{1.25+2\alpha(0.05)}\left(\dfrac{t}{1.25+2\alpha(0.05)}\right)^{\beta-1} \\ \dfrac{\beta}{1.45+2(1-\alpha)(0.05)}\left(\dfrac{t}{1.45+2(1-\alpha)(0.05)}\right)^{\beta-1}, \dfrac{\beta}{1.35+2(1-\alpha)(0.05)}\left(\dfrac{t}{1.35+2(1-\alpha)(0.05)}\right)^{\beta-1} \end{cases} \quad .....(28)$$

Fuzzy mean time to failure.

$$M\tilde{T}TF[\alpha](\beta) = \{\theta\Gamma(1+\beta^{-1}) \mid \theta \in \tilde{\theta}[\alpha]\}$$

$$= \begin{cases} \left[\left(1.25+2\alpha(0.05)\right)\Gamma(1+\beta^{-1}) \quad , \quad \left(1.50-2\alpha(0.05)\right)\Gamma(1+\beta^{-1})\right] \\ \left[\left(1.35+2(1-\alpha)(0.05)\right)\Gamma(1+\beta^{-1}) \ , \left(1.45+2(1-\alpha)(0.05)\right)\Gamma(1+\beta^{-1})\right] \end{cases} \quad ......(29)$$

The lowest value of the function FMTTF is reached when the parameter β is approximately 2.166, as determined by the properties of the gamma function

$$M\tilde{T}TF[\alpha](2.166) = M\tilde{T}TF[\alpha](\beta) = \{\theta\Gamma(1+\beta^{-1}) \mid \theta \in \tilde{\theta}[\alpha]\}$$

$$= \begin{cases} \left[\left(1.25+2\alpha(0.05)\right)\Gamma(1.46168) \quad , \quad \left(1.50-2\alpha(0.05)\right)\Gamma(1.46168)\right] \\ \left[\left(1.35+2(1-\alpha)(0.05)\right)\Gamma(1.46168) \ , \left(1.45+2(1-\alpha)(0.05)\right)\Gamma(1.46168)\right] \end{cases}$$

$$= \begin{cases} \left[\left(1.25+2\alpha(0.05)\right)\left(0.8856\right) \quad , \quad \left(1.50-2\alpha(0.05)\right)\left(0.8856\right)\right] \\ \left[\left(1.35+2(1-\alpha)(0.05)\right)\left(0.8856\right) \ , \left(1.45+2(1-\alpha)(0.05)\right)\left(0.8856\right)\right] \end{cases}$$

$$= \begin{cases} \left[\left(1.1070+0.0886\alpha\right) \ , \left(1.3284-0.0886\alpha\right)\right] \\ \left[\left(1.2841+0.0886\alpha\right) \ , \left(1.3727+0.0886\alpha\right)\right] \end{cases} \quad ......(30)$$

The membership function derived from this is as follows

$$\xi_{\tilde{S}(t_0)}(x) = \begin{cases} \left(\dfrac{x-1.107}{0.04428}\right) & \text{for} \quad 1.107 \le x \le 1.15128 \\ \left(\dfrac{x-1.15128}{0.04428}\right) & \text{for} \quad 1.15128 \le x \le 1.19556 \\ 1 & \text{for} \quad x = 1.19556 \\ \left(\dfrac{1.23984-x}{0.04428}\right) & \text{for} \quad 1.19556 \le x \le 1.23984 \\ \left(\dfrac{1.28412-x}{0.04428}\right) & \text{for} \quad 1.23984 \le x \le 1.28412 \end{cases} \quad ....(21)$$

Conclusion

The study focuses on advancing the Weibull distribution by integrating fuzzy reliability and hazard functions, complete with their α-cuts. This approach is particularly useful when both randomness and fuzziness affect the lifetimes of components and their parameters, where traditional statistical methods may not suffice. The study develops a method for assessing the dependability of fuzzy systems by applying fuzzy set theory and fuzzy probability theory. It uses a pentagonal fuzzy number to denote the magnitude factor, and suggests that future research might examine the shape and location parameters as fuzzy variables, either independently or in combination. Additionally, there is a call for more in-depth research into significant aspects of fuzzy reliability theory, such as the average remaining lifespan.

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