

Effects Of Three-Tier Teaching Model For Teaching Mathematics In Context On Pupils' Performance In Fractions In Cape Coast Metropolis

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Abstract: This paper describes the outcomes of introducing the three-tier teaching model for teaching mathematics in context. The model, an innovative approach to mathematics teaching, links school-taught mathematical concepts to the everyday experiences of learners. The study was grounded in cultural-historical theory. The study investigated the effect of the three-tier teaching model for teaching mathematics in context on pupils' performance in fractions in Cape Coast Metropolis (CCM). The study used multistage sampling technique selecting 434 primary one pupils in the Cape Coast Metropolis. The study administered performance tests to pupils in 12 primary schools in the CCM. We analysed the quantitative data from 434 primary school pupils using frequency counts, mean, standard deviation, independent sample t-test, and paired sample t-test. We analysed the contents of the interview transcript and the narrative discussion in the focus group sections to obtain qualitative data. The results indicated that the three-tier teaching model for teaching mathematics in context's pedagogy was more effective than the conventional approach, which reflects the normal approach used by teachers in teaching fractions at the primary school level. This study recommends that mathematics teachers at the Basic One level should implement the three-tier teaching model for teaching mathematics in context as an innovative pedagogical strategy to enhance the contextual teaching and learning of fractions.

Keywords: Effect of three-tier teaching model, teaching fractions in context, Cape Coast Metropolis, pupils' performance in fractions.

INTRODUCTION

In this study, we conceptualised a fraction as part of any object. Fractions are cultural concepts that have played a significant role in human history and continue to be an essential part of various cultures around the world (Bishop, 1988; Planas & Llinares, 2021; Rosa & Barros, 2020). While the mathematical concept of fractions is universal, their understanding and significance can vary across different societies (Bishop, 1988; Davis, 2010). Fractions' historical development is one aspect of their cultural context. Ancient civilisations, such as the Egyptians, Papua New Guinea, and Mesopotamians, were among the first to develop mathematical systems that included fractions (Saxe, 2002). These early civilisations used fractions for practical purposes, such as measuring land, dividing resources, and conducting trade (Davis, 2010). Many cultures have also incorporated fractions into everyday language and expressions. For example, idioms and proverbs often incorporate fractional orientations (Borba & Greer, 2016; Davis, 2016). Phrases such as "a fraction of a second", "a piece of pie", or "in half a heartbeat" commonly convey a sense of smallness, division, or speed in English. Similar idiomatic expressions related to fractions exist in other languages, reflecting the cultural significance of this mathematical concept (Saxe, 1994).

Literature says that fractions have cultural relevance in various areas, such as art, cuisine, and music (Planas & Llinares, 2021). In art, the concept of fractions is fundamental for achieving balanced compositions, proportions, and perspective. Cooking and culinary traditions use fractions to measure ingredients and ensure precise ratios (Saxe, 2002). Additionally, music relies heavily on fractions, as the notation and timing of musical notes are based on fractions and their relationships (Planas & Llinares, 2021). Different cultures also tie fractions to symbolic meanings (Saxe, 1994). Saxe recognises that in

Chinese culture, the number eight is considered very lucky because it sounds like the word "wealth" or "prosperity." Because of this association, fractions involving the number eight, such as $\frac{1}{8}$ or $\frac{7}{8}$, may be symbolically significant. In some African cultures, traditional crafts and patterns often incorporate fractional measurements. For example, in weaving or beadwork, the use of fractions can represent balance and harmony within the design. Some Native American cultures use fractions in their storytelling and symbolism. For instance, many Native American spiritual traditions may use the fraction $\frac{1}{4}$ to symbolise the four cardinal directions. Certain numerical patterns or fractions have religious or spiritual significance in some societies. For instance, the division of time into fractions, such as hours, minutes, and seconds, has influenced cultural perceptions of punctuality, efficiency, and time management (Brislin & Kim, 2003). Social norms and customs, exemplified in sharing and distribution, can be associated with fractions (Davis, 2013; Rosa & Barros, 2020). In many cultures, the idea of fair division and equitable sharing is deeply ingrained. Fractions serve to fairly divide resources among individuals or groups, ensuring everyone receives an appropriate portion (Davis, 2010). It is important to note that while fractions have cultural significance (i.e., in cooking and baking, shopping, farming, time management, measurements, finances, medication dosage, sports and fitness, statistics and probability, sewing crafts, travelling, sharing and dividing, grading and assessment, medical, and health), their specific interpretations and uses can differ widely across cultures (Saxe, 2002). Therefore, cultural diversity shapes our understanding, application, and representation of mathematical concepts like fractions (Bishop, 1988; Davis, 2017a, 2017b). Therefore, studying fractions as a cultural concept requires exploring the unique perspectives and practices of different societies around the world and connecting such experiences to the school concepts of mathematics to make the school concepts of mathematics meaningful and relevant. Fractions can manifest themselves in various ways.

LITERATURE REVIEW

Fractional Constructs

This paper presents key fractional concepts that represent fractions. Fraction as part of a whole: according to Kieren (1980) and Steffe and Olive (2010), using the part-whole construct is an effective way of teaching fractions to pupils. You can represent part-whole concepts by shading a region or highlighting a subset within a group of objects. For example, $\frac{3}{5}$ of the pupils in a class turned in their assignments. When interpreting a fraction as part of a whole, the denominator indicates the number of equal parts in the whole, while the numerator indicates the number of those parts removed from the total equal number of parts. In this scenario, we can divide a whole into several equal parts and extract some of them as portions of the total. Area models, which partition a shape into equal parts, mostly represent fractions as parts of the whole. For instance, we can divide the area of a rectangle into two equal parts. We must extend the concept of a fraction as a whole to other practical problems involving length, volume, and groups of objects, not just the area concept. As a result, we can have a third of the length, a quarter of the volume, and half of the pupils in class (Davis, 2010, 2017b). We can also use it in more practical contexts, like a third of a sugar cane, a quarter of a bucket of water, or half of a class of pupils. In more practical problems, all these concepts show how a fraction can represent a portion of a larger concept. Fraction as a measure involves identifying a length, then using that length as a measurement piece to determine the length of an object (Kieren, 1980). For example, in the fraction $\frac{5}{8}$, you can use the unit fraction $\frac{1}{8}$ as the chosen length and then count or measure to show that it takes five of those to get to $\frac{5}{8}$. This concept, as in part-whole situations, emphasises how much rather than how many parts (Martinie, 2007). The concepts and process of rectilinear measurement heavily influence fraction as a unit of measurement. Rectilinear measurements commonly use scales (Kieren, 1980). When measuring an interval with a ruler, the units of measurement are typically centimetres. This means that each centimetre represents one-hundredth of a metre in length, and measuring from the beginning of the interval to its endpoint tells us how many units the endpoint is away from zero. To get a more precise measurement, we could divide the centimetres into smaller, equal parts as millimetres. We can divide the distance between 0 and 1 into 5 parts on the number line, yielding one-fifth as a unit of measurement; we can use the number line instead of a ruler. For example, a $\frac{1}{5}$ point indicates that the point is one unit away from zero. Pupils can further subdivide Point $\frac{1}{5}$ into smaller parts. Fraction as a division: consider the idea of splitting Gh20.00 among five

people. Although this is not a part-whole situation, it still means that each person will receive one-fifth ($1/5$) of the money, or 4 Ghana cedis. Unfortunately, most divisions are unrelated to fractions. Meanwhile, pupils often bring ideas from their cultural context to school, undermining the effectiveness of context-based teaching (Davis, 2010, 2017b). We learned the popular convention of 'a over b' in primary school, meaning 'a divided by b,' where 'b' does not equal zero. For example, the fraction one-third signifies 1 divided by 3, that is, $\frac{1}{3} = 1 \div 3$. We typically utilize this concept to convert improper fractions into mixed numerals. For instance, $\frac{13}{2} = 13 \div 2 = 6\frac{1}{2}$. In this case, the fraction represents the outcome of the division. Fractions as an operator: fractions, such as $4/5$ of 20 square feet or $2/3$ of the audience holding flags, can represent an operation. These situations indicated a fraction of a whole number, and pupils may be able to figure out the answer using mental mathematics. School curricula underemphasise this concept (Usiskin, 2007). In this case, we use the fraction to 'operate' on a quantity. It implies that the fraction now behaves as if it were a function. Simply knowing how to represent fractions does not imply that pupils will be able to operate with them at an advanced level of mathematics (Davis, 2010). Another context that utilises fractions is the concept of ratios (Lamon, 2006). A fraction of a fourth, for example, can indicate that an event is likely one in four. It is possible to have part-part and part-whole ratios. The ratio $3/4$ could refer to the proportion of jacket-wearers to non-jacket-wearers or to total class members. We frequently use ratios, illustrated with discrete items, to express the quantitative relationship between two groups (Kieren, 1980). Davis (2010) defines a ratio as the frequency with which one amount either contains or contains another. The description of a consistent correlation in the rise or fall of measurements, like the rate of time or length, is crucial. These rates are crucial in understanding and quantifying changes over time or distance, providing a clear indication of the speed or intensity of growth or decline in a given measure.

Challenges in Learning and Teaching Fractions

As pupils advance from primary to higher grade levels, misunderstandings greatly impede their ability to comprehend fractions (Mitchell & Horne, 2010). Research suggests that teachers often introduce fractions to pupils as an abstract concept, disconnected from their real-life experiences (Lamon, 2006; Clarke & Roche, 2014; Davis, 2010). This disconnection makes it challenging for young learners to understand fractions. Davis (2010, 2017b) observed that the teachers' failure to contextualise teaching may contribute to pupils' difficulties with fractions. He goes on to explain that the introduction of fractions in the early grades hinders pupils' ability to develop a conceptual understanding of the concept. As a result, pupils in this area continue to struggle with poor performance. For example, Pean and Stephens (2004) found that grade 8 pupils are unable to determine whether $2/3$ or $3/5$ is greater. Student 2 said that two-thirds was larger because "three-fifths is two numbers away from being a whole, and two-thirds is one number away from being a whole" (p. 434). Post and Cramer (1987) also revealed that grade 4 pupils see $3/4$ and $2/3$ as equal fractions because "the difference between numerator and denominator in each fraction was one." (p. 33). This type of misconception is also commonly observed among early grade learners when learning fractions, and it may significantly impact their understanding of other mathematical concepts. Cramer and Wyberg (2009), in their study, also indicated that when pupils were asked to determine which is greater, $3/4$ or $5/12$, they identified $3/4$ to be bigger than $5/12$ because "5/12 still has 7 more to go" as opposed to $3/4$, which has "one more to go". So, it should be bigger" (p. 241). These fractional thinking misconceptions may affect more than 25% of our pupils, and studies indicate that they are prevalent in Years 4, 6, and 8 (Pean & Stephens, 2004; Mitchell & Horne, 2010).

Also, Cramer and Whitney (2010) found that many pupils do not recognise the numerator-denominator ratio as a single value. For example, it is difficult for them to recognise that $3/5$ is a single number. They recommend that pupils use a number line to find fractional values. Pupils can enjoy this as a fun warm-up activity every day, with specific values written on a classroom number line or in their mathematics notebooks. Residual thinking and benchmarking could be used to close these fractional gaps (Clarke & Roche, 2014). Comparing how much each fraction deviates from one is the essence of residual thinking, while benchmark thinking involves comparing fractions to a common benchmark, typically $1/2$ or 1, and using equivalent fractions to simplify the comparison. For instance, $1/8$ and $1/6$ represent the residuals for $7/8$ and $5/6$, respectively. In the case of $3/4$ and $2/5$, we can designate $1/2$ as the common benchmark

and conclude that $\frac{3}{4}$ is greater than $\frac{1}{2}$ and $\frac{2}{5}$ is less than $\frac{1}{2}$; therefore, $\frac{3}{4}$ exceeds $\frac{2}{5}$. Pupils may also struggle with fractions due to the multiple interpretations we attribute to fractions, which include part-whole, operator, division, ratio, measurement, and so on (McNamara & Shaughnessy, 2010). The pupil generalises their whole-number knowledge. The teacher must assist pupils in distinguishing between similar fractions and whole numbers. This would assist pupils in not confusing whole number concepts with fractions. We measure the fraction on the number line to various levels of precision, such as the nearest tenth of a centimetre. Unless they are discussing ratios or probability, pupils should avoid using the phrases "three out of five" or "three over five". Instead, say "three-fifths" (Siebert & Gaskin, 2006). Pupils do not understand that $\frac{2}{3}$ refers to three equal parts (although not necessarily equal-shaped objects). The researchers suggest that teachers ask pupils to identify fractions using various manipulatives and paper. Allow learners to appreciate fractions as a division of any object into equal parts by presenting problems that do not already have all partitions drawn. Because 5 is less than 10, pupils believe that a fraction like $\frac{1}{5}$ is less than a fraction like $\frac{1}{10}$. Mitchell and Horne (2010) also discovered that teachers could also impart the inverse principle, which states that a larger denominator corresponds to a smaller fraction. When teachers teach such rules without allowing pupils to conceptualise and explain why, they may cause them to overgeneralise, leading them to believe $\frac{1}{5}$ is greater than $\frac{7}{10}$. We recommend that teachers use a variety of visuals and contexts to demonstrate different aspects of the whole. For example, ask pupils whether they would rather spend half an hour, a quarter of an hour, or a tenth of an hour outside school. Consider the concept of equitable distribution. Is it fair that Kofi gets one fourth of the bread and Ama gets one eighth? We should instruct pupils to explain why this situation is unfair and identify who receives the majority share (Davis, 2010). Examples of these contexts will assist pupils in properly comparing fractional objects. A pupil may incorrectly apply the whole-number operation "rules" to fractions, such as $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$. This was a completely erroneous relationship. The researchers recommend that pupils use various visuals in the learners culture to learn fractions in context. Teachers should pay close attention to whether or not pupils' answers emerge from conceptual thinking (Davis, 2010). Pupils who make these mistakes do not understand fractions in terms of relationships. They will continue to make errors by emphasising whole number concepts until they grasp fractional learning (Siegler et al., 2010). In the early grades of school, pupils begin to use the concept of parts and wholes, beginning with halves and progressing to quarters and eighths (NaCCA, 2019). Some pupils will only notice the number of parts, ignoring the fact that the parts must be equal (Davis, 2010, 2017b). These pupils may have challenges in developing appropriate understandings, such as "the greater the number of parts, the smaller the parts." However, the misconception that a fraction is defined solely by its number of parts may persist. When such pupils examine a written fraction, they may mistake the denominator for the number of parts, failing to realise that the fraction label only applies if the parts are equal. Davis' research in 2017b expanded the scope of teaching fractions in the early grades by presenting pupils with three sticks, each divided into two parts with only one divided into exact halves, and asking them to identify the fractions in each scenario. This was done to ascertain the pupils' understanding of half, and it was found that more than three-quarters of the pupils in grade 2 were unable to correctly identify the appropriate fraction for half. Davis (2017b) suggested a more comprehensive way to help learners develop proper conceptual links when learning mathematical concepts like fractions. According to him, it is crucial for mathematics teachers to teach in a context that allows pupils to integrate their everyday experiences into their understanding of mathematical concepts taught in school. As part of the learning process, it is natural for pupils to make mistakes and try out new strategies. However, a gap in conceptual understanding of fractions may form if incorrect fractional thinking becomes evidence of a misunderstanding. In general, we cannot correct misconceptions unless we explicitly address them through innovative teaching and learning (Davis, 2010). As a result, teachers must be aware of common misconceptions and seek evidence of them in their pupils' thinking. Identifying misconceptions in learning mathematics necessitates careful observation or purposeful diagnostic assessment (Cramer & Whitney, 2010). Because pupils build on prior knowledge, when they encounter situations involving fractions, they naturally apply what they know about whole numbers to solve the problems.

Rationale and Context for the Present Study

The International Association for the Valuation of Educational Achievement (IEA) of the USA conducted the Trends in International Mathematics and Science Study (TIMSS) in 2003, and the results indicated that Ghanaian grade 8 pupils performed poorly in the mathematics achievement test (TIMSS, 2003), ranking 45th out of 46 participating countries. The Trends in International Mathematics and Science Study (TIMSS, 2007) conducted a similar study in 2007, ranking Ghana's eighth-graders 47th out of 48 participating countries. The situation worsened in 2011, when the Trends in International Mathematics and Science Study (TIMSS, 2011) ranked Ghana 76th out of 76 participating nations. After the first three appearances in TIMSS, Ghana has stayed out of the subsequent ones. This points to the fact that Ghanaian learners have limited subject knowledge, applications, and reasoning skills that they needed for problem solving (Anamuah-Mensah et al., 2004; Davis, 2010). TIMSS (2011) emphasised that teaching methods in countries where pupils were not able to exhibit maximum application and reasoning skills, like Ghana, were mainly the teaching by telling approach, which mainly results in procedural knowledge and instrumental understanding. Literature has it that the challenge of learning fractions persists in many countries, particularly in sub-Saharan African countries like Ghana (TIMSS, 2003; 2007; 2011). In 2013, when JHS pupils in Ghana were asked to subtract a mixed fraction from another mixed fraction, most candidates could not do the simplification well and hence failed to answer the question well (WAEC, 2013). In 2015, JHS pupils faced a fractional word problem that required them to calculate the product of $\frac{2}{5}$ and $\frac{2}{3}$. Most of the candidates chose to add instead of multiplying the two fractions (WAEC, 2015). This situation became worse in the 2016 BECE, when more than half of the candidates were unable to recognise decimal fractions and common fraction equivalences for halves in a question (WAEC, 2016).

According to the literature (Davis & Chaiklin, 2015; Davis, 2017b), inappropriate teaching methods in mathematics education led to these low achievement and retention rates. The usual ways teachers introduce pupils in the early grades to mathematical concepts, especially fractions in Ghanaian primary schools, do not allow learners to connect fractions to everyday realities (Davis, 2010, 2017b). This makes pupils perceive home and school mathematics to be completely disjoint. This research looked into how a new method affected pupils' performance in representing fractions in Cape Coast Metropolis (CCM), which is in the southern belt of Ghana. The goal was to learn more about how to teach mathematics in real-life situations by using the three-tier teaching model for teaching mathematics in context. This study suggests that the three-tier model for teaching mathematics in context, grounded in cultural-historical theory, enhances students' ability to recognise and name fractions by linking mathematical concepts to their everyday experiences, social interactions, and language development within their learning potential.

Cultural-Historical Theory

The cultural-historical theory of learning, grounded in the work of Lev Vygotsky, provides a rich framework for understanding the processes that underpin cognitive development. Central to this theory are three interrelated concepts: social interaction, language, and the zone of proximal development (ZPD). Each of these plays a foundational role in shaping how individuals acquire knowledge, develop skills, and internalise the cultural tools necessary for higher-order thinking. Vygotsky (1978) asserted that learning is inherently a social process, unfolding on two levels: the interpersonal and the intrapersonal. Initially, knowledge is co-constructed through interactions with others, particularly with more knowledgeable peers or adults, before it is internalised by the learner. This dual-stage process underscores the importance of collaborative learning environments where social interaction serves not only as a context for learning but also as a catalyst for cognitive development. Through shared activities and guided participation, learners are exposed to new ways of thinking, problem-solving strategies, and cultural norms, which they gradually adopt and adapt as their own. Language is another crucial component within Vygotsky's framework, functioning both as a medium of communication and a tool of thought. It serves as the primary semiotic system through which individuals engage with their environment, negotiate meaning, and regulate their own cognitive processes. In Vygotsky's view, language mediates both social and individual functioning, playing a central role in transforming external dialogues into internal reasoning. This linguistic mediation is particularly important in educational settings, where teachers and peers use language to scaffold learning and support students in making sense of complex concepts. At the heart of Vygotsky's theory lies the concept of the zone of proximal development, which refers to the distance between what a learner

can accomplish independently and what they can achieve with the assistance of a more capable individual. The ZPD highlights the potential for growth that lies just beyond the learner's current abilities, making it a powerful guide for instructional planning. Vygotsky (1978) observed that assessing a child's ZPD offers a more accurate measure of their cognitive development than evaluations based solely on independent performance. This perspective highlights the importance of differentiated instruction and targeted support that align with each learner's developmental stage, enabling them to progress through increasingly challenging tasks with appropriate guidance. Importantly, cultural-historical theory also recognises the influence of cultural and intercultural contexts on intellectual development. Learning does not occur in a vacuum; it is shaped by the values, beliefs, practices, and tools of the surrounding culture. As such, teachers are encouraged to consider students' diverse cultural backgrounds when designing and implementing instructional activities like the three-tier teaching model for teaching mathematics in context's approach.

Three-tier Teaching Model for Teaching Mathematics in Context (Davis, 2010)

Davis (2010, 2017b) suggested three levels (i.e., enculturation, transition and acculturation) that are necessary and sufficient for teaching mathematics in context. Teaching mathematics in context is a pedagogical approach that aims to enhance pupils' understanding and application of mathematical concepts by drawing on learners' everyday realities to scaffold the school concept of mathematics. This approach contrasts with traditional methods where mathematics is often taught as abstract concepts and disconnected from pupils' everyday experiences (Davis & Chaiklin, 2015; Davis, 2010). Teaching mathematics in context is a pedagogical approach that aims to enhance pupils' understanding and application of mathematical concepts by presenting them in real-life situations or meaningful contexts (Davis & Chaiklin, 2015). This approach stands in stark contrast to traditional methods that often teach mathematics as abstract concepts, disconnected from pupils' everyday experiences. Davis and Chaiklin discuss the importance of teaching mathematics in context and how it has become a central issue in education worldwide. Davis developed the three-tier teaching model for teaching mathematics in context in 2010 and validated it in Davis (2017b) as an innovative pedagogical approach to improve mathematics instruction for primary pupils. The model focuses on teaching mathematics within relevant and practical contexts to enhance pupils' understanding and engagement with the subject. The model's approach drew on Vygotsky (1987) and Lancy's theory of cognitive development (Lancy, 1983). The three-tier teaching model for teaching mathematics in context's approach acknowledges the use of learners' everyday knowledge, social contexts, and language to enhance their understanding and engagement. Presenting mathematical concepts in a language that pupils can easily relate to and understand can reduce barriers to learning and promote a more profound understanding of the subject matter. We present the stages of Davis' three-tier teaching model for teaching mathematics in contexts approach in this study below.

The enculturation stage involves introducing pupils to their own mathematical culture, with a focus on understanding their social and cultural backgrounds related to mathematics (Bishop, 1988; Davis, 2010). Recognising that mathematics is not an isolated subject but interconnected with the learners' lives and experiences. Teachers may encourage pupils to share how they use mathematics in their daily lives, within their families, communities, and cultural practices. During this stage, the goal is to integrate the pupils' cultural experiences with mathematics into the classroom learning environment. Teachers can design lessons and activities that draw upon the pupils' everyday mathematical practices and problem-solving techniques. Teachers can incorporate traditional methods of counting or calculating from a particular cultural group into mathematics lessons to explore different ways of approaching problems. These practices can help demystify the abstract nature of mathematics and make it more relatable and accessible for pupils who might otherwise find it abstract or disconnected from their lives (Davis, 2017b). In the current study, the teacher starts by reviewing the pupils' prior understanding of splitting objects into two halves. The teacher forms small groups of four to six pupils and demonstrates breaking three sticks into two parts, with only one of them being exactly half. In each case, the teacher asks the pupils to name and describe the fractions, expressing their understanding of halves from their sociocultural contexts. The model's developer advises against introducing scientific concepts at this stage. Pupils then consider how tourists from Europe or America might identify halves in their home countries. This exercise helps pupils think about communicating the concept of halves to outsiders in a comprehensible way (Davis, 2010).



Figure 1: Identification of fraction of a stick (Davis, 2010)

Transition stage: The transition stage serves as the interface between enculturating pupils into their everyday mathematics (i.e., out-of-school mathematics) and acculturating pupils into international mathematics (which is mainly in-school mathematics). The transition stage is conceptualised as the vehicle through which the teacher consciously moves the learner from their everyday concepts (not formal in nature) to the scientific concepts (formal in nature). During the transition period, pupils are given the opportunity to contemplate how tourists might identify halves by reviewing their previous knowledge on obtaining halves in their sociocultural settings. Pupils also explore how halves are obtained in other cultures, leading them to conceptualise "half" as "chemu pɛ", or division into two equal parts, a prerequisite for the school concept of halves (Davis, 2010, 2017b). The teacher guides pupils to understand that halves from an international perspective mean equal halves. It is emphasised that while Ghana has its methods, it is also essential to learn international standards.

Acculturation stage: This stage starts by revisiting the initial sticks used in the lesson. Pupils, in their groups, re-examine the sticks to identify which one actually represents half from an international perspective. The teacher provides worksheets with cultural artefacts, asking pupils to shade half of them based on "chemu pɛ." This hands-on activity reinforces the idea of halves. Pupils are given further worksheets with plane shapes to shade half of each according to the international perspective. The teacher checks pupils' understanding by asking if the shaded portions represent halves. Finally, the teacher guides the pupils to represent halves symbolically. The teacher explains that the fraction $\frac{1}{2}$ signifies one of two equal portions, solidifying the pupils' understanding of halves both practically and symbolically. Through this teaching model, pupils engage with the concept of halves from their cultural context, transition to understanding it in an international context, and finally, acculturate the knowledge into a comprehensive understanding, both practically and symbolically. This method aims to provide a holistic learning experience that combines cultural relevance with universal mathematical principles. Hence, the third stage of Davis' three-tier teaching model for teaching mathematics in context must be informed by the first stage with positive transitional experiences (stage two). These three constructs (i.e., enculturation, transition and acculturation) must not be seen in isolation but rather as interrelated concepts that are necessary and sufficient for teaching mathematics in context (Davis, 2010). We must guide learners through these three stages of Davis' three-tier teaching model for teaching mathematics in context in an intellectually honest manner (Davis, 2010).

The Conventional Approach (NaCCA, 2019)

The revised curriculum by NaCCA (2019) has significantly changed the introduction of fractions in early education. It now introduces pupils to the concept of halves in Basic One, a shift from the previous curriculum (NaCCA, 2012) that introduced fractions in Basic Two. The conventional approach, reflecting typical classroom practices, integrates a series of educational indicators and exemplars aimed at aiding pupils in understanding and working with the fraction one-half. This approach combines tangible and graphical representations to help pupils grasp the concept of one-half and develop the ability to count in halves. The mathematics textbooks for Basic One are also reflective of the conventional approach in Ghana (Baah-Yeboah, 2019). The first key indicator focuses on understanding one half. A series of

exemplars facilitate this understanding. The first exemplar involves the use of concrete objects, such as oranges, to illustrate the concept. Teachers demonstrate that dividing a whole object into two equal parts yields one-half, allowing pupils to physically interact and observe the division process. The second exemplar employs pictorial representations to reinforce the notion of one-half. Teachers present images or diagrams of objects divided into two equal parts, helping pupils visually connect with the concept. The third exemplar involves sorting fractions using pictorial representations. The task involves presenting pupils with various images or diagrams of fractions and asking them to categorise them as half or not. This activity reinforces their understanding of what a half looks like from a visual perspective. The second indicator emphasises counting by halves and includes two examples. The first entails using concrete objects. Teachers present multiple concrete items, such as half oranges, guiding pupils in counting these objects in halves. To help pupils understand counting in halves through a hands-on approach, they use language such as "one-half, two halves, three halves". The second example involves counting halves using pictorial representations. Teachers show images or diagrams representing halves and ask pupils to count these using the same language, thereby reinforcing the counting concept visually. The teacher then concludes the lesson by showing the pupils how to write half the fraction symbolically on the board.

Research Questions / Hypotheses

The study was guided by the following research questions and research hypotheses:

1. What is the performance of the pupils in the identification of fractions in the pretest in the CCM?
2. What is the performance of pupils in the identification of fractions in the posttest in CCM?
3. What is pupils' performance in identification of fractions by context of groups in the posttest in CCM?
4. What is pupils' performance in identification of fractions by control and experimental groups in the posttest in CCM?

H₀1: There is no statistically significant difference between pupils' performance in the control group and those from the experimental group in the pretest in CCM.

H₀2: There is no statistically significant difference between pupils' performance in the control group and those from the experimental group in the posttest in CCM.

H₀3: There is no statistically significant difference in pretest and posttest scores for fraction identification between the control and experimental groups.

RESEARCH METHODS

Research Design and Participants

The study employed a sequential explanatory mixed-methods research design, which included a quasi-experimental pretest-posttest control group design (Creswell & Creswell, 2018). This research design was deemed appropriate for this study as it aimed to investigate the impact of the three-tier teaching model for teaching mathematics in context on the performance of primary pupils in identification of fractions. This was achieved by comparing the conventional instructional method, which involves using the NaCCA (2019) mathematics curriculum, which mirrors teachers' usual classroom practices, with the three-tier teaching model for teaching mathematics in context's approach, which links school concepts to learners' everyday experiences. Furthermore, we used the pupils' intact classes to avoid disrupting the school's classroom assignment system (Cohen et al., 2011). In this study, we randomly assigned schools to either the experimental or control groups, and each participant underwent a pretest and a posttest to evaluate their performance in identifying fractions. We chose the research participants using a multistage sampling procedure. In the first stage, we used purposive sampling techniques to select the Central Region, out of the 16 regions in Ghana, for participation in the study (Davis & Gbormittah, 2023). This selection was done because the Central Region is the educational hub of Ghana, home to numerous basic schools and the two main universities in the country. Second, the researchers used a convenient sampling technique to select the Cape Coast Metropolis (CCM) in Ghana's Central Region. The researchers live in the Central Region of Ghana; therefore, conducting the study in the region would make it easy for the researchers to access the participants while also ensuring cost and time effectiveness (Yin, 2009). Third, the researchers selected 12 schools by sampling intact classes using simple random sampling techniques.

They divided the schools into two groups: the experimental group, which included six schools teaching mathematics in context using the three-tier teaching model for teaching mathematics in context, and the control group, which comprised six schools teaching mathematics using the conventional approach, reflecting the teachers' usual classroom practices. The fourth stage involved a purposeful selection of Basic One mathematics teachers and their pupils. We chose Basic One mathematics teachers and their pupils for this study because that is the level primary school pupils are to be first introduced to fractions at in Ghanaian primary schools (NaCCA, 2019). In all, 12 teachers and 434 pupils participated in the study at CCM. Finally, six teachers were purposely selected and interviewed.

Data Collection Instruments

The instruments used for this study were Pupils' Fraction Identification Worksheet (PIFW)-Pretest, Pupils' Fraction Identification Worksheet (PIFW)-Posttest, and Semi-structured Interview Guide for Teachers (SIGT). The PIFW-pretest and PIFW-posttest were based on a literature review (TIMSS, 2003, 2007, 2011) and consisted of ten questions/items for identifying fractions. The first four questions/sub-questions of both the PIFW-pretest and PIFW-posttest were of low cognitive order (i.e., pupils were either to name, shade, or circle without doing much synthesis of ideas), while the remaining six were of high cognitive order (i.e., pupils were to apply or synthesise their concept of halves in the quest to get them right). For items 1, 3a, 3b, 3c, 3d, 3e, and 3f, pupils were asked to circle the pictures or figures that correctly represent halves. They were also required to demonstrate their understanding of halves by shading or painting half of the picture or figure in item 2. Additionally, pupils were asked to name the fractions given in items 4a and 4b symbolically. Each task was marked out of two points, except for item 2, which was worth four points. Both the pretest and posttest were marked out of a total of 22 points each. These items or questions required pupils to demonstrate their understanding of halves, thirds, and fourths. We distributed the instrument to other experts to verify its compliance with the content and face validity standards. We conducted a pilot test in Abura-Asebu/Kwamankese District, using 60 Basic One pupils who share similar characteristics with those in Cape Coast Metropolis. We excluded this school from the main study. We reconstructed items that the pupils found unclear and conducted the interview to elicit details and explanations of the broader picture that emerged in the quantitative phase of the study. Cronbach's alpha was 0.82, indicating high internal consistency and strong reliability of the instrument (Pallant, 2005). Semi-structured interview was conducted with teachers from the experimental group to assess their perceptions of the three-tier teaching model for teaching mathematics in context used in teaching fractions.

Data Collection and Analysis Procedure

The study received ethical approval from the University of Cape Coast Institutional Review Board (UCCIRB/CES/2020/74). Participant anonymity was ensured, and withdrawal from the study was permitted without penalty. Researchers obtained also consent from primary school heads, teachers, and pupils. They conducted a pretest and posttest to assess the effectiveness of the three-tier teaching model for teaching mathematics in context. They taught the control group twice using conventional instruction and the experimental group twice using the three-tier teaching model for teaching mathematics in context. We gave teachers in the experimental group a one-week workshop on using the three-tier teaching model for teaching mathematics in context. We administered an equivalent test to both groups as a posttest. We coded and analysed the collected quantitative data using frequency counts and descriptive and inferential statistics, specifically independent samples t-tests and paired samples t-tests, while we analysed the qualitative data using content analyses and narrative discussion. We used frequency counts (percentages), the mean, and the standard deviation to address research questions 1 and 2. We used content analysis to address research question 4. We used the independent samples t-test to address hypotheses 1 and 2 and the paired samples t-test to address hypothesis 3. We conducted the inferential analysis with a 0.05 error margin.

Biographical Data of the Participants

Table 1 shows the biographical data of the participants in CCM.

Table 1: Biographical Data of the Pupils who Participated in the Study

		Frequency	%
Gender	Male	209	48.2
	Female	225	51.8
	Total	434	100.0
Age	6	295	68.0
	7	75	17.3
	8	64	14.7
	Total	434	100
Groups	Control group	220	50.7
	Experimental group	214	49.3
	Total	434	100.0











The biographical data in Table 1 show that the study conducted in the CCM in the southern belt of Ghana had a nearly equal representation of males and females, with 48.2% being males and 51.8% being females. This figure indicates a relatively balanced gender distribution among the participants. Additionally, the biographical data in Table 1 suggests that the number of pupils who participated in the study from the control group (N = 220) was slightly more than those in the experimental group (N = 214) within the district. This result implies that the sample size for the study was roughly equivalent between the two groups. Furthermore, there were an almost equal number of participants from the different schools in the district. The average age of the pupils in the district is 6.5 years. The pupils have to stay over in grade 1 due to the COVID-19 pandemic.

RESULTS

RQ1: *What is the performance of the pupils in the identification of fractions in the pretest in the CCM?*

Table 2 shows the descriptive statistics of pupils' performance in *identification of fractions* in the pretest in CCM.

Table 2: Descriptive Statistics of Pupils' Performance in *Identification of Fractions* in the Pretest (N = 434)

Item	Right N (%)	Wrong N (%)	M	SD
1 	228(52.5)	206(47.5)	1.1	1.0
2 	16(3.7)	418(96.3)	0.2	0.8
3a 	228(52.5)	206(47.5)	1.1	1.0
3b 	212(48.8)	222(51.2)	1.0	1.0
3c 	0(0.0)	434(100.0)	0.0	0.0
3d 	0(0.0)	434(100.0)	0.0	0.0
3e 	219(50.5)	215(49.5)	1.0	1.0
3f 	0(0.0)	434(100.0)	0.0	0.0
4a 	84(19.4)	350(80.6)	0.4	0.8
4b 	16(3.7)	418(96.3)	0.1	0.4
Overall	23(5.3)	411(94.7)	4.2	4.1











The results presented in Table 2 indicate that the pupils in the CCM of Ghana's southern belt performed poorly in the identification of fractions during the pretest. The pupils' mean score was 4.2 out of 22.0, with a standard deviation of 4.1. This information suggests that the majority of pupils, specifically 411 (94.7%), failed the pretest, while only a small number of pupils, 23 (5.3%), managed to pass. Furthermore, the analysis of the results shows that the pupils in the CCM had a challenging time with items 2, 3d, 3e,

3f, 4a, and 4b, which asked them to use what they knew about halves in ways like shading or making halves, counting the number of shaded parts in each picture or diagram, or giving the correct fractional names for the given pictures or diagrams. These findings suggest that a solid understanding of fraction identification is crucial for correctly comprehending and applying concepts related to halves. TIMSS's reports in 2003, 2007, and 2011 confirmed these results that Ghanaian grade 8 pupils have limited content knowledge needed to apply their knowledge to solving real-life problems (Davis, 2010; Clarke & Roche, 2014). Based on these results, it can be inferred that pupils in the CCM will need to develop a strong foundation in identification of fractions to improve their performance and grasp more complex fraction-related concepts.

RQ2: What is the performance of pupils in the identification of fractions in the posttest in CCM?

Table 3 shows the descriptive statistics of pupils' performance in identification of fractions in the posttest in CCM.

Table 3: Descriptive Statistics of Pupils' Performance in Identification of Fractions in the Posttest (N = 434)











Item	Right N (%)	Wrong N (%)	M	SD
1 	100(100.0)	0(0)	2.0	0.0
2 	350(80.6)	84(19.4)	3.2	1.6
3a 	433(99.8)	1(0.2)	2.0	0.1
3b 	433(99.8)	1(0.2)	2.0	0.1
3c 	233(53.7)	201(46.3)	1.1	1.0
3d 	191(44.0)	243(56.0)	0.9	1.0
3e 	363(83.6)	71(16.4)	1.7	0.7
3f 	68(15.7)	366(84.3)	0.3	0.7
4a 	290(66.8)	144(33.2)	1.3	0.9
4b 	259(59.7)	175(40.3)	1.2	1.0
Overall	370(85.3)	64(14.7)	15.7	4.7

The numbers in Table 3 (M = 15.7, SD = 4.7) show that most of the pupils, or 370 out of the total number of participants, were able to do the parts of the posttest that didn't require them to use a lot of mental processing power. This information suggests that most of the participants were successful in answering questions that did not require them to apply their knowledge of halves to identify fractions. However, the results also indicate that there is still a group of pupils who struggle with identifying halves. In item 3e, 71 (16.4%) participants incorrectly selected the picture or diagram as one-half ($1/2$), which is incorrect. Literature (Cramer & Whitney, 2010; Davis, 2017b) has shown that pupils face challenges when dealing with mathematical tasks involving fractions that require high cognitive skills for solving them, primarily because teachers often fail to connect these concepts to learners' everyday realities or prior experiences. Pupils' inability to apply their content knowledge to solving real-life problems could cause mathematical anxiety among them (Aksoy & Yazlik, 2017; Amuah et al., 2019). This finding suggests that these individuals lacked a solid understanding of the concept of identifying halves that they needed to have when applying it to complex tasks or real-life examples. In other words, the findings suggest that while the majority of the pupils were able to handle items that did not require high cognitive skills in identifying halves, there is still a portion of pupils who faced difficulties in correctly identifying halves, as evidenced by their incorrect responses in item 3e.

RQ3: What is pupils' performance in identification of fractions by control and experimental groups in the posttest in CCM?

Table 4 shows control and experimental pupils' performance in identification of fraction in the posttest in CCM.

Table 4: Pupils Performance in Identification of Fraction in the Posttest by Context of Groups

Item	Control (N = 220)				Experimental (N = 214)			
	Right N (%)	Wrong N (%)	M	SD	Right N (%)	Wrong N (%)	M	SD
1 	220(100.0)	0(0.0)	2.0	0.0	214(100.0)	0(0.0)	2.0	0.0
2 	136(61.8)	84(38.2)	2.5	1.9	214(100.0)	0(0.0)	4.0	0.0
3a 	219(99.5)	1(0.5)	2.0	0.1	214(100.0)	0(0.0)	2.0	0.0
3b 	219(99.5)	1(0.5)	2.0	0.1	214(100.0)	0(0.0)	2.0	0.0
3c 	37(16.80)	183(83.2)	0.3	0.7	196(91.60)	18(8.4)	1.8	0.6
3d 	1(0.5)	219(99.5)	0.0	0.1	190(88.80)	24(11.2)	1.8	0.6
3e 	149(67.7)	71(32.3)	1.4	1.0	214(100.0)	0(0.0)	2.0	0.0
3f 	0(0.0)	220(100.0)	0.0	0.0	68(31.8)	146(68.2)	0.6	0.9
4a 	76(34.5)	144(65.5)	0.7	1.0	214(100.0)	0(0.0)	2.0	0.0
4b 	67(30.50)	153(69.5)	0.6	1.0	192(89.7)	22(10.3)	1.8	0.6
Overall	156(70.9)	64(29.1)	11.50	1.8	214(100.0)	0(0.0)	20.0	1.7

The results (control group: $M = 11.5$ out of 22, $SD = 1.8$; experimental group: $M = 20.0$, $SD = 1.7$) in Table 4 show the descriptive statistics of the pupils' performance in the identification of fractions in both control and experimental groups in the posttest from CCM. The results indicate that the pupils in the control group had an average score of 11.5 out of 22, with a standard deviation of 1.8. This information indicates that, on average, the pupils in the control group had a moderate level of knowledge in identifying fractions, while their counterparts in the experimental group had a significantly higher average score of 20.0 out of 22, with a slightly lower standard deviation of 1.7. This evidence suggests that the pupils in the experimental group performed well in the identification of fractions compared to the control group. The results also showed that in the control group, 64 pupils out of the 220 failed to pass the posttest in the identification of fractions. On the other hand, all pupils in the experimental group passed the posttest. This evidence indicates that the experimental group had a 100% pass rate, while the control group had a pass rate below 72.0%. The results further reveal that a majority of the pupils in the control group had difficulty applying their understanding of halves to identifying specific items (3c, 3d, 3e, 3f, 4a, and 4b) during the posttest. For example, none of the pupils in the control group correctly identified item 3f, which required identifying the total number of shaded parts as half of the total number of parts. In contrast, 68 out of the 214 pupils in the experimental group correctly solved this question using their understanding of halves. These findings suggest that the experimental group, which was taught using a special method (a three-tier teaching model for teaching fractions in context), seemed to perform better on tasks (3c, 3d, 3e, and 3f) that needed more advanced thinking skills to identify fractions compared to the control group.

H₀₁: There is no statistically significant difference between pupils' performance in the control group and those from the experimental group in the pretest in CCM.

Table 5 shows difference in pupils' performance in the control group and those from the experimental group in the pretest in CCM.

Table 5: Mean Difference between the Control and Experimental Group in the Pretest in CCM

	Levene's Test for Equality of Variances		t-test for Equality of Means			95% confidence interval of the difference	
	F	Sig.	T	df	Sig. (2-tailed)	Lower	Upper
Equal variances assumed	2.166	0.142	-0.254	432	0.8	-0.816	0.629
Equal variances not assumed			-0.254	431.973	0.8	-0.815	0.629

Table 5 presents the pretest analysis comparing the control and experimental groups prior to the implementation of the three-tier teaching model for teaching mathematics in context's pedagogy. The aim was to establish whether both groups were equivalent at baseline. Levene's test indicated no significant difference in variances ($F = 2.166$, $p = 0.142$), allowing for the assumption of equal variances in the t-test. The independent samples t-test showed no statistically significant difference between the groups ($t(432) = -0.254$, $p = 0.80$), with the confidence interval for the mean difference ranging from -0.816 to 0.629. This outcome confirms that the two groups began the study on comparable footing, with no meaningful difference in their initial performance. This similarity at the start of the study makes the results more reliable, meaning that any differences seen later in the posttest can be linked to the three-tier teaching model for teaching mathematics in context intervention instead of differences that were already there.

H₀₂: There is no statistically significant difference between pupils' performance in the control group and those from the experimental group in the posttest.

Table 6 shows difference in pupils' performance in the control group and those from the experimental group in the posttest in CCM.

Table 6: Mean Difference between the Control and Experimental Group in the Posttest in CCM

	Levene's Test for Equality of Variances		t-test for Equality of Means			95% confidence interval of the difference	
	F	Sig.	T	Df	Sig. (2-tailed)	Lower	Upper
Equal variances assumed	0.487	0.50	-50.091	432	0.0	-8.939	-8.264
Equal variances not assumed			-50.117	431.957	0.0	-8.939	-8.264

Levene's Test in Table 6 showed that the assumption of equal variances was not violated ($F = 0.487$, $p = 0.50$), which supports using the row that assumes equal variances. The t-test results revealed a statistically significant difference in mean scores ($t(432) = -50.091$, $p < 0.001$), with the experimental group performing markedly better than the control group. The confidence interval for the mean difference (-8.939 to -8.264) confirms both the magnitude and reliability of this effect. These results lead to the rejection of the null hypothesis, indicating that the three-tier teaching model for teaching mathematics in context significantly improved pupils' conceptual understanding and posttest performance compared to traditional methods.

H₀₃: There is no statistically significant difference between the control and experimental group's pretest and posttest scores in identification of fractions.

Table 7 shows difference in the control group's pretest and posttest scores in identification of fractions.

Table 7: A Paired Samples t-test for the control and experimental groups in CCM

		Paired Differences							
		Mean	SD	Std. Error Mean	95% Confidence Interval of the Difference		T	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	posttest – pretest	7.464	4.186	0.282	6.908	8.020	26.449	209	0.0
Pair 2	Posttest- pretest	15.972	4.172	0.285	15.410	16.534	56.001	213	0.0

Paired sample t-tests in Table 7 indicated significant improvements in fraction identification for both control and experimental groups ($p < .05$). The control group showed a mean gain of 7.46 points ($SD = 4.19$), while the experimental group improved by 15.97 points ($SD = 4.17$). Effect sizes were notably large, with Cohen's d of 4.19 for the control group and 7.17 for the experimental group, reflecting substantial learning gains. Although both interventions were effective, the experimental approach yielded more than twice the improvement, demonstrating its superior impact on students' understanding of fractions.

RQ4: How do teachers perceive the three-tier teaching model for teaching mathematics in context's approach in the teaching of identification of fractions in CCM? The teachers' perception about the use of the three-tier teaching model for teaching mathematics in context was sought, and they expressed their opinion about the use of the three-tier model for teaching mathematics in context the identification of fractions. They attributed the improved performance of the pupils in the treatment group primarily to the model's strength. For example, teacher one (T1) said, "Ok, it helped us because our new curriculum says our teaching should be child-centred, and this model is more child-centred than we used to teach them." This suggests that the three-tier model teaching model for teaching mathematics in context supports learner-centred pedagogy. Teacher two (T2) shared a similar thought by saying that "it makes teaching easier and interesting for the learners since they were involved and they have to work on something. It makes the lesson engaging for them" (T2). Another strength of the model mentioned by the teachers was that it offers flexibility in the use of teaching and learning materials. The teachers assert that this approach enhances pupils' comprehension and strengthens their understanding of fractions. This is an excerpt of what a teacher said: "I think the approach that we used in teaching them is different from the one we were using at first. Using the sticks to introduce the lesson to them was different from the other one, and the understanding was better than the other one." (T3). Though the teachers acknowledged the effectiveness of the three-tier model teaching model for teaching mathematics in context, they identified the difficulty in managing pupils' behaviour as a key challenge that may serve as a barrier to its use. Importantly, they highlighted the difficulty in controlling class as problematic. For example, one teacher noted the pupils' inability to regulate themselves and work with instructions, saying that, "When I gave the worksheets out to them, it was quite difficult controlling them. They won't wait for you to tell them what to do; some have even started doing the work on the worksheets. It was quite challenging for me to maintain control over that aspect" (T5).

DISCUSSION

Pretest results from pupils in the CCM of Ghana's southern belt revealed significant difficulties in identifying fractions, particularly halves, indicating limited conceptual understanding. This finding aligns with existing literature, which consistently identifies fractions as a challenging area for elementary learners (Clarke & Roche, 2014; Davis, 2017b; Aksoy & Yazlik, 2017; Elias et al., 2020; Gbormittah, 2022). Research by Davis (2010, 2016) further highlights the persistent struggle among Ghanaian primary pupils to grasp and apply fraction concepts, particularly in contexts such as measurement and operations with mixed fractions. However, posttest results showed improvement, with many pupils accurately recognizing halves in tasks requiring lower cognitive demand. This suggests that while foundational understanding was beginning to develop, pupils were not yet equipped to handle more complex applications. These

findings underscore the need for sustained, progressive practice with fractions, allowing learners to consolidate basic concepts before engaging with more advanced content (Soni & Okamoto, 2020; Venkat et al., 2021). So, building on this foundation, pupils will be able to expand their cognitive capacities and comfortably tackle more challenging concepts in mathematics in the future. Conversely, they were not able to handle those activities requiring higher cognitive skills in the posttest, as seen in their incorrect responses to items that required the use of deep structures. The performance of pupils who received instruction using the conventional approach demonstrated this more clearly. McNamara and Shaughnessy (2010) found that teachers may introduce concepts to pupils solely as an area concept, such as considering a larger shape like a circle and shading a specific portion of it, implying that the shaded area represents a fraction of the total area of the circle. Part-whole conceptions of halves heavily influenced the control group's instruction, potentially explaining the difficulties pupils encountered when using the conventional approach, a practice that is typical in most developing countries (Elias et al., 2020). For example, pupils in the control group (i.e., those taught using the NaCCA (2019) mathematics curriculum approach, which reflects teachers' usual practices in the classroom) had a moderate level of knowledge in identifying fractions. Because fractions are multifaceted, the conventional approach is insufficient for helping pupils develop a deeper understanding of fractions. Davis (2010) argued that expanding the pupils' understanding of halves beyond the area model to include length and capacity (volume) is crucial. These allow them to accommodate halves in a variety of contexts. Their counterparts in the experimental group (those taught using the three-tier teaching model for teaching mathematics in context approach for teaching mathematics in context, which connects school concepts of mathematics to learners' everyday experiences) had a significantly higher level of knowledge in the identification of fractions. With the three-tier teaching model for teaching mathematics in context approach, pupils received innovative instruction using visual aids, hands-on activities, and culturally orientated artefacts that improved their understanding of halves concepts. The potential impact of the two instructional approaches explains pupils' performance in identification of fractions (Boyce & Moss, 2017). Davis and Chaiklin (2015) recommended that teachers use a more innovative approach that integrates the cultural practices of the pupils into the school's teaching and learning of mathematics at the primary school level. In other words, the learners' social and cultural contexts should influence their mathematics learning, making the subject more relevant and meaningful to them (Vygotsky, 1978). As in the case of the three-tier teaching model for teaching mathematics in context approach, by relating fractions to real-world events, pupils have a greater understanding of the concepts and are able to apply them more effectively. The study's findings highlight that using contextualised teaching methods like three-tier teaching model for teaching mathematics in contexts that links school-based concepts of fractions to learners' everyday experiences can significantly improve students' understanding and performance in mathematics, particularly in identifying fractions (Davis, 2017b; Stevens et al., 2020). This approach aligns with Vygotsky's cultural-historical theory, which emphasizes the importance of social interaction, language, and the Zone of Proximal Development (ZPD) in learning (Vygotsky, 1978). When fraction concepts are taught through familiar real-life contexts, learners are more likely to engage in meaningful discussions, collaborate with peers, and internalise new knowledge through language. These interactions provide a strong foundation for learners to understand mathematical concepts in ways that are personally relevant and cognitively accessible. Furthermore, contextualised teaching through the use of the three-tier teaching model for teaching mathematics in context supports learners within their ZPD by bridging the gap between what they already know and what they are capable of learning with guidance. Real-world examples using cultural artifacts reduce abstraction, allowing teachers to scaffold instruction effectively and support students' progression from informal understanding to formal mathematical reasoning. Although the conventional approach had a positive effect on pupils' performance, it is also worth noting that the approach recorded a significant minority failure rate of approximately 29.10% while their counterpart taught using the three-tier teaching model for teaching mathematics in context and recorded no failure rate. This indicates that the three-tier teaching model for teaching mathematics in context had a more positive impact on the pupils' performance in identifying fractions, leading to higher outcomes compared to the conventional approach (i.e., those taught using the NaCCA (2019) mathematics curriculum approach that reflects the normal practices of teachers). Davis observed that, this lack of surprise stems

from the teachers' failure to connect the school concept of fractions with learners' everyday experiences (Davis, 2013, 2017b). The study indicates that a pedagogical approach to teaching might also have implications for how teachers assess their pupils' learning in relation to fractions (Davis & Gbormittah, 2023; Tsai & Li, 2017). Moreover, teachers' general impressions or perceptions about the use of the model highlighted the strengths of the three-tier teaching model for teaching mathematics in context over the conventional approach. They acknowledged that the three-tier teaching model for teaching mathematics in context improves the teaching and learning of fractions and recommended the model to be used by other teachers in teaching mathematics. This study also highlighted how the cultural contexts of learners facilitate their understanding of fraction identification, along with the implications for teacher pedagogy.

CONCLUSIONS

The study concluded that the pupils had low knowledge in the identification of fractions due to their low performance in the pretest. The study found that the majority of the pupils struggled with correctly identifying halves when confronted with tasks that demanded higher cognitive skills in the posttest. In the posttest, the pupils in the control group, who received instruction using the conventional method which reflects the usual practices of the teachers in the classroom, demonstrated a moderate level of knowledge in fraction identification, compared to their counterparts in the experimental group, who received guidance using the three-tier teaching model for mathematics in context approach.

The performance of pupils in both the control and experimental groups was similar during the pretest; however, there was a significant difference in the identification of fractions between the control group and the experimental group, with the experimental group performing better in the posttest. Although the conventional approach had a positive effect on the pupils' performances, it is also worth noting that it recorded a significant minority failure rate. There was a strong relationship between the three-tier teaching model for teaching mathematics in context and pupils' performance in identifying and naming fractions. This study suggests that a cultural-historical driven approach like the three-tier teaching model for teaching mathematics in context is more effective in explaining and influencing pupils' fraction identification skills. The teachers viewed the three-tier teaching model for teaching mathematics in context as an innovative approach that leverages learners' everyday experiences to scaffold the school concepts of fractions.

Recommendation for Policy and Teaching

The study found that the experimental group demonstrated significantly greater proficiency in identifying and naming fractions, highlighting the effectiveness of the three-tier teaching model for teaching mathematics in context. Although the conventional method yielded gains, a notable minority failure rate accompanied it. These findings underscore the importance of adopting innovative instructional strategies such as the three-tier teaching model for teaching mathematics in context pedagogy and call for greater attention to how fractions are taught in Basic One classroom.

Funding

This research project has no outside funding; instead, it is being driven by the researchers' passion and commitment to extend the understanding of teaching and learning fractions in context through the use of the three-tier teaching model for teaching mathematics in context's approach to improve pupils' performance in identification of fractions.

Ethical Approval

Prior to collecting data, this research study received clearance from the Institutional Review Board (IRB) of the University of Cape Coast with approval with ID: UCCIRB/CES/2020/74. All research procedures involving human participants were conducted in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Declaration of Helsinki and its later amendments or comparable ethical standards.

Informed Consent

The study adhered strictly to all ethical standards. Consent forms were distributed to parents and guardians through class teachers, and only pupils with documented consent were included. Participation

was voluntary, with confidentiality and anonymity assured, and participants were free to withdraw at any time without penalty.

Conflict of Interest There are no possible conflicts of interest.

Authors' Contributions Author 1 conceptualized the study, study design, collected and analysed the data, and drafted the manuscript. Author 2 contributed to the study design, assisted with data interpretation, and critically revised the manuscript. Both authors read and approved the final version of the manuscript.

Data Availability Statement The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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