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A Sustainable And Environmentally Conscious Inventory Model With TPD Demand For Deteriorating Pharmaceutical Products Of Fixed Lifetime

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Abstract
In this paper, an ecologically friendly inventory model was worked out for the pharmacy industry, where the management of stocks is even more difficult in the case of perishable stock and time- and price-variable demand. Normal models can typically do poorly with the dual difficulty of variable demand and decay, resulting in needlessly uncompliant stocking or stingy stockouts of stocks. The presented model brings together the time- and price-determined demand and time-varying deterioration rates and presents an empirical time and volume of orders optimization model. It can manage stocks effectively with realistic data and use superior mathematical methods to minimize wastages and meet the objectives of environmental sustainability. It defends itself by providing case studies and simulation outcomes that support its solid and robust nature, with the ability to generate savings of great magnitude. Conclusively, this research work serves a contribution to pharmacy supply chains by demanding the need to cognitively balance between economic optimality and ecological responsibilities.
Keywords: *Pharmaceutical products, Deterioration, shelf life, Time and price varying demand, and sustainability*.

1. INTRODUCTION:

The issue of inventory management is very critical in any industry. In the competitive pharmaceutical industry of today, they need inventory management to help maintain revenue and the assurance of life-saving drugs. Such an investigation presents a framework of inventory management specifically in the pharmaceutical industry, where the time-sensitive nature of the goods and unpredictable demand based on various reasons create significant problems in inventory management. Conventional inventory models are often unreliable in meeting the twofold complexity of time-varying demand and the deterioration of goods, leading to overstocking and the possibility of stockouts. This model uses a mix of dynamic demand curves and variable deterioration rates to incorporate them into an encompassing framework to handle an optimal order size and timing to maximize the efficiency of the stock management, and in compliance with the environmental sustainability goals. The model has great cost savings potential, which provides valid justifications based on simulations and case studies, and thus makes it believable to adopt into practice using actual world data and using highly sophisticated mathematical techniques. This study finally supports the fact that there is an inalienable need to strike a balance between economic effectiveness and environmental responsibility in the management of drug supply chains.

1.1. Literature Review on Diverse Inventory Models Addressing Deterioration, Sustainability, and Environmental Considerations

This literature review aims to explore the evolution and current state of inventory control models tailored to the pharmaceutical industry. It examines both traditional mathematical approaches and more recent advancements incorporating sustainability and risk management. Ghare and Schrader (1963) brought a great milestone inventory model concerning the items that have exponentially deteriorating models. This work was the first to formalize the math of item decay, thus facilitating further research. Fries (1975) further adapted this model to suit perishable products that have a limited shelf life. The researchers also suggested an ideal ordering policy, and one factor that needs to be considered is the timing as a way of reducing spoilage and related expenses. As studied by Mandal and Phaujdar (1989) examined the dependence between the consumption rate and the stocks. They presented a dependency in which demand was boosted by the greater availability of stocks in their inventory model. Ray and Chaudhuri (1997) came up with an economic order quantity (EOQ) model with stock-dependent demand and such aspects of importance as inflation and time-discounting. Non-linear stock-dependent holding costs have developed as suggested by Giri and Chaudhuri (1998), which takes into consideration cost variation with stock. Balkhi and Benkherouf (2004) augmented the classical inventory models by adding time-varying demand rates and which helps to support the realism of inventory systems with time-varying demand rates. Their practice is subject to dynamic analysis of the level of the inventory, which makes it more precise in planning in non-stationary consumption environments. Deng et al. (2007) narrowed this down to ramp-type demand, which incorporates the surge in demand at a given time of the year. He et al. (2010) came up with a production-inventory model that was aimed at addressing multiple-market demands. Their model handled the complexity of the management of inventories of perishable items sold in diverse markets. Sarkar et al. (2012) formulated a time-optimal policy on the inventory of deteriorating materials containing a time-square demand and time-related partial backlogging. This model was useful since it discussed how one can adopt the replenishment strategies throughout the life of the item, and it considers changing demand and partial backorders. Venkateswarlu and Mohan (2013) analysed the model of time-varying deterioration inventory problem with the price-dependent quadratic demand and salvage value of the inventory it is important to understand that both price and time should also be incorporated in the demand forecasting and the inventory planning. Sarkar and Sarkar (2013) developed an inventory model with complete backlogging, time-varying deterioration, and stock-dependent demand. They worked on the interactive nature of inventory level and customer demand, where the focus was on the effects of changes in stock levels on customer demands in the future. Yadav and Devi (2013) introduced an inventory model where they included the aspect of flexibility in the volume held, random deterioration, and an exponentially increasing demand rate. The model hopes to reduce the incurred total inventory cost, considering the uncertainties in the deterioration and responsiveness to the changing market demand. The analysis helps develop better responsive and flexible inventory management techniques in uncertain and rapidly changing conditions. Chen et al. (2013) discussed an approach to carbon-constrained Economic Order Quantity (EOQ), providing an example of how carbon emissions and environment-related issues can become a part of the production and inventory decision-making process. Bozorgi et al. (2014)came up with a model of inventory regarding cold items, where the emissions and the cost related to emissions are taken into consideration, which highlights the need to ensure the efficiency of the operation and sustainability of the environment. Ahmed et al. (2017) investigated the effects of carbon tax and the uncertainty on economic policies within the second-generation biofuels supply chains. In their work, they discussed the possibility that inventory and production strategy to be affected by environmental cost, which is the cost of carbon emission. Tripathi et al. (2017) have examined the time-dependent variable deterioration process, as well as the cost of production, to make coordinated decisions in product replenishment and production scheduling of a deteriorating inventory. Malik et al. (2018) generalized the inventory management analysis to support a time-varying demand of non-immediate degrading items. Their analysis took into consideration the lifetime limit of items used, whereby their items had usability limits before becoming unusable. This model took into consideration the changing nature of demand with time, the impact that time-dependent change has on the global inventory policy. Canyakmaz et al. (2019) examined the uncertainty of stochastic price-dependent demand in the inventory model and presented a new study that addresses the dynamic interaction between price variations and the control of inventory levels. Chowdhury and Ghosh (2022) have considered a production-inventory model of perishable items characterized by the demand-dependent rate of production, abandonment, and holding costs dependent on the anticipated shortage. This model gave a flexible approach to managing variable demand and production limitations. Kumar et al. (2023) proposed their fuzzy inventory model that takes into account the deteriorating items but has a regular seasonal pattern of demand. The uncertainty is taken into account in the model with regard to inventory parameters employing fuzzy set theory, and the optimality criterion is the overall cost. Numerical examples confirm the usefulness of the model in operating perishable items that are subject to cyclic demand in stochastic settings. Kumar et al. (2024) suggested a fuzzy inventory model that includes demands that are time dependent (TPD) in addition to inflation and the prices of carbon emissions, and the partial backordering. The model equally combines both economic and environmental considerations with consideration of parameter uncertainty, applying the fuzzy set theory. It shows how the sustainable inventory decisions can be enhanced in natural conditions of financial and environmental constraints.

1.2. Pharmaceutical Inventory Models

Unlike general consumer goods, pharmaceutical items are often perishable, regulated, and subject to unpredictable demand fluctuations. Moreover, the risks associated with stockouts or overstocking in this sector can directly impact public health outcomes and operational costs. Recognizing these challenges, a wide array of inventory control models has been developed to address specific issues in pharmaceutical logistics, including product expiry, demand variability, regulatory constraints, and the need for cold chain storage. Uthayakumar and Priyan (2013) gave a larger picture of the supply chain of the pharmaceutical industry and how to manage the inventory practice, one that provided the optimization methods that may be employed in the pharmaceutical industry, and the medical practice to optimize their performance. Jaberidoost et al. (2013) evaluated that in order to overcome the pharmaceutical inventory management problems effectively, it is essential to take a comprehensive approach. Such a plan should combine innovative predicting techniques, dynamic tracking of stocks, and cooperation with vendors and distributors. Also, one should keep in mind the existence of threats to the pharmaceutical supply chains that are able to interfere with the provision of medicines, including their quantity, quality, and timely supply. Sana et al. (2015) examined the areas of optimal replenishment approaches to pharmaceutical goods and the effect that sales plans and team approaches could have on the level of inventory. Operation and marketing factors led to the addition of a very unique perspective in this research. In the work by Uthayakumar and Karuppasamy (2016), the authors developed an original model that has been applied to the healthcare industry, where quadratic demand and linear holding costs were discussed with the consequences of shortages. This was very instrumental in the determination of the inventory strategies in the health care sector, where the demand and supply could differ vastly. Uthayakumar and Karuppasamy (2017a) prolonged their research to variable degrading pharmaceutical products, where the demand was time dependent, and holding costs in a trade credit environment. Of importance to note about this model is that it contained the issue of trade credit, which influences inventory management decisions. Uthayakumar and Karuppasamy (2017b) investigated the pharmaceutical inventory under stochastic demand, holding costs, and partial backlogging, where, in particular, the permitted delay in payments was active. The design of this model was targeted to maximize the management of inventory and encompass the production of financial flexibility in payment. The pharmaceutical industry came to mind because the mechanism in charge of inventory has to contend with the potential risk of adverse obsolescence and spoilage, and pharmaceutical goods are not infinitely shelf-stable as well as prone to degrade when kept in the wrong environment (Bucalo & Jereb, 2017). Such limitations require a more accurate and sensitive management of inventories to guarantee the safety of products, and it should reduce losses. Uthayakumar and Tharani (2018) have investigated the inventory model of deteriorating drugs with the complete backlogging system. This applies to those industries where stock availability is very important, even when there are challenges like the deterioration of stock. Items based on a periodic demand pattern by season. Uncertainty in the parameters involved in inventory is included by application of fuzzy set theory in the model, and the model aims at minimizing the total cost. Examples of numerical experiments confirm the viability of the model to deal with perishable products and products that have cyclic demands in uncertain conditions. As noted by Maihami and Ghalehkhondabi (2019), an effective management of pharmaceutical inventories appears to be a daunting challenge, and a comprehensive strategy is required, which should take into consideration the sensitive characteristics of the industry. The model developed by Balugani et al. (2019) is quite different because of the numerous essential items that distinguish traditional models in the inventories of pharmaceuticals: the perishability of the substantive that a pharmaceutical possesses, a strict set of regulatory requirements that it has to meet in the context of storage and handling, and the overarching ethical responsibility of adhering to safety and efficacy of medicine as well as the concern with patient safety. Rastogi and Singh (2019) concentrated on the management of pharmaceutical inventory with the accent on price-sensitive demands in addition to the partial backlogging with the impact of learning effects. It was an important study because it involved human and operational learning, which allowed the developmentof better policies regarding replacement and the management of stocks in the course of time. Ellison and Cook (2020) examined that in most supply chain decisions, and particularly in the humanitarian context, there is a limited amount of analytics to assist those decisions, as such decisions are large, complex, and often lacking in reliable data. Kumar et al. (2021) discussed that the ethical aspect of pharmaceutical inventory management cannot be overvalued because patient well-being and health outcomes depend on the availability of essential medicines to the maximum. Rathipriya et al. (2022) attempted to conceptualize the complex interaction of price characteristics in which the effect of market forces, patent expirations, and the entry of generic substitutes trigger the occurrence of prices. The pharmaceutical supply chains are complex, in which a sequence of activities is undertaken with sometimes interlinked, and the complexities just increased to the many challenges being experienced by the pharmaceutical firms throughout the world. The focus of Hansen et al. (2023) concerned conventional inventory management practices that tend to be insufficient in terms of managing the complexity of a pharmaceutical supply chain, which is marked by long lead time, complex distribution channels, and the possibility of interruption through a change in regulation or unanticipated events.In this study, a mathematical representation is created to study an inventory system regarding pharma products that are deteriorating with a state of carbon emission costs. The model consists of a time-dependent deterioration rate and the demand, which varies with time and decreases exponentially with price in a linear manner. The intention is to get expressions of important inventory parameters, which include costs in the environment.

2. Assumptions and notations:

2.1. Notations

A = Ordering Cost per order

C_H = Annual Holding Cost per year

C_p = Purchase price per unit

M = Maximum lifetime of a product (Expiry Date of Pharmaceutical products)

$\theta(t)$ = Deterioration rate

T = Total period (cycle length)

Q = Maximum inventory level

μ =Fixed parameter of demand

ν = parameter which varies with time

ω = parameter that varies with price

ρ = emissions per deteriorated item (kg/unit)

C_D= Deterioration cost

C_T= Carbon charge ($/kg)

A_0= Fixed ordering cost for carbon emission

H_0= Fixed Holding cost for carbon emission

2.2. **Assumptions:**

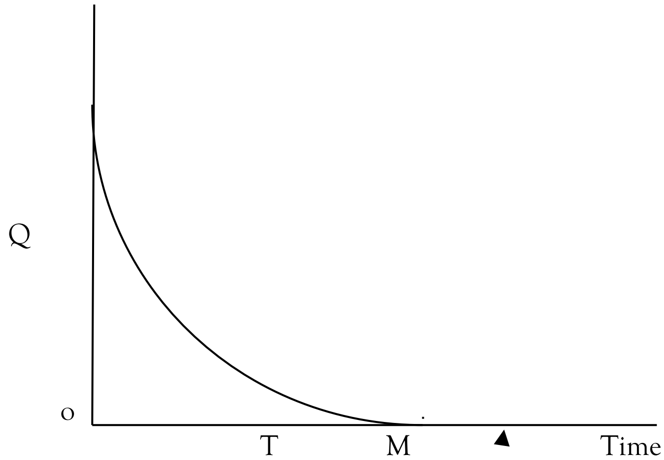
- The demand $D(p,t)=\left(\mu+\nu t\right)e^{-\omega p}$ is linearly dependent on time and exponentially declining with price. Where μ is a fixed parameter of demand, ν is a parameter that varies with time, and ω is a parameter that varies with price.

- Holding cost is constant.
- The deterioration rate is time-dependent as $\theta(t) = \frac{1}{1+M-t}$ where M > t and M is the maximum lifetime of products at which the total on-hand inventory deteriorates, when t increases, $\theta(t)$ increases and $\lim_{t \to M} \theta(t) \to 1$.
- Instantaneous replenishment rate.
- Zero lead time.
- Shortages are not allowed.
- Time horizon is finite.

3. Mathematical Formulation

An ordinary differential equation (ODE) defines the inventory level and takes into consideration the time-variant deterioration and demand. Assume I (t) is the inventory at any time t in the interval [0, T], T being the periodicity. Both demand and deterioration influence change in the inventory over a period of time. With the instantaneous replenishment, zero lead time, there are no shortages, and including the fact that there is only one warehouse, the Inventory management system may be written as the following differential equation:

$$\frac{dI(t)}{dt} + \theta(t) I(t) = -(\mu + \nu t) e^{-\omega p}, 0 \le t \le T \quad (1)$$



Inventory Level at any time t

The solution to equation (1) is

$$I(t) = e^{-\omega p}(1+M-t)\left[(\mu+\nu+\nu M)\log\left(\frac{1+M-t}{1+M-T}\right) + \nu(t-T)\right] \quad (2)$$

The Pharmaceutical Holding cost in the interval [0,T] is denoted by HC and can be written as

$$HC = \int_0^T C_H . I(t) dt$$

$$HC = C_H e^{-\omega p}\left[\begin{array}{l}\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)\left\{\left(1+M\right)T-\frac{T^2}{2}\right\}+\frac{(\mu+\nu+\nu M)(1+M-T)}{2}\\ \left\{\frac{T^2}{2}-\left(1+M\right)T-\left(1+M\right)^2\log\left(\frac{1+M-T}{1+M}\right)\right\}+\nu\left\{\frac{T^3}{6}-\left(1+M\right)\frac{T^2}{2}\right\}\end{array}\right] \quad (3)$$

Deterioration cost for the cycle [0, T] is

$$DC = \int_0^T C_D I(t)\theta(t)dt$$

$$DC = C_D e^{-\omega p}\left[\begin{array}{l}T\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)-\left(\mu+\nu+\nu M\right)\\ (1+M-T)\left\{T+\left(1+M\right)\log\left(\frac{1+M-T}{1+M}\right)\right\}-\frac{\nu T^2}{2}\end{array}\right] \quad (4)$$

The initial order quantity is obtained from equation (2) by putting t=0 in the case 0 ≤ t ≤ T; thus, the initial order quantity is

$$Q = -e^{-\omega p}\left(1+M\right)\left[\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)+\nu T\right] \quad (5)$$

Purchase Cost for the inventory Q is

$$PC = C_p.Q$$

$$PC = -C_p e^{-\omega p}(1+M)\left[\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)+\nu T\right] \quad (6)$$

Total Sales Revenue for inventory Q is

$$SR = p\int_0^T D(p,t)dt$$

$$SR = p\int_0^T \left(\mu+\nu t\right)e^{-\omega p}dt$$

$$SR = pe^{-\omega p}\left(\mu T+\frac{\nu T^2}{2}\right) \quad (7)$$

Total profit per unit time

Tp = $\frac{1}{T}$ [Sales revenue - Ordering Cost - Holding Cost - Deterioration Cost- Purchase Cost]

$$Tp = \frac{1}{T}\left[SR - A - HC - DC - PC\right]$$

$$Tp=\frac{1}{T}\Bigg[pe^{-\omega p}\left(\mu T+\frac{\nu T^2}{2}\right)-A-C_He^{-\omega p}\Bigg(\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)\left\{\left(1+M\right)T-\frac{T^2}{2}\right\}$$
$$+\frac{\left(\mu+\nu+\nu M\right)\left(1+M-T\right)}{2}\left\{\frac{T^2}{2}-\left(1+M\right)T-\left(1+M\right)^2\log\left(\frac{1+M-T}{1+M}\right)\right\}+$$
$$\nu\left\{\frac{T^3}{6}-\left(1+M\right)\frac{T^2}{2}\right\}\Bigg)-C_De^{-\omega p}\Bigg(T\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)-\left(\mu+\nu+\nu M\right)$$
$$\left(1+M-T\right)\left\{T+\left(1+M\right)\log\left(\frac{1+M-T}{1+M}\right)-\frac{\nu T^2}{2}\right\}\Bigg) \tag{8}$$
$$+C_pe^{-\omega p}\left(1+M\right)\left[\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)+\nu T\right]\Bigg]$$

Total Carbon Emission Cost under tax = $C_T * C_E$

$$=C_T*(A_0+H_0*AI+\rho*DI)$$

$$C_T*C_E=C_T*\left[A_0+H_0*\mathrm{e}^{-\omega p}\left[\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)\left\{\left(1+M\right)T-\frac{T^2}{2}\right\}\right]+\rho*DI\right] \tag{9}$$

Now Total profit under carbon emission

$$Tp=\frac{1}{T}\Bigg[pe^{-\omega p}\left(\mu T+\frac{\nu T^2}{2}\right)-A-C_He^{-\omega p}\Bigg(\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)$$
$$\left\{\left(1+M\right)T-\frac{T^2}{2}\right\}+\frac{\left(\mu+\nu+\nu M\right)\left(1+M-T\right)}{2}\left\{\frac{T^2}{2}-\left(1+M\right)T-\left(1+M\right)^2\log\left(\frac{1+M-T}{1+M}\right)\right\}$$
$$+\nu\left\{\frac{T^3}{6}-\left(1+M\right)\frac{T^2}{2}\right\}\Bigg)-C_De^{-\omega p}\Bigg(T\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)-\left(\mu+\nu+\nu M\right)$$
$$\left(1+M-T\right)\left\{T+\left(1+M\right)\log\left(\frac{1+M-T}{1+M}\right)-\frac{\nu T^2}{2}\right\}\Bigg)+C_pe^{-\omega p}\left(1+M\right)\Big(\left(\mu+\nu+\nu M\right)$$
$$\log\left(\frac{1+M-T}{1+M}\right)+\nu T\Big)-C_T\Bigg(A_0+H_0e^{-\omega p}\Bigg(\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)\left\{\left(1+M\right)T-\frac{T^2}{2}\right\}$$
$$+\frac{\left(\mu+\nu+\nu M\right)\left(1+M-T\right)}{2}\left\{\frac{T^2}{2}-\left(1+M\right)T-\left(1+M\right)^2\log\left(\frac{1+M-T}{1+M}\right)\right\}$$
$$+\nu\left\{\frac{T^3}{6}-\left(1+M\right)\frac{T^2}{2}\right\}\Bigg)+\rho\Bigg(e^{-\omega p}\Bigg(T\left(\mu+\nu+\nu M\right)\log\left(\frac{1+M-T}{1+M}\right)-$$
$$\left(\mu+\nu+\nu M\right)\left(1+M-T\right)\left(T+\left(1+M\right)\log\left(\frac{1+M-T}{1+M}\right)-\frac{\nu T^2}{2}\right)\Bigg)\Bigg)\Bigg)\Bigg] \tag{10}$$

4. Solution Methodology

Tp is the function of p and T, so any value of Tp satisfies the necessary conditions for the total profit per unit time (10) to be maximized.

The necessary and sufficient conditions for the total profit Tp to be maximum are

$\frac{\partial Tp(p,T)}{\partial T}=0$ and $\frac{\partial Tp(p,T)}{\partial p}=0$

And $$\frac{\partial^2 Tp(p,T)}{\partial T^2}<0,\ \frac{\partial^2 Tp(p,T)}{\partial p^2}<0$$

And $$\left(\frac{\partial^2 Tp(p,T)}{\partial T^2}\right)\left(\frac{\partial^2 Tp(p,T)}{\partial p^2}\right)-\left(\frac{\partial^2 Tp(p,T)}{\partial p\partial T}\right)^2<0$$

4.1. Numerical Example

A pharmaceutical company seeks to develop a sustainable inventory strategy for managing a drug with a limited shelf life and associated environmental impact. The objective is to determine the optimal order quantity and cycle length that minimize the total cost, considering both economic and environmental factors. The ordering cost (A) is \$60 per order, the annual holding(C_H) cost is \$2 per unit per unit time, and the purchase price(C_p) is \$40 per unit. The drug has a maximum usable life of one year, with a deterioration rate (θ) to be time. where μ = 100, v = 5, ω = 0.02, and p is the selling price. Additional cost components include a deterioration cost (C_D) of \$0.5 per unit, carbon emissions (ρ) of 0.02 kg per deteriorated unit, a carbon tax (C_T) of \$2 per kg, and fixed carbon-related ordering (A_0) and holding costs (H_0) of \$5 and \$2, respectively. The task is to compute the total cost by combining all relevant components—ordering, holding, purchasing, deterioration, and environmental—and determine the optimal values of order quantity (Q) and cycle length (T) that maximize this total profit. Sensitivity analysis to the carbon tax and deterioration rate should also be conducted to understand their impact on the overall sustainability of the inventory policy.

The solution to this example is obtained with the help of Wolfram Mathematica 13.2. The optimal value of selling price (*p**), cycle length (**T***), and total profit (**Tp***) are **97.0043**, **0.633359,** and **619.313,** respectively.

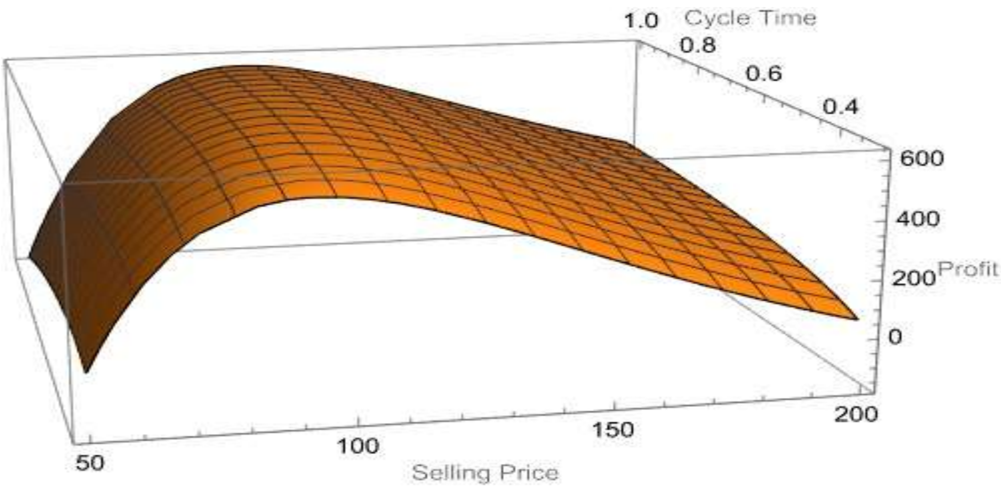


Figure 1: Graphical representation of Profit

4.2. Sensitivity Analysis

The sensitivity analysis using one parameter while keeping all the other parameters reiterate was carried out to determine the robustness of the proposed model. Individual adjustments of all the selected parameters by 10 percent and 20 percent up and down were made to determine how changes in a parameter affect the results of the model. Such a one-at-a-time (OAT) method can assist in presenting the parameters that exert the most important impact on system performance, hence providing guidelines on the stability and reliability of its model in different situations.

Table: Sensitivity Analysis

Changing Parameter	Variation percentage	Variation Value	P	T	Tp
A	-20%	48	96.3739	0.584685	639.01
	-10%	54	96.6966	0.609817	628.965
	0%	60	97.0043	0.633359	619.313
	+10%	66	97.2989	0.655515	610.003
	+20%	72	97.5819	0.676447	600.994
μ	-20%	80	97.873	0.701377	477.444
	-10%	90	97.4027	0.664883	548.046
	0%	100	97.0043	0.633359	619.313
	+10%	110	96.6615	0.605801	691.147
	+20%	120	96.3628	0.581462	763.475
ν	-20%	4	96.9699	0.627835	616.485
	-10%	4.5	96.987	0.63059	617.896
	0%	5	97.0043	0.633359	619.313
	+10%	5.5	97.0217	0.636142	620.734
	+20%	6	97.0393	0.63894	622.161
ω	-20%	0.016	108.968	0.592055	991.133
	-10%	0.018	102.279	0.611862	780.501
	0%	0.020	97.0043	0.633359	619.313
	+10%	0.022	92.7653	0.656402	493.594
	+20%	0.024	89.3097	0.68092	394.063
C_H	-20%	1.6	97.0175	0.628734	618.223
	-10%	1.8	97.0109	0.631028	618.776
	0%	2	97.0043	0.633359	619.313
	+10%	2.2	96.9976	0.635729	619.864

	+20%	2.4	96.9911	0.638139	620.419
C_p	-20%	32	87.7365	0.669568	774.701
	-10%	36	92.3563	0.647747	693.149
	0%	40	97.0043	0.633359	619.313
	+10%	44	101.684	0.624257	552.342
	+20%	48	106.4	0.619132	491.534
C_D	-20%	0.40	97.0056	0.632488	619.126
	-10%	0.45	97.0049	0.632923	619.219
	0%	0.50	97.0043	0.633359	619.313
	+10%	0.55	97.0036	0.633797	619.407
	+20%	0.60	97.0003	0.634236	619.5
C_T	-20%	1.6	96.9278	0.617342	620.514
	-10%	1.8	96.9664	0.625279	619.891
	0%	2.0	97.0043	0.633359	619.313
	+10%	2.2	97.0413	0.641591	618.78
	+20%	2.4	97.0775	0.649984	618.292

4.3. Observations of Sensitivity Analysis: Observation parameter-wise of the sensitivity analysis is below

1. *Ordering Cost (A)*: The total cost has a declining trend as the ordering cost increases. As the ordering cost moves in the opposite direction and rises by 20 percent (48 to 72), the total cost drops by 10 percent (639.01 to 600.99). This opposite-to-common-sense outcome implies that when the ordering cost becomes higher, fewer, yet larger orders will be given and the length of the order cycle (P) will increase, hence, less often the ordering costs will be incurred and the total cost will be lower.
2. *Demand Parameter* (***μ***): There exists a close positive correlation that exists between demand parameter (***μ***) and the total cost. The total cost increases up to 3 times as the value of ***μ*** (80 to 120) increases (from 477.44 to 763.47). This is in measure to the sensitivity of the inventory system to the change in demand. The increased demand will result in an increased number of replenishments, and this means more procurement, holding, and operational costs.
3. *Time Varying Demand Parameter (v)*: The change in the time-varying parameter v that is used to play within the demand does not show a significant impact on the total cost. The overall cost does not vary appreciably from 616.49 to 622.16, within the range of the tested cohorts, indicating that the system is fairly resilient in the face of temporal variation of demand when this parameter is in play.
4. *Seasonality Parameter (w)*: As the level of w increases, the value of total cost reduces significantly. In particular, the T decreases when 0.016 to 0.024, going down, from 991.13 to 394.06. This means that introducing seasonal adjustments in demand should contribute to the synchronization of the

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inventories at a lower cost. Therefore, omega is very crucial in the course of cost optimization of time-variant demand models.

5. *Holding Cost (CH)*: The annual change in holding cost only induces a relatively small change in the total cost by altering it to 620.42 when the holding cost changes by 0.8 (1.6 to 2.4). It means that the model is quite stable in its response to spikes in CH in the range under examination, which is a sign of stability in the system of managing storage and the costs related to it.
6. *Purchase Price (Cp)*: The interesting feature to note is that the purchase price and the total cost are inverted. When Cp rises (32 to 48), the cost of the total drops (774.70 to 491.53). This may be because of the buying in bulk advantages or vendor-related incentives, which equal the unit cost, portraying positive cost behavior in the cases of increasing prices of the purchase.
7. *Deterioration Cost (CD)*: Changes in the deterioration cost (CD) indicate an insignificant difference in the overall cost. The values of T do not change drastically (almost staying at 2934.2), which suggests that the targeted parameter range is sufficient to cover the deterioration cost in the system and does not have a noticeable influence on the cost economy.
8. *Carbon Tax (CT)*: A slight U-shaped behavior is noted with a change in CT. The lowest cost is where CT is close to the value of 2.0. Minimal changes in CT cause a marginal increment in the total cost. This implies that the impact is small, showing that emissions (though important environmentally) are not the dominant cost driver within the current scale of CT.

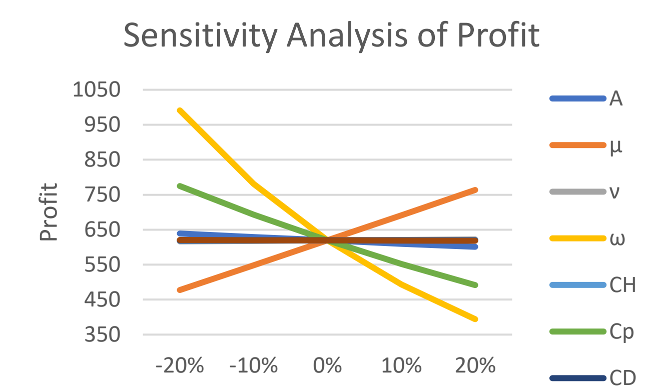

Figure 2. Graphical Representation of Profit Analysis

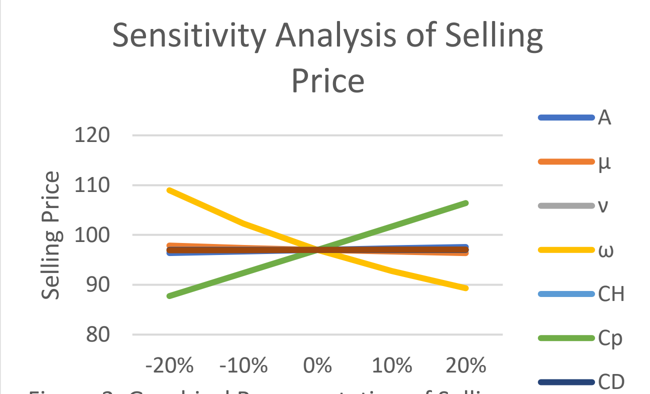

Figure 3. Graphical Representation of Selling Price Analysis

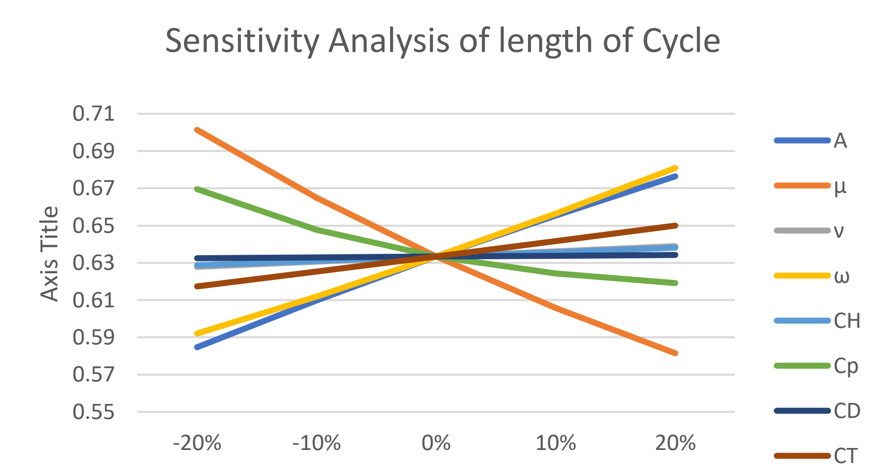

Figure 4. Graphical Representation of Selling Price Analysis

5. Managerial Insights

Apart from the conclusions made by this study, there are certain strategic insights with regard to pharmaceutical inventory management. On the one hand, boosting the ordering cost can also reduce the overall costs because of fewer but bigger orders, which is why managers are supposed to streamline the frequency of the orders and discuss the concept of bulk purchases. Demand intensity (mu) was found to be another large cost driver, and the need to have good forecasts and dynamic demand adjustment mechanisms was emphasized in order to avoid stockouts on the one hand and overstocking of goods on the other hand. The seasonal demand variations which are embodied in the parameter (omega) are also of substantive importance to cost savings, implying that it would be more efficient than otherwise, to ensure that the timing of procurement is pegged in line with seasonality demand. Interestingly, a higher price of purchase price (Cp) can lower total costs because of the incentives provided by the suppliers, or economies of scale, and that is why it is wise to negotiate a long-term and volume-based contract. The impact of Carbon Tax (CT) is minimal, slight change in this cost create a marginal increment in total cost. Moreover, the model has a high tolerance to alterations of small parameters of the cost, such as holding cost (CH), time-based demand (v), and deterioration cost (CD), and managers can prioritize other features of the cost. Finally, the approach of the model to environmentally friendly frames helps to contribute sustainability to the competitive advantage; pharmaceutical companies can not only cut down the costs of their operations but also enhance both environmentally and socially.

6. CONCLUSION

The paper has made a conceptual proposal of a new and greener inventory system under specific conditions of the pharmaceutical sector. The model thus considers the perishability to be coupled with a two-fold problem of dynamic demand behavior (both time and price-based demands) and time-varying deterioration. Differently with conventional methods, the model handles in a proper manner the problems associated with the management of inventory found in the real world as it optimizes the order quantity, and this is done under varying conditions and cycle times. Backed by empirical data and sensitivity analysis, the model showed high adaptability and robustness, further providing a balanced answer with the minimization of total costs that comply with the sustainability goals. Simulation results confirm the effectiveness of the model in terms of minimizing wastage, eliminating stockouts, and encouraging more sustainable inventory behavior. This has mainly helped fuel the increased need for sustainable logistics in healthcare supply chains.

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