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A Mathematical Study of the Impact of Geometric Location on Climate Change in India Using the Finite Element Method

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Abstract
India's climate varies significantly across its vast geographical expanse, owing to its diverse topography, ranging from the towering Himalayan Mountains in the north to the coastal plains in the south. The mountains, plateaus, deserts and coastlines winds all have an impact on India's climate. Numerous adverse impacts on agriculture, water resources, forests and biodiversity, health, coastal management and temperature are linked to climate change. Our main objective is to investigate how temperature variations in the surrounding area effect climate change. The temperature variations in different parts of India affect the air flow's intensity. Heat transfer happens between the land, sea and atmosphere because the country's environment frequently fluctuates between extremely scorching summers, frigid winters and unpredictable monsoons. In this paper, the Finite Element Method is used to study the effect of Himalayan and costal region on climate change of India. Our main focus is to study the climate change occurs due to the temperature difference at the surrounded region. Intensity of flow of the air varies due to the changes in temperature at various location in India.
Keywords: *Climate, Season, Weather, Dirichlet boundary Conditions, Neumann boundary Condition, Finite Element Method.*

INTRODUCTION

India's climate is greatly influenced by its topography, since geographical features such as mountains, plateaus, deserts and coastlines affect the country's temperatures, rainfall and wind patterns. India's topography is highly diverse and can be broadly divided into five major regions likes the Himalayan Range, the Northern Plains, the Deccan Plateau, the Indian Desert Region and the Coastal Plains (Western Ghats and Eastern Ghats). The country's physical, climatic and cultural features are influenced differently by each of these regions. Mathematical modelling to acquire a better understanding of heat transfer in the climate of India. We use the Finite Element Method (FEM), an effective mathematical technique that is capable of resolving challenging physical problems. We develop a model to investigate steady-state heat distribution using the Poisson equation. In order to examine how heat changes over time, we also construct time-dependent models. When together, these methods help in the explanation of the dynamic processes which impact India's climate and offer practical advice for important sectors like agriculture and disaster management. Climate modelling is a significant difficulty in India due to its huge and diverse topography, which is formed by the massive Himalayas, vast oceans, and numerous terrains. We must create tools for properly understanding how heat transfer happens between the land, sea and atmosphere because the country's environment frequently fluctuates between extremely scorching summers, frigid winters and unpredictable monsoons. A novel approach to thermal modelling has been created to address this. This approach examines the climate in detail using various boundary conditions. It performs especially well in places where a lot of variables come together, including the coastal regions where land and sea combine to produce complex heat patterns and the snow-covered Himalayas, which have a significant impact on heat distribution.

LITERATURE REVIEW

A literature review offers a new way to solve the selected problem. Sangam, Shivappa, and Savitha (2019) represented a methodological approach focused on climate change and global warming studies as a Scientometric approach during 2001-2016.[1] Issac Newton and Gomathi (2018) focused on the scientometric analysis of global warming from 2008-2010.[2] Lukwale, Sophia, and Sife, Alfred (2017) assessed research trends on climate change literature in Tanzania between 2006 and 2016.[3] The study analyzed the growth, collaboration between authors, and distribution of subject categories on climate change. Haunschild et al. (2016) have studied climate change research publications; they aim to display relevant literature in bibliometrics and present several quantitative data.[4] Ali, Hydar, and Adithyakumari (2015) analyzed the results of the biodiversity field conducted from 2003 to 2012 with various parameters, including citation impact, international collaboration, and subject field contributions.[5] Saravanan et al. (2014) analyzed climate change research in five scientific networks in developing countries like Argentina, Brazil, China, India, and Mexico.[6] Venkatesan et al. (2013) analyzed climate change research using the Web of Science database between 1999 and 2012. The publications based on year-wise growth, country, language, and types of documents.[7] Nabout, Joao, et al. (2012) conducted a scient metric analysis of the global climate change (GCC) literature to identify the patterns, trends, and biases in this research field.[8] Jinfeng Li et al. (2011) evaluated the global scientific results of climate change research for the past 18 years. They characterized the research methods and research trends. Alex and PreedipBalaji (2010) analyzed the results of climate change in India for 2005 - 2009 based on the ISI Science Citation Index. The research emphasizes the growth in the field of climate change for five years.[9] Stanhill, Gerald (2001) analyzed the climate change science as a scientometric study. The annual summaries of scientific publications on climate change, published in the Bulletin of the "American Meteorological Society" were an indicator of its growth.[10].

Mathematical Model

For the simplicity, we use unsteady heat equation for finding the change of temperature at a point due to change in surrounding temperature $\frac{\partial T}{\partial t} - c^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = f(x, y)$. Boundary of the domain is non-local. Let Ω be a bounded domain in R^2 with piecewise smooth boundary Γ. We consider a linear self-adjoint elliptic equation of second order in Ω. We consider the Poisson equation as a steady state heat equation $-c^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = f(x, y)$. If we prescribe on Γ_0, a part of Γ, the following boundary condition holds $T = c$ on Γ_0 and $\int_{\Gamma_0} \frac{\partial T}{\partial \eta} ds = A_0$. η is the unit outward normal vector on Γ_0. Define non-local boundary condition of Dirichlet type on $Q = \Omega \cdot (0, T) \rightarrow q = \Gamma_0 \cdot (0, T)$.

Here we take two types of boundary conditions namely Dirichlet boundary condition and Neumann boundary condition. In Dirichlet boundary conditions, the value of the solution is specified directly at the boundary of the computational domain. Dirichlet boundary conditions often correspond to situations where the value of the solution is known or fixed at the boundary, such as fixing the temperature at a boundary or specifying a fixed concentration of a substance. In Neumann boundary conditions, the derivative of the solution normal to the boundary is specified. Neumann boundary conditions often correspond to situations where the heat flux of a quantity across the boundary is known or specified, rather than the value of the quantity itself.

Problem Formulation

Here, a simple case of unsteady state heat flow problem in a India's domain shown in Figure 1.

Figure 1: Structure of an Indian Map

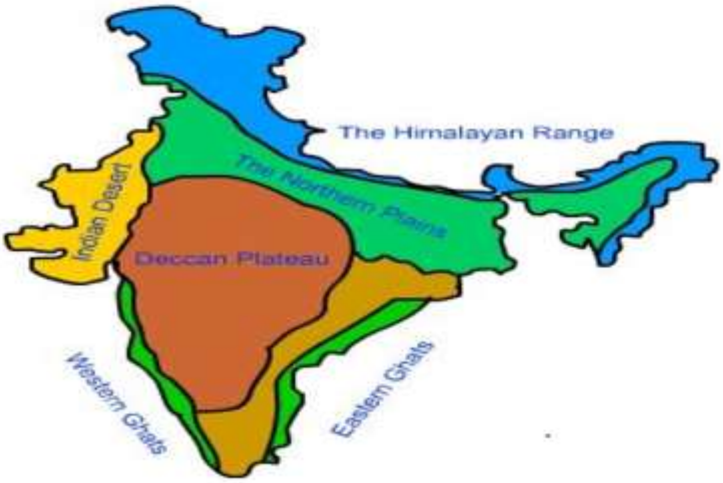

Figure 2: India's Topography Graph

The Himalayan Range (The Northern Mountains)

In the north, the Himalayas comprise a powerful mountain range that crosses states like Jammu & Kashmir, Himachal Pradesh, Uttarakhand, Sikkim and Arunachal Pradesh. The Greater Himalayas (Himadri), which include some of the highest peaks, the Lesser Himalayas (Himachal), which have valleys and hill stations and the Shivaliks, which are foothill ranges are divided into three parallel groups. They affect rainfall patterns.

During winter, cold and dense Siberian winds tend to flow southward because of the pressure gradient force

$$F_p = -\frac{1}{\rho}\nabla p. \tag{1}$$

If the Himalayas did not exist, these katabatic winds (u) would easily penetrate into the Indo–Gangetic Plains, causing a drastic fall in temperature. However, the Himalayas act as a solid boundary condition in the Navier–Stokes framework, enforcing $u_n = 0$. (no normal velocity through the mountain wall).
This effectively blocks cold advection, allowing the plains and even the Deccan Plateau to retain radiative heat, which results in warmer winters.

The situation changes in the summer when the Himalayas are hit by wet monsoon winds ($q \uparrow$) from the Indian Ocean.

The continuity equation $\nabla \cdot \vec{u} = 0$ states that upward velocity $(w > 0)$ is the result of horizontal wind convergence.
At the moist adiabatic lapse rate $\frac{dT}{dz} \approx -\Gamma_m$, the air cools as it rises.

Condensation happens when actual humidity surpasses saturation level $q > q_s(T)$ (actual humidity $>$ saturation). Latent heat is released during this process, increasing throughout the northeastern basins and the Indo–Gangetic Plains buoyancy $B = g\frac{\Delta\theta_v}{\theta_v}$.

On the leeward side, after condensation, the air descends over the Tibetan Plateau and Ladakh. As it sinks, it undergoes adiabatic warming at the dry lapse rate $\frac{dT}{dz} = +\Gamma_d(\Gamma_d \approx 9.8^{\circ}\frac{C}{km})$. This warming reduces relative humidity sharply, resulting in extremely dry conditions. Consequently, Ladakh and adjoining regions experience a cold desert climate, highlighting the critical role of the Himalayas in shaping both rainfall distribution and aridity.

The Northern Plains:

South of the Himalayas are the Northern Plains, one of the most populated and fertile areas of the planet covering the states of Haryana, Punjab, Uttar Pradesh, Bihar, West Bengal and Assam. Sugarcane, rice and wheat are among the major crops grown in this region, which has earned it the title of "food bowl of India."

The Indian monsoon system converges on the Northern Plains. The moisture monsoon winds move eastward across the plains by the large pressure differential created by the Thar Desert's intense heating during the summer. The flat terrain ensures that horizontal transport predominates over vertical effects in terms of the Navier-Stokes momentum balance,

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right) = -\nabla p + \rho g, \tag{2}$$

During the monsoon season, this causes rainfall to be distributed evenly and widely throughout northern India. However, the same plains display a different climatic behaviour throughout the winter. Cold air is trapped close to the surface because to severe thermal inversion, which causes temperature to rise with height $\frac{dT}{dz} > 0$.

As a result, persistent layers of fog and haze occur, especially over Punjab, Haryana, Uttar Pradesh, and Bihar, giving the area a distinctive seasonal weather phenomenon.

The Deccan Plateau (The Peninsular Plateau):
A large portion of central and southern India, including the states of Maharashtra, Madhya Pradesh, Telangana, Andhra Pradesh, Karnataka, and Tamil Nadu, is covered by the Deccan Plateau, one of India's oldest landforms. The Eastern and Western Ghats are mountain ranges that run parallel to the coastlines and surround the plateau. Whereas the Eastern Ghats are lower and irregular, the Western Ghats are higher and continuous, affecting monsoon rainfall. They are home to many national parks and sanctuary for wildlife and are together rich in biodiversity. Seas surround the Deccan Plateau on three sides. In contrast to northern India, the climate in the plateau is temperate. Because of its elevation and location in relation to the Western Ghats, the Deccan Plateau has a distinct climate. The plateau is often colder than the nearby lowlands because of its elevation above sea level, and the dry adiabatic lapse rate indicates that the temperature is dropping:

$$\Delta T \approx \Gamma_d \cdot \Delta z, \tag{3}$$

Where $\Gamma_d = 9.8^{\circ}\,C/km$. On the windward side (Kerala and coastal Karnataka), orographic rainfall occurs as a result of the southwest monsoon winds' forced climb over the Western Ghats. But as the winds move down the leeward side, they cover areas like Telangana, Maharashtra, and the interior of Karnataka. Most of their moisture is lost when they warm adiabatically, forming a semi-arid rain shadow area. This difference can be articulated using the moisture budget formula:

$$\frac{dq}{dt} = E - P, \tag{4}$$

where precipitation (P) greatly exceeds evaporation (E) on the windward side $(P \gg E)$, while on the leeward side precipitation is nearly zero $(P \approx 0)$, making evaporation dominate. As a result, the Deccan Plateau has a climate marked by moderate temperatures, lower humidity, and limited rainfall, highlighting the strong control of topography and fluid dynamics on regional climate.

Indian Desert (Thar Desert):
The Indian Desert, which is situated in western Rajasthan, is distinguished by its extreme climate, sand dunes, and arid conditions. Water scarcity is widespread, and there is minimal vegetation. Despite these challenges, irrigation projects like the Indira Gandhi Canal allow the desert to sustain human communities, agriculture, and cultural traditions. Rajasthan's desert heats up quickly in the summer. Rainfall in India is greatly influenced by this. However, because of its arid nature, the desert itself receives relatively little rainfall.

Due to its extreme heat, the Indian Desert is a major contributor to the country's summer climate. The desert surface is quite hot during the hottest summer months, creating a significant sensible heat flux that is given by

$$Q_H = \rho c_p u(T_s - T_a) \tag{5}$$

When intense heat transfer results from the significant temperature differential between the heated surface (T_s) and the surrounding air (T_a). The ideal gas law $(p = \rho RT)$ states that a strong thermal low-pressure zone $(p \ll 1000\text{hPa})$ is created when air temperature rises and density (ρ) falls. The pressure gradient force is greatly increased by this low pressure, drawing moisture-laden southwest monsoon winds into India from the Bay of Bengal and Arabian Sea. Additionally, buoyancy-driven convection is produced by the intense heating.

$$B = ga(T - T_{ref}), \tag{6}$$

which lifts hot air rapidly, producing dust storms, local turbulence, and unstable atmospheric conditions during the pre-monsoon season. Thus, the Indian Desert acts as a thermal engine, initiating and sustaining the large-scale monsoon circulation.

The Coastal Plains:

The Eastern Coastal Plain along the Bay of Bengal and the Western Coastal Plain along the Arabian Sea are India's two main coastal plains. The Eastern Coastal Plain is wider and separated into the Northern circular and Coromandel coasts, while the Western Coastal Plain is narrower and separated into the Konkan, Kannad and Malabar coastlines. Cyclones are also common in the coastal plains, especially along the east coast. The Arabian Sea on the west and the Bay of Bengal on the east have a significant ocean effect, giving the coastal regions of India a unique climate. Heavy orographic rainfall occurs in areas like Kerala and the Konkan coast due to the Western Ghats' presence forcing moisture-laden monsoon winds upward along the West Coast. The sea breeze–land breeze cycle also significantly shapes the region. During the day, the land heats up more quickly than the sea $(T_{land} > T_{sea})$, causing low pressure over the land and drawing cooler sea breeze air $(T_{land} < T_{sea})$.

This pressure difference can be expressed as $\Delta p = \rho R \Delta T$, Warm Sea surface temperatures (SSTs) have a greater impact on the East Coast, which faces the Bay of Bengal. Since warm SSTs supply the energy for storm development, cyclones commonly affect this area. The link between Clausius and Clopeyron,

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}, \tag{7}$$

demonstrates how the saturation vapor pressure, or e_s, rises with temperature, increasing the amount of moisture that can be held in the atmosphere. Because of this, cyclones in this area are frequently accompanied by heavy rains and powerful gusts. Therefore, coastal regions are crucial zones of India's climate system, with the East Coast being more susceptible to cyclone-induced variability while the West Coast climate is controlled by orographic uplift and sea-land breeze circulation.

Case-1 Winter Season

From December to February, India experiences winter, which is marked by low temperatures and dry, cold winds that originate from Central Asia and travel to North-East India. Because of the large desert region in North-West India, it continues to lose its temperature. There is a coastline boundary in south India. There is frequently a noticeable temperature difference between the sea and the land in the winter. During winter, the land cools more quickly than the water, which causes land breezes to form and blow from land to sea. High-pressure systems frequently rule northern India throughout the winter and reach the Bay of Bengal. Based on position and severity, these systems can strengthen northeasterly winds or produce calm conditions along the shore.

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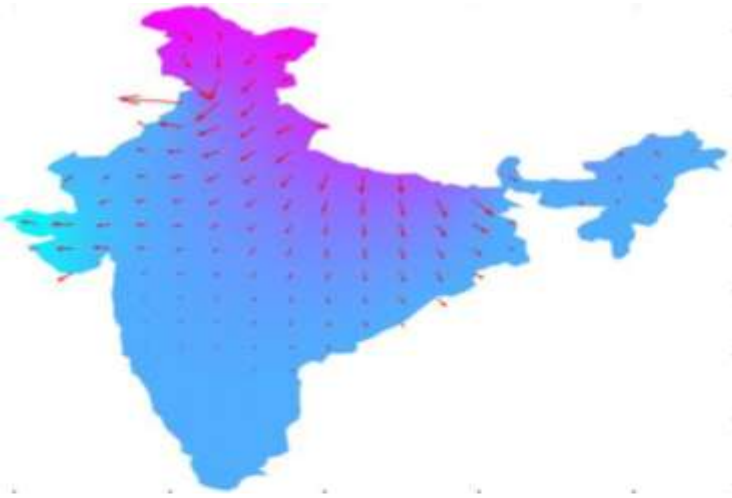

Figure 3: Wind Flow in Winter Season

Case-2 Summer Season

The summer season in India, lasting from March to May, brings warm and dry weather to the country. It is intriguing, how the sun's position and geographical factors contribute to the variation in temperatures across different regions. Additionally, learning about the impact of tropical cyclones and the positive influence of sporadic rainfall on agriculture, particularly the mango and coffee crops, highlights the resilience and adaptability of the Indian ecosystem. During the summer season, the coastal boundaries of India primarily experience southwest winds associated with the southwest monsoon, as well as sea breezes during the daytime. During the daytime, especially in clear weather conditions, sea breezes develop along the coastlines. As the land heats up more rapidly than the sea, warm air rises over the land, creating a low-pressure area. Cooler air from the sea then flows inland to replace the rising warm air, creating a sea breeze. Sea breezes bestow relief from the heat and humidity in coastal areas during summer.

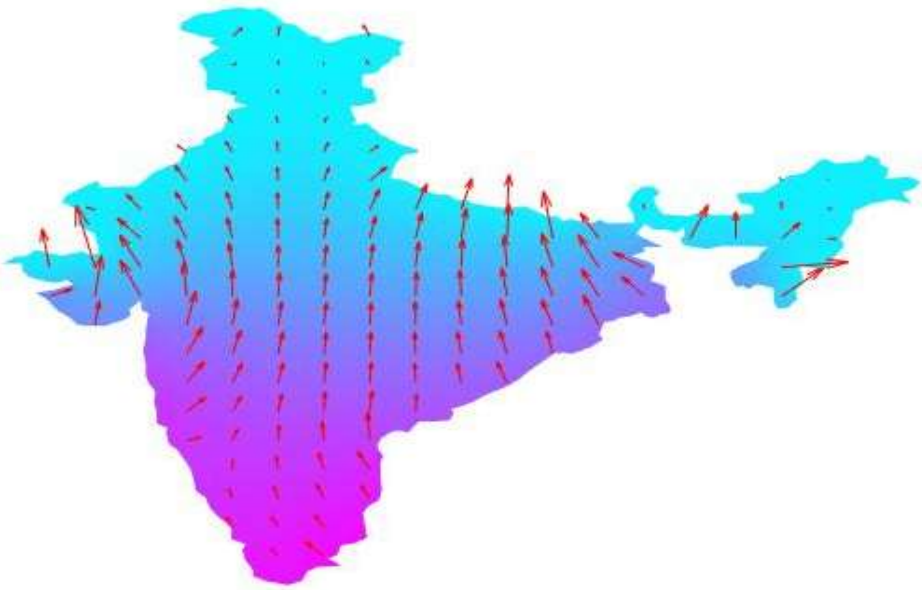

Figure 3: Wind Flow in Summer Season

Case-3 Monsoon Season

The monsoon season in India, ranging from June to September is significant for agriculture. The southwest monsoon winds bring vital rainfall to almost all parts of the country. The complex interplay of factors such as high temperatures, low-pressure systems, and the influence of geographical features provides the distribution of rainfall across different regions. The cyclones originating from the Arabian Sea and the Bay of Bengal intensify the significance of the monsoon. However, the uneven distribution of rainfall highlights the diverse and intricate nature of the monsoon system and its impact on the country's ecosystem and agriculture. During the monsoon season, there is often a significant temperature contrast between the land and sea. This contrast contributes to the development of low-pressure systems over land, which helps draw in moist air from the oceans, enhancing the strength of the monsoon winds. This wind pattern is characterized by moist air masses developing from the Indian Ocean, which flow towards the Indian subcontinent. Along the east coast (Bay of Bengal coast), the wind flow during the monsoon season is generally more complex. Initially, there are prevailing south-westerly winds from the Indian Ocean, which can bring moisture to the eastern coastal areas.

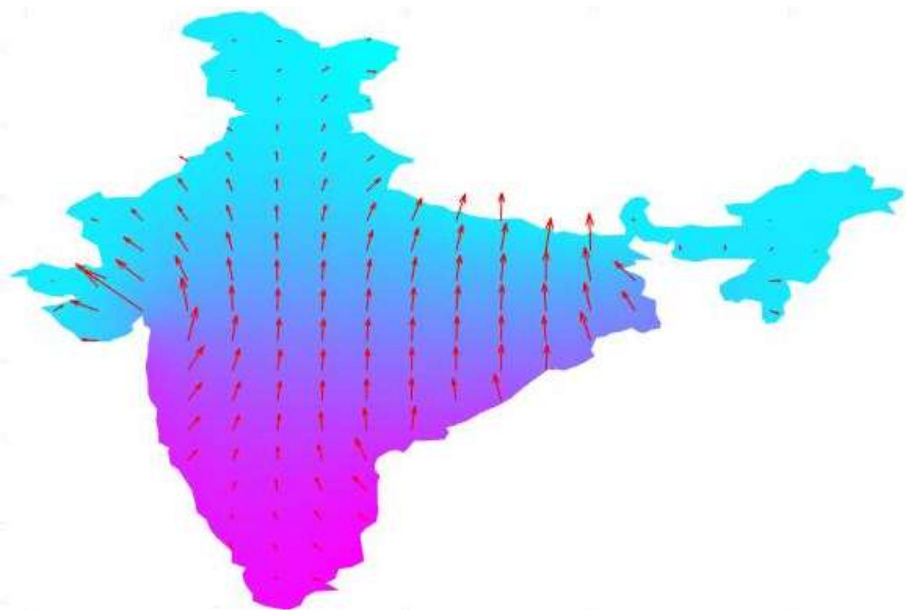

Figure 4: Wind Flow in Monsoon Season

CONCLUSION

India's climate and topography are closely connected. Mountains, plains, plateaus, deserts, and coastlines make up India's diverse topography, which shapes the nation's climate, agriculture, natural resources, and cultural diversity. A balanced climate system is produced throughout India by the interaction between wind flow and India's topography. While the Himalayas absorb the moist monsoon currents and block the cold Siberian winds, the Indo-Gangetic Plains and the Himalayas combine to make the monsoon a success. These winds attract deep into the subcontinent by low pressure created by the plains heating up in the summer, ensuring widespread rainfall. In a similar way, the Peninsular Plateau and the Thar Desert affect how rain falls. While the plateau's higher elevation receives monsoon winds, the desert's intense summer heat generates low pressure. These winds are split up, especially in the Western and Eastern Ghats, leaving some areas relatively dry and under rain shadow while making others highly wet. Finally, the Coastal Plains act as the monsoon system's entry points and regulators. The monsoon season begins with the first raindrops along the coast. India's diverse yet connected climate is the result of the interaction between these geographical elements and the wind flow system. Thus, the wind flow system in each area doesn't function independently. It influences and sustains the climate in other areas.

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