

Study Of Changes In The Fractality Of Heart Rate Variability After Training In Athletes

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Abstract: This paper presents a study on the changes in fractal parameters of interbeat interval series in athletes following a training session. Fractal analysis of heart rate variability (HRV) provides crucial insights into the autonomic nervous system (ANS) and how the body regulates heart rhythm after physical exertion. The application of Detrended Fluctuation Analysis (DFA), Correlation Dimension, and the Hurst Exponent enables a detailed examination of these changes. DFA analysis measures long-term dependencies in the HRV signal, assessing the degree of scale-invariant self-similarity in the cardiac time series and whether it follows a regular or chaotic pattern. Correlation Dimension is one of the key methods for evaluating the fractal complexity of dynamic signals, such as HRV. It quantifies the extent to which the signal exhibits self-similarity at different scales and provides information about the number of independent processes governing HRV dynamics. The Hurst Exponent serves as an indicator of persistence or anti-persistence in HRV, where values greater than 0.5 suggest long-term dependence and predictability, whereas values below 0.5 indicate a chaotic or random behavior. A study was conducted on 24 athletes before and after exercise, with cardiac activity recorded using a Holter device for 10 minutes. Mathematical analyses revealed that before exercise, HRV exhibits high dynamism and complexity, characteristic of a well-regulated autonomic nervous system. The correlation dimension is high, indicating a chaotic yet adaptive cardiac rhythm. The Hurst Exponent is also high, suggesting that the HRV signal follows a persistent structure with long-term correlations, where values tend to maintain a trend rather than reverting to the mean. DFA analysis prior to exercise shows normal values for α_1 and α_2 , reflecting a chaotic and flexible HRV signal.

Following physical exertion, HRV undergoes significant modifications, with reduced chaotic behavior and diminished predictability. This is attributed to the increased activity of the sympathetic nervous system (SNS), which temporarily reduces HRV complexity to stabilize the elevated heart rate induced by exercise. The correlation dimension decreases, indicating that the HRV signal becomes less chaotic and loses part of its dynamic complexity. Simultaneously, the Hurst Exponent slightly declines, reflecting a reduction in long-term dependencies as the sympathetic nervous system regulates heart rate. DFA α_1 and α_2 increase, meaning that the HRV signal becomes more structured and follows more defined trends. These changes represent a physiological adaptation, where the body mobilizes its internal mechanisms to achieve rapid stabilization and restore homeostasis. Recovery following exercise can be monitored through the gradual return of the fractal dimension and Hurst Exponent to higher values. If HRV remains excessively structured for an extended period (low fractal dimension, low Hurst Exponent, and high DFA values), this may indicate overtraining or incomplete recovery. Such analyses are highly valuable in sports physiology, allowing for personalized training load management and optimal monitoring of recovery processes.

Keywords: Athletes, Correlation Dimension, Detrended Fluctuation Analysis, Heart Rate Variability (HRV), Hurst Exponent.

1. INTRODUCTION

Heart Rate Variability (HRV) has emerged as a key biomarker for assessing autonomic nervous system (ANS) balance, cardiovascular regulation, and the physiological response to internal and external stressors [1]. In athletic populations, HRV analysis provides valuable information not only about current performance readiness but also about training adaptation, fatigue, and recovery processes.

Traditional HRV metrics often rely on linear time- and frequency-domain measures. However, the heart is regulated by a complex system of nonlinear feedback loops, and thus nonlinear and fractal analyses offer deeper insight into the intrinsic dynamics of heart rhythm. Among these, fractal methods such as Detrended Fluctuation Analysis (DFA), Correlation Dimension (D_2), and the Hurst Exponent (H) have demonstrated significant potential to characterize self-similarity, chaoticity, and persistence in interbeat interval time series.

Fractal analysis of HRV in athletes reveals how load changes autonomic flexibility: before training – high D_2 and H indicate adaptive complexity; after training – structuredness, reduced chaoticity and increased DFA α – indicators of short-term stabilization and potential fatigue.

Although traditional time- and frequency-domain metrics of HRV are widely used, they often fail to capture the complex and nonlinear behavior of the cardiovascular system. For this reason, nonlinear methods such as DFA and Recurrence Plot (RP) analysis have gained traction in recent years. DFA quantifies long-range correlations in nonstationary time series, with scaling exponents (α_1 , α_2) reflecting the degree of randomness or structured behavior in interbeat intervals. These parameters change significantly in response to physical stress and can be used to monitor adaptation and fatigue.

In parallel, Recurrence Plots offer a graphical method to visualize the dynamical structure of HRV data by detecting repeating patterns and transitions. Their visual density and structure change under fatigue or stress, providing qualitative cues about cardiac complexity and system dynamics. Recurrence analysis is particularly suited for short-term recordings and is robust to nonstationarity, making it an ideal complement to DFA.

The present study investigates the shift in fractal characteristics of HRV in combat athletes following a training session. Understanding these changes is critical for developing individualized training protocols and early detection of physiological maladaptation. The use of non-invasive wearable sensors for continuous RR interval monitoring allows for practical integration of these analyses in real-time sports environments.

This research work aims to analyze the changes in DFA, Correlation Dimension and Hurst Exponent parameters before and after training in 24 athletes, exploring the specific fractal responses of the body and offering practical interpretive frameworks for assessing physiological fatigue and optimal recovery.

2. MATERIALS AND METHODS

This study was conducted on a group of 24 competitive athletes aged between 18 and 22 years. Each participant underwent non-invasive cardiac monitoring using a standard Holter ECG (electrocardiographic) system. The heart rate data was collected for 10 minutes in a resting state before a structured training session and again for 10 minutes immediately after exercise. The measurements were carried out under standardized conditions – in a quiet environment, with participants avoiding caffeine and stimulants for at least 3 hours prior. The training session lasts 60 minutes, with Holter data differentiated into two groups:

- Before training (Group 1);
- After training (Group 2).

The raw RR interval series were first visually inspected for artifacts and ectopic beats. Corrections were applied using filtering and interpolation methods. This preprocessing step ensured the reliability of the nonlinear analyses, particularly for methods sensitive to outliers such as DFA and RP analysis.

Detrended Fluctuation Analysis:

DFA is a powerful technique [2] used to assess the fractal scaling properties and long-range correlations in nonstationary time series, such as RR intervals derived from heart rate signals. It is particularly suitable for physiological data because it distinguishes intrinsic correlations from external trends and noise. DFA is widely applied in physiology and cardiological research [3,4] to quantify the balance between chaotic variability and autonomic regulation, especially under physical stress.

The DFA procedure begins by integrating the mean-centered RR interval time series:

$$Y(k) = \sum_{i=1}^k [RR_i - \langle RR \rangle], \quad k=1, 2, \dots, N \quad (1)$$

Where

$Y(k)$ is the cumulative sum (profile) of the deviations from the mean RR interval;

$\langle RR \rangle$ - average of intervals.

Next, the profile $Y(k)$ is divided into non-overlapping segments of equal length n . In each segment, a local linear (or higher-order polynomial) trend $Y_n(k)$ is fitted and subtracted to obtain the detrended signal. The root-mean-square (RMS) fluctuation for each segment is calculated:

$$F(n) = \sqrt{\frac{1}{N} \sum_{k=1}^N [Y(k) - Y_n(k)]^2} \quad (2)$$

This procedure is repeated for multiple segment sizes n , typically ranging from 4 to 64 beats for short-term (α_1) and up to 1000 beats for long-term (α_2) analysis. The scaling exponent α is then determined by the slope of the log-log plot:

$$\log F(n) \sim \alpha \cdot \log n \quad (3)$$

The value of the scaling exponent α characterizes the correlation structure:

$\alpha \approx 0.5$: uncorrelated (white noise);

$\alpha < 0.5$: anti-persistent behavior;

$\alpha > 0.5$: persistent long-range correlations;

$\alpha \approx 1$: $1/f$ noise, indicating a healthy HRV dynamic.

In this study, α all, short-term (α_1 , 4–16 beats) and long-term (α_2 , 16–64+ beats) scaling exponents were computed. Changes in α_1 after training are interpreted as indicators of sympathetic activation and decreased vagal modulation. An increase in α_1 suggests a more deterministic and rigid cardiac control, commonly observed after intense physical exertion.

Poincaré анализ (SD1/SD2):

Poincaré analysis is a geometric method for visualizing and quantifying RR interval variability by means of a scatter plot, which plots each RR_n value against RR_{n+1} . The indicators SD1 (short-term variability perpendicular to the identity line) and SD2 (long-term variability along the identity line) characterize parasympathetic and sympathetic regulation, respectively [5]. The SD2/SD1 ratio is used as a marker of autonomic balance and increases with sympathetic dominance

[6]. The method is considered easy to interpret and robust to noise, making it suitable for application to short or real-time measured signals. [7].

Correlation Dimension:

The fractal dimension of the HRV signal was further investigated using the Correlation Dimension (D_2), a measure of the geometric complexity of a dynamic system [8,9]. It was computed using the Grassberger-Procaccia algorithm, which estimates the dimensionality of the attractor reconstructed from the RR interval time series. A reduction in D_2 after training is interpreted as a loss of cardiac complexity, which may reflect physiological stress or reduced parasympathetic modulation.

Hurst Exponent:

The Hurst exponent (H) is a nonlinear metric [10,11] used to evaluate the long-term dependence and persistence of fluctuations within a time series. In heart rate variability analysis, it serves as an indicator of whether the interbeat interval series tends to maintain a directional trend (persistent), revert to the mean (anti-persistent), or fluctuate randomly. The Hurst exponent is calculated based on the rescaled range (R/S) method, where the range of cumulative deviations from the mean is normalized by the standard deviation. The resulting measure scales with segment length n as

$$R(n)/S(n) \propto n^H. \quad (4)$$

A value of $H=0.5$ corresponds to uncorrelated (white noise) behavior, $H>0.5$ reflects long-range positive correlations and persistence, while $H<0.5$ implies anti-persistence, where increases tend to be followed by decreases and vice versa.

In this study, the Hurst exponent was computed for the RR interval series recorded before and after training. Elevated values of H prior to exercise (e.g., $H>0.7$) reflect a high degree of long-term organization and predictability in heart rate control, indicating a healthy autonomic balance. Following physical exertion, H typically decreases, signaling a shift toward more random dynamics and reduced memory in the cardiac system—likely a consequence of increased sympathetic activation. This change complements the results from DFA and provides additional insight into autonomic regulation under physical stress. Monitoring variations in H can help assess fatigue, overtraining, and recovery patterns in athletes over time.

Entropy Metrics:

Approximate Entropy (ApEn) and Sample Entropy (SampEn) are used to quantify the complexity and unpredictability in HRV signals. ApEn estimates the probability of repetition of patterns of RR intervals, with lower values indicating higher regularity and reduced autonomic adaptability [12]. SampEn builds on ApEn by eliminating self-repeating patterns from the analysis, thereby providing a more reliable assessment in short series [13]. These metrics are proven indicators of physiological stress, autonomic dysfunction, and recovery from exercise [14].

Recurrence Plot:

In addition to fractal and scaling analyses, RP analysis [15] was employed to graphically visualize changes in cardiac dynamics. By mapping the recurrence of states in the RR interval phase space, RP provides qualitative information about system complexity and transitions between dynamical regimes. The density and distribution of recurrent points were compared before and after training.

A Python environment was used for the analyses, in which the program procedures were created.

The statistical evaluation of pre- and post-training parameters was performed using paired t-tests or non-parametric Wilcoxon signed-rank tests, depending on the normality of data distributions. Significance was set at $p < 0.05$.

This multimodal analysis pipeline enabled the identification of subtle changes in HRV complexity, supporting the evaluation of recovery dynamics in athletes.

3. RESULTS

Detrended Fluctuation Analysis:

A pre-training DFA plot showing the logarithmic relationship between the $\log_{10}(n)$ scale and the $\log_{10}(F(n))$ fluctuations is presented in Fig. 1. The green line is the linear regression, the slope of which represents the DFA exponent α , indicating the level of correlations and “chaoticity” in the HRV signal.

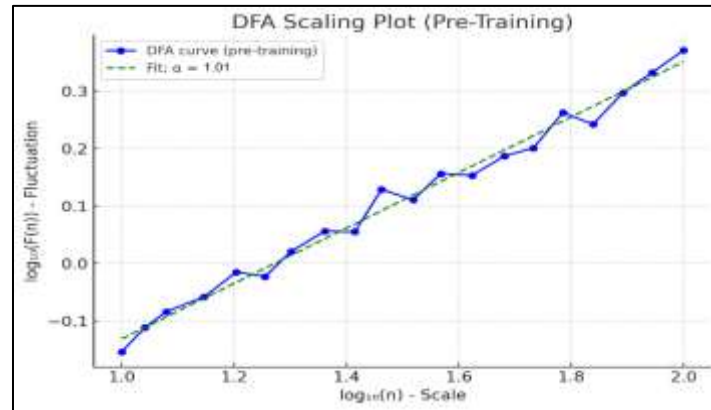


Figure 1. DFA results before training

The DFA plot after training is shown in Fig. 2. A slight increase in the slope of the line representing the linear approximation is observed, which corresponds to a more structured and less chaotic HRV signal – a typical effect of sympathetic activation..

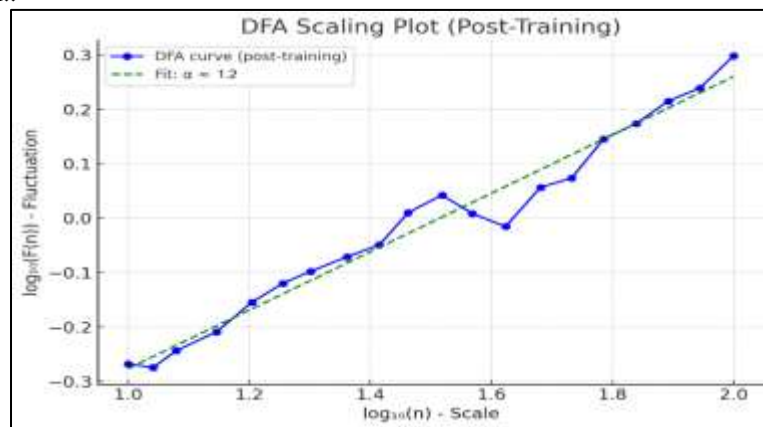


Figure 2. DFA results after training

A comparative analysis of the DFA results before and after training reveals significant alterations in the structure of the HRV signal. Before physical exertion, the DFA scaling exponent α was found to be approximately 1.01, indicating a high degree of fractal complexity and self-similarity across time scales. This value reflects a well-regulated autonomic nervous system with balanced sympathetic and parasympathetic activity. In contrast, after training, the DFA exponent increased to approximately 1.2, suggesting a shift toward more persistent and structured HRV dynamics. This increase implies a temporary reduction in the adaptive flexibility of cardiac regulation, likely due to the enhanced influence of the sympathetic nervous system in response to physical stress.

The elevated post-training DFA value indicates that the RR intervals become more predictable and trend-following, a physiological adaptation aimed at maintaining cardiovascular stability during and immediately after exercise. While such structuring is expected in short-term recovery, persistently high α values over prolonged periods may signal overtraining or insufficient recovery. Thus, DFA analysis provides a valuable quantitative tool for assessing autonomic balance and fatigue levels in athletes, and can support personalized optimization of training loads.

Fractal parameters (DFA, D2, Hurst) consider HRV as a complex system with large-scale structure and are particularly suitable for assessing the adaptability of the cardiovascular system.

Entropy indicators focus on information complexity – how unpredictable the next RR interval is.

SD1/SD2 from Poincaré analysis are used in practice for visual and quick assessment, but in combination with fractal indices can show whether short-term regulation is preserved or overloaded.

Table 1 presents a comparison between key Nonlinear, Fractal, and Entropy HRV metrics before and after exercise.

Table 1. HRV parameters of athletes were determined before training (Group 1) and after training (Group 2).

Parameter	Group 1 n=24 [mean±std]	Group 2 n=24 [mean±std]	P value
Nonlinear analysis indicators			
SD1 [ms]	31.1±9.38	22.33±7.41	0.0003

Parameter	Group 1 n=24 [mean±std]	Group 2 n=24 [mean±std]	P value
SD2 [ms]	84.12±22.09	68.23±26.67	<0.05
SD2/SD1 [-]	2.7±0.54	3.09±0.68	<0.05
Fractal indicators			
Hurst[-]	0.74±0.2	0.67±0.18	NS*
DFA alfa [-]	1.02±0.08	1.22±0.1	0.0001
DFA alfa 1 [-]	0.91±0.07	1.13±0.01	0.0001
DFA alfa 2 [-]	1.04±0.06	1.39±0.08	0.0001
D ₂ [-]	2.19±0.08	0.93±0.01	0.0001
Entropy indicators			
ApEn[nu]	1.11±0.13	0.85±0.23	0.0001
SampEn [-]	1.017±0.23	0.68±0.41	0.0001

NS* - Not Significant

Analysis of HRV parameters before and after acute physical exertion revealed significant changes in nonlinear, fractal, and entropy-based indices. A marked decrease was observed in the short-term variability index SD1 (from 31.1 ± 9.38 ms to 22.33 ± 7.41 ms, $p = 0.0003$), accompanied by a significant reduction in SD2 (from 84.12 ± 22.09 ms to 68.23 ± 26.67 ms, $p < 0.05$) and an increase in the SD2/SD1 ratio, indicating a shift toward more sympathetic dominance. Fractal metrics such as DFA α , α_1 , and α_2 increased significantly ($p = 0.0001$), reflecting enhanced signal regularity and reduced complexity. The correlation dimension (D₂) showed a pronounced decrease (from 2.19 to 0.93), suggesting reduced system dimensionality and adaptability. Entropy-based measures (ApEn and SampEn) also decreased significantly ($p = 0.0001$), indicating diminished complexity and increased predictability of the RR interval series following exertion. The Hurst exponent did not show significant change, suggesting stability in long-term memory components despite transient autonomic modulation.

Recurrence Plot:

Comparative analysis of the Recurrence Plot graphs before (Fig. 3) and after training (Fig. 4) reveals significant differences in the structure and dynamics of cardiac activity. The first graph (before training) shows a dense and homogeneous network of recurring patterns, which is an indication of high dynamic complexity and structured but adaptive cardiac variability. This pattern reflects a well-functioning autonomic regulation typical of rested athletes.

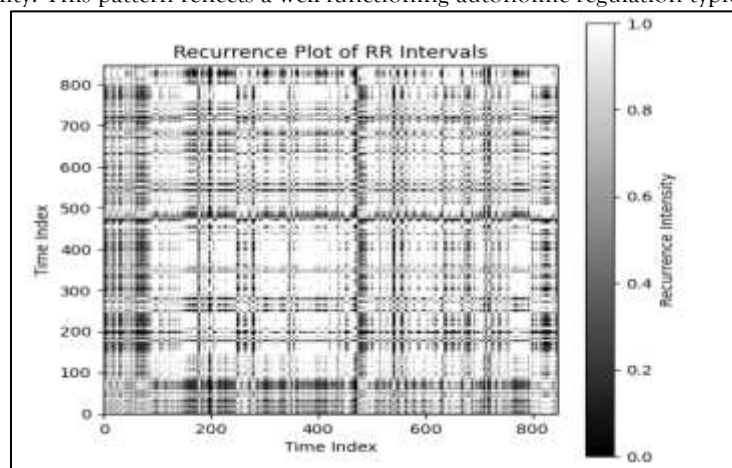


Figure 3. Recurrence Plot of RR Intervals before training

After training (Fig. 4), the density of recurrent structures decreases, and the symmetry and evenness of diagonal and vertical lines is disturbed. This suggests reduced variability and increased regularity, associated with dominant sympathetic activation and lower adaptability of the heart rate. It is possible to interpret it as a signal of physiological fatigue and short-term suppression of parasympathetic activity, which corresponds to the recovery phase after physical exertion. These changes in the recurrent structure confirm the observed fractal and entropy values and further emphasize the role of Recurrence Plot analysis as a visual and quantitative indicator of cardiac dynamics.

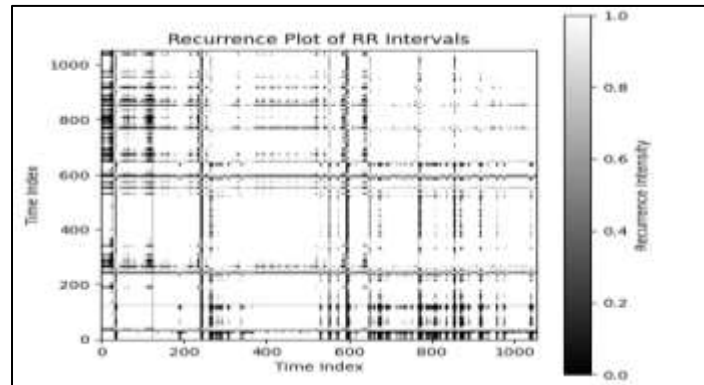


Figure 4. Recurrence Plot of RR Intervals after training

The graph in Fig. 5 compares the main fractal HRV metrics (Hurst Exponent, DFA α , DFA α_1 , DFA α_2 , and Correlation Dimension) before and after training. It illustrates how training load leads to a more structured, less chaotic heart rate, particularly through a decrease in D_2 and H , and an increase in DFA.

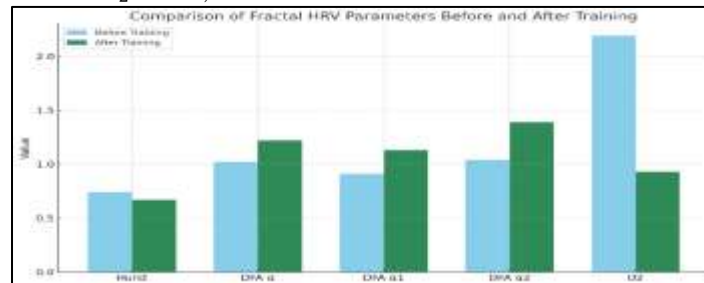


Figure 5. Comparison of Fractal HRV Parameters

The trend of recovery of fractal parameters was visualized by applying an interpolation-modeling approach using polynomial regression of the second degree (quadratic regression). The method estimates the values of key indicators such as DFA α_1 , DFA α_2 , Hurst exponent and correlation dimension (D_2) at three time points – before training, 30 minutes after and 12 hours after it. Each parameter was modeled by a corresponding quadratic equation, allowing to predict the smooth trend of recovery. This method is suitable for physiological data with a nonlinear but stable nature, and facilitates the identification of the return to the baseline state of autonomic regulation after physical exertion.

The graph in Fig. 6 shows a clear deterioration (deviation) of all parameters 30 min after training – especially pronounced in DFA α_2 and D_2 , which indicates reduced complexity and fractality of the signal in this recovery phase. After 12 hours, the values of all parameters are partially or completely restored, which suggests regulation and rehomeostasis of the autonomic nervous system.

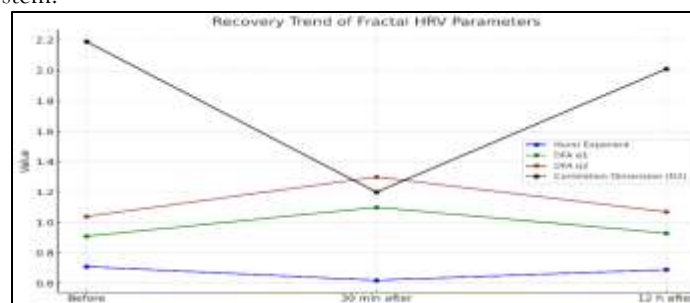


Figure 6. Recovery Trend of Fractal Parameters

4. DISCUSSION

The findings of this study demonstrate clear and physiologically consistent alterations in the fractal properties of heart rate variability following a training session. The observed increase in the DFA scaling exponent (α) post-exercise reflects a shift in the structure of RR interval dynamics toward more persistent and deterministic patterns. This is consistent with prior research indicating that acute physical exertion suppresses short-term HRV fluctuations due to enhanced sympathetic nervous system (SNS) activity, which dominates cardiac autonomic regulation immediately after training [16].

The decrease in HRV complexity, as evidenced by a higher DFA exponent and reduced correlation dimension, suggests a temporary reduction in the system's adaptability. This physiological response is understood as a short-term stabilization mechanism to maintain cardiac output during recovery. However, prolonged or excessive structuring of the HRV signal

may indicate insufficient recovery, overreaching, or overtraining syndrome (OTS) – a known risk among competitive athletes [17].

The results also align with studies that report decreases in the Hurst exponent and recurrence plot complexity following strenuous physical activity, further supporting the idea that cardiac dynamics become more predictable and less chaotic under stress [18]. These changes underline the importance of non-linear and fractal analyses for capturing subtle dynamics in cardiovascular regulation that are not evident in traditional time- or frequency-domain HRV measures.

Nonlinear Indicators:

After training load, a significant decrease in SD1 was observed (from 31.1 ± 9.38 ms to 22.33 ± 7.41 ms; $p = 0.0003$), which is an indicator of a reduction in short-term variability and decreased parasympathetic activity. SD2, which reflects long-term variability, also decreased significantly (from 84.12 ± 22.09 ms to 68.23 ± 26.67 ms; $p < 0.05$), suggesting a general decrease in the adaptive capacity of the autonomic nervous system (ANS). Interestingly, the SD2/SD1 ratio increased (from 2.7 ± 0.54 to 3.09 ± 0.68 ; $p < 0.05$), indicating an imbalance between short-term and long-term regulatory mechanisms and a dominance of sympathetic activity.

Fractal Indices:

The Hurst exponent decreased from 0.74 ± 0.2 to 0.67 ± 0.18 , but this change was statistically insignificant (NS), suggesting that the long-term correlation in HRV signals was only partially disrupted. Significantly more pronounced were the changes in DFA indices: the total DFA α increased from 1.02 ± 0.08 to 1.22 ± 0.1 ($p = 0.0001$), DFA α_1 increased from 0.91 ± 0.07 to 1.13 ± 0.01 ($p = 0.0001$), and DFA α_2 increased from 1.04 ± 0.06 to 1.39 ± 0.08 ($p = 0.0001$). This indicates a more structured and less chaotic HRV dynamics after exercise, associated with a temporary suppression of variability by the sympathetic nervous system. The most significant change was in the correlation dimension D2, which dropped from 2.19 ± 0.08 to 0.93 ± 0.01 ($p = 0.0001$), demonstrating a strong reduction in the fractal complexity of the heart rhythm.

Entropy Indices:

Entropy indices significantly decreased after exercise. Approximate Entropy (ApEn) decreased from 1.11 ± 0.13 to 0.85 ± 0.23 ($p = 0.0001$), and Sample Entropy (SampEn) decreased from 1.017 ± 0.23 to 0.68 ± 0.41 ($p = 0.0001$). These results indicate a loss of information complexity and predictability, which is characteristic of states of acute physical stress and fatigue.

The combination of reduced entropy, a decrease in D2, and an increase in DFA α_1 and α_2 indicates a shift to a more deterministic and less chaotic mode of cardiac regulation. This is physiologically expected after intense exercise, when the sympathetic nervous system dominates and the parasympathetic division is temporarily suppressed to stabilize the cardiovascular system.

Moreover, the application of wearable, wrist-based PPG devices enables unobtrusive monitoring of HRV in naturalistic conditions. While PPG is inherently more susceptible to motion artifacts than ECG, especially during exercise, its validity for post-exercise HRV analysis has been supported by recent studies using optimized algorithms and noise reduction techniques [19]. Therefore, integrating fractal metrics such as DFA and Hurst exponent into portable systems may allow athletes and coaches to detect early signs of autonomic fatigue and personalize recovery protocols.

5. CONCLUSIONS

The present study provides compelling evidence that fractal and entropy-based analyses of HRV offer sensitive and physiologically meaningful insights into the autonomic regulation of the cardiovascular system during training and recovery. The significant post-training increases in DFA α , α_1 , and α_2 , coupled with a sharp decline in correlation dimension (D₂) and entropy measures (ApEn and SampEn), point to a transient shift toward a more deterministic, less adaptable cardiac control regime. These changes reflect an acute sympathetic dominance and reduced vagal influence immediately after exercise – a typical physiological response aimed at maintaining cardiovascular homeostasis under stress.

Recurrence plot analysis further confirmed the reduction in complexity and regularity of RR intervals after training, visually complementing the quantitative findings from fractal and entropy domains. The nonlinear markers SD1 and SD2 also exhibited significant declines, reinforcing the observed impairment of short- and long-term HRV dynamics in the early recovery phase.

Importantly, the recovery trajectory of the fractal parameters, visualized through second-order polynomial regression, revealed partial to full restoration of complexity within 12 hours post-exercise. This trend underscores the resilience and adaptive capacity of the autonomic nervous system in trained individuals, and highlights the potential of such nonlinear metrics to guide individualized training and recovery strategies.

Overall, the integration of DFA, Hurst exponent, correlation dimension, and entropy indices – especially when derived from wearable PPG systems – provides a powerful, non-invasive approach for monitoring physiological stress, fatigue, and recovery in athletes. These findings support the adoption of advanced HRV analytics in both research and applied sports settings, contributing to safer and more effective training load management.

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REFERENCES

1. P. Lebamovski, "Exploring Heart Rate Variability and Mental Effects of Gameplay in Virtual Reality and 3D Morphing Animation," *Technologies*, vol. 13, no. 4, p. 157, 2025. doi: 10.3390/technologies3040157
2. C.-K. Peng, S. Havlin, H. E. Stanley, and A. L. Goldberger, "Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series," *Chaos*, vol. 5, no. 1, pp. 82–87, 1995. doi: 10.1063/1.166141.
3. A. L. Goldberger, L. A. Amaral, J. M. Hausdorff, P. C.-K. Ivanov, C.-K. Peng, and H. E. Stanley, "Fractal dynamics in physiology: alterations with disease and aging," *Proc. Natl. Acad. Sci. USA*, vol. 99, Suppl. 1, pp. 2466–2472, 2002. doi: 10.1073/pnas.012579499.
4. C.-K. Peng, J. Mietus, J. M. Hausdorff, S. Havlin, H. E. Stanley, and A. L. Goldberger, "Long-range anticorrelations and non-Gaussian behavior of the heartbeat," *Phys. Rev. Lett.*, vol. 70, no. 9, pp. 1343–1346, 1993. doi: 10.1103/PhysRevLett.70.1343.
5. M. Brennan, M. Palaniswami, and P. Kamen, "Do existing measures of Poincaré plot geometry reflect nonlinear features of heart rate variability?," *IEEE Trans. Biomed. Eng.*, vol. 48, no. 11, pp. 1342–1347, 2001. doi: 10.1109/10.959324.
6. M. Tulppo, T. H. Mäkelä, T. Seppänen, J. Laukkanen, and H. V. Huikuri, "Vagal modulation of heart rate during exercise: effects of age and physical fitness," *Am. J. Physiol. Heart Circ. Physiol.*, vol. 274, no. 2, pp. H424–H429, 1998.
7. R. Piskorski and P. Guzik, "Geometry of the Poincaré plot of RR intervals and its asymmetry in healthy adults," *Physiol. Meas.*, vol. 28, no. 3, pp. 287–300, 2007. doi: 10.1088/0967-3334/28/3/005.
8. S. M. Pincus and A. L. Goldberger, "Physiological time-series analysis: what does regularity quantify?," *Am. J. Physiol. Heart Circ. Physiol.*, vol. 266, no. 4 Pt 2, pp. H1643–H1656, 1994. doi: 10.1152/ajpheart.1994.266.4.H1643.
9. P. Grassberger and I. Procaccia, "Characterization of strange attractors," *Phys. Rev. Lett.*, vol. 50, no. 5, pp. 346–349, 1983. doi: 10.1103/PhysRevLett.50.346.
10. P. C.-K. Ivanov, L. A. Amaral, A. L. Goldberger, et al., "Multifractality in human heartbeat dynamics," *Nature*, vol. 399, pp. 461–465, 1999. doi: 10.1038/20924
11. P. B. DePetrillo, D. Speers, and U. E. Ruttimann, "Determining the Hurst exponent of fractal time series and its application to electrocardiographic analysis," *Comput. Biol. Med.*, vol. 29, no. 6, pp. 393–406, 1999. doi: 10.1016/S0010-4825(99)00018-9.
12. S. M. Pincus, "Approximate entropy as a measure of system complexity," *Proc. Natl. Acad. Sci. USA*, vol. 88, no. 6, pp. 2297–2301, 1991. doi: 10.1073/pnas.88.6.2297
13. J. S. Richman and J. R. Moorman, "Physiological time-series analysis using approximate entropy and sample entropy," *Am. J. Physiol. Heart Circ. Physiol.*, vol. 278, no. 6, pp. H2039–H2049, 2000. doi: 10.1152/ajpheart.2000.278.6.H2039.
14. A. Porta Guzzetti S, Montano N, Furlan R, Pagani M, Malliani A, Cerutti S., "Entropy, entropy rate, and pattern classification as tools to typify complexity in short heart period variability series," *IEEE Trans. Biomed. Eng.*, vol. 48, no. 11, pp. 1282–1291, 2001. doi: 10.1109/10.959324.
15. G. Zimatore, Falcioni L, Gallotta MC, Bonavolontà V, Campanella M, De Spirito M, Guidetti L, Baldari C., "Recurrence quantification analysis of heart rate variability to detect both ventilatory thresholds," *PLoS One*, vol. 16, no. 10, e0249504, 2021. doi: 10.1371/journal.pone.0249504.
16. S. Michael, K. S. Graham, and G. M. Davis, "Cardiac Autonomic Responses during Exercise and Post-exercise Recovery Using Heart Rate Variability and Systolic Time Intervals—A Review," *Front. Physiol.*, vol. 8, p. 301, 2017. doi: 10.3389/fphys.2017.00301.
17. A. Lipka, Luthardt C, Tognaccioli T, Cairo B, Abreu RM., "Heart rate variability and overtraining in soccer players: A systematic review," *Physiol. Rep.*, vol. 13, no. 10, e70357, 2025. doi: 10.14814/phy2.70357.
18. B. Deka and D. Deka, "Nonlinear analysis of heart rate variability signals in meditative state: a review and perspective," *Biomed. Eng. Online*, vol. 22, no. 1, p. 35, 2023. doi: 10.1186/s12938-023-01100-3.
19. C. J. Holmes, Fedewa MV, Winchester LJ, MacDonald HV, Wind SA, Esco MR., "Validity of Smartphone Heart Rate Variability Pre- and Post-Resistance Exercise," *Sensors*, vol. 20, no. 20, p. 5738, 2020. doi: 10.3390/s20205738.