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Contra $\boldsymbol{M}$-continuous Maps and Contra $\boldsymbol{M}$-irresolute Maps in Fuzzy Hypersoft Topological Spaces

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Abstract
The purpose of this paper is to introduce and study fuzzy hypersoft contra M continuous maps and fuzzy hypersoft contra M irresolute maps in fuzzy hypersoft topological spaces with examples. Further, we derive some useful results and Properties related to them.
Keywords: *fuzzy hypersoft contra M continuous maps, fuzzy hypersoft contra M irresolute maps.*
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1 INTRODUCTION

The real world decision making problems in medical diagnosis, engineering, economics, management, computer science, artificial intelligence, social sciences, environmental science and sociology contains more uncertain and inadequate data. The traditional mathematical methods cannot deal with these kind of problems due to the imprecise data. To deal the problems with uncertainty, Zadeh [12] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang [6] as fuzzy topological space. Molodstov [8] introduced a new mathematical tool, soft set theory in 1999 to deal uncertainties in which a soft set is a collection of approximate descriptions of an object. Soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory have several applications in different fields such as decision making, optimization, forecasting, data analysis etc. Shabir and Naz [9] presented soft topological spaces. Smarandache [10] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in the decision-making issues involving different kind of attributes. Abbas et. al. [1] defined the basic operations on hypersoft sets and hypersoft point in all the universe of discourses. Ajay and Charisma [3] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ajay et.al. [4] defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision making. Aras and Bayramov [5] introduced neutrosophic soft continuity in neutrosophic soft topological spaces. Ahsan et. al. [2] studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes. Vadivel et. al. [11] introduced Gerenalized fuzzy contra e continuous in fuzzy topological spaces.
In this paper, we develop the concept of fuzzy hypersoft contra M continuity in fuzzy hypersoft topological spaces and some of their properties are analyzed with examples. Added to that, we discuss some characterizations and properties of fuzzy hypersoft contra M irresolute maps.

2 PRELIMINARIES

Definition 2.1 *[12] Let $\mathfrak{Z}$ be an initial universe. A function λ from $\mathfrak{Z}$ into the unit interval I is called a fuzzy set in $\mathfrak{Z}$. For every $\mathfrak{z} \in \mathfrak{Z}, \lambda(\mathfrak{z}) \in I$ is called the grade of membership of $\mathfrak{z}$ in λ. Some authors say that λ is a fuzzy subset of $\mathfrak{Z}$ instead of saying that λ is a fuzzy set in $\mathfrak{Z}$. The class of all fuzzy sets from $\mathfrak{Z}$ into the closed unit interval I will be denoted by $I^{\mathfrak{Z}}$.*

Definition 2.2 *[8] Let $\mathfrak{Z}$ be an initial universe, $\mathfrak{R}$ be a set of parameters and $\mathcal{P}(\mathfrak{Z})$ be the power set of $\mathfrak{Z}$. A pair $(\tilde{I}, \mathfrak{R})$ is called the a soft set over $\mathfrak{Z}$ where $\tilde{I}$ is a mapping $\tilde{I}: \mathfrak{R} \to \mathcal{P}(\mathfrak{R})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{R}$.*

Definition 2.3 *[10] Let $\mathfrak{Z}$ be an initial universe and $\mathcal{P}(\mathfrak{Z})$ be the power set of $\mathfrak{Z}$. Consider $\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3, \ldots, \mathfrak{r}_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $\mathfrak{R}_1, \mathfrak{R}_2, \ldots, \mathfrak{R}_n$ with $\mathfrak{R}_i \cap \mathfrak{R}_j = \emptyset$, for $i \neq j$ and $i, j \in \{1,2,\ldots,n\}$. Then the pair $(\tilde{I}, \mathfrak{R}_1 \times \mathfrak{R}_2 \times \ldots \times \mathfrak{R}_n)$ where $\tilde{I}: \mathfrak{R}_1 \times \mathfrak{R}_2 \times \ldots \times \mathfrak{R}_n \to \mathcal{P}(\mathfrak{Z})$ is called a hypersoft set over $\mathfrak{Z}$.*

Definition 2.4 *[1] Let $\mathfrak{Z}$ be an initial universal set and $\mathfrak{R}_1, \mathfrak{R}_2, \ldots, \mathfrak{R}_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{Z})$ be the set of all fuzzy sets of $\mathfrak{Z}$. Let $\mathfrak{S}_i$ be the nonempty subset of the pair $\mathfrak{R}_i$ for each $i = 1,2,\ldots,n$. A fuzzy hypersoft set (briefly, $FHSs$) over $\mathfrak{Z}$ is defined as the pair $(\tilde{I}, \mathfrak{S}_1 \times \mathfrak{S}_2 \times \ldots \times \mathfrak{S}_n)$ where $\tilde{I}: \mathfrak{S}_1 \times \mathfrak{S}_2 \times \ldots \times \mathfrak{S}_n \to \mathcal{P}(\mathfrak{Z})$ and $\tilde{I}(\mathfrak{S}_1 \times \mathfrak{S}_2 \times \ldots \times \mathfrak{S}_n) = \{(\mathfrak{r}, \langle \mathfrak{z}, \mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) \rangle : \mathfrak{z} \in \mathfrak{Z}) : \mathfrak{r} \in \mathfrak{s}_1 \times \mathfrak{s}_2 \times \ldots \times \mathfrak{s}_n \subseteq \mathfrak{R}_1 \times \mathfrak{R}_2 \times \ldots \times \mathfrak{R}_n\}$ where $\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z})$ is the membership value such that $\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) \in [0,1]$.*

Definition 2.5 *[1] Let $\mathfrak{M}$ be an universal set and $(\tilde{I}, \Delta_1)$ and $(\tilde{J}, \Delta_2)$ be two $FHSs$'s over $\mathfrak{M}$. Then $(\tilde{I}, \Delta_1)$ is the fuzzy hypersoft subset of $(\tilde{J}, \Delta_2)$ if $\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) \leq \mu_{\tilde{J}(\mathfrak{r})}(\mathfrak{z})$. It is denoted by $(\tilde{I}, \Delta_1) \subseteq (\tilde{J}, \Delta_2)$.*

Definition 2.6 *[1] Let $\mathfrak{Z}$ be an universal set and $(\tilde{I}, \Delta_1)$ and $(\tilde{J}, \Delta_2)$ be $FHSs$'s over $\mathfrak{Z}$. $(\tilde{I}, \Delta_1)$ is equal to $(\tilde{J}, \Delta_1)$ if $\mu_{\tilde{I}(\ \mathfrak{r}))}(\mathfrak{z}) = \mu_{\tilde{J}(\ \mathfrak{r}))}(\mathfrak{z})$.*

Definition 2.7 *[1] A $FHSs$ $(\tilde{I}, \Delta)$ over the universe set $\mathfrak{Z}$ is said to be null fuzzy hypersoft set if $\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) = 0$, $\forall \mathfrak{r} \in \Delta$ and $\mathfrak{z} \in \mathfrak{Z}$. It is denoted by $\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}$. A $FHSs$ $(\tilde{J}, \Delta)$ over the universal set $\mathfrak{Z}$ is said to be absolute fuzzy hypersoft set if $\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) = 1$ $\forall \mathfrak{r} \in \Delta$ and $\mathfrak{z} \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{Z},\mathfrak{R})}$. Clearly, $\tilde{0}^c_{(\mathfrak{Z},\mathfrak{R})} = \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}$ and $\tilde{1}^c_{(\mathfrak{Z},\mathfrak{R})} = \tilde{0}_{(\mathfrak{Z},\mathfrak{R})}$.*

Definition 2.8 *[1] Let $\mathfrak{Z}$ be an universal set and $(\tilde{I}, \Delta)$ be $FHSs$ over $\mathfrak{Z}$. $(\tilde{I}, \Delta)^C$ is the complement of $(\tilde{I}, \Delta)$ if $\mu^C_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) = \tilde{1}^c_{(\mathfrak{Z},\mathfrak{R})} - \mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z})$ where $\forall \mathfrak{r} \in \Delta$ and $\forall \mathfrak{z} \in \mathfrak{Z}$. It is clear that $((\tilde{I}, \Delta)^C)^C = (\tilde{I}, \Delta)$.*

Definition 2.9 *[1] Let $\mathfrak{Z}$ be the universal set and $(\tilde{I}, \Delta_1)$ and $(\tilde{J}, \Delta_2)$ be $FHSs$'s over $\mathfrak{Z}$. Extended union $(\tilde{I}, \Delta_1) \cup (\tilde{J}, \Delta_2)$ is defined as*

$$\mu\left((\tilde{I}, \Delta_1) \cup (\tilde{J}, \Delta_2)\right) = \begin{cases} \mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) & \text{if} \quad q \in \Delta_1 - \Delta_2 \\ \mu_{\tilde{J}(\mathfrak{r})}(\mathfrak{z}) & \text{if} \quad \mathfrak{r} \in \Delta_2 - \Delta_1 \\ \max\{\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}), \mu_{\tilde{J}(\mathfrak{r})}(\mathfrak{z})\} & \text{if } \mathfrak{r} \in \Delta_1 \cap \Delta_2 \end{cases}$$

Definition 2.10 *[1, 3] Let $\mathfrak{Z}$ be the universal set and $(\tilde{I}, \Delta_1)$ and $(\tilde{J}, \Delta_2)$ be $FHSs$'s over $\mathfrak{Z}$. Extended intersection $(\tilde{I}, \Delta_1) \cap (\tilde{J}, \Delta_2)$ is defined as*

$$\mu((\tilde{I}, \Delta_1) \cap (\tilde{J}, \Delta_2)) = \begin{cases} \mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}) & if \mathfrak{r} \in \Delta_1 - \Delta_2 \\ \mu_{\tilde{J}(\mathfrak{r})}(\mathfrak{z}) & if \mathfrak{r} \in \Delta_2 - \Delta_1 \\ \min\{\mu_{\tilde{I}(\mathfrak{r})}(\mathfrak{z}), \mu_{\tilde{J}(\mathfrak{r})}(\mathfrak{z})\} & if \mathfrak{r} \in \Delta_1 \cap \Delta_2 \end{cases}$$

Definition 2.11 *[3] Let $(\mathfrak{Z}, \mathfrak{R})$ be the family of all $FHSs$'s over the universe set $\mathfrak{Z}$ and $\tau \subseteq FHSs(\mathfrak{Z}, \mathfrak{R})$. Then τ is said to be a fuzzy hypersoft topology (briefly, $FHSt$) on $\mathfrak{Z}$ if*

1. $\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}$ and $\tilde{1}_{(\mathfrak{Z},\mathfrak{R})}$ belongs to τ
2. the union of any number of $FHSs$'s in τ belongs to τ
3. the intersection of finite number of $FHSs$'s in τ belongs to τ.

Then $(\mathfrak{Z}, \mathfrak{R}, \tau)$ is called a fuzzy hypersoft toplogical space (briefly, $FHSts$) over $\mathfrak{Z}$. Each member of τ is said to be fuzzy hypersoft open set (briefly, $FHSos$). A $FHSs$ $(\tilde{I},\Delta)$ is called a fuzzy hypersoft closed set (briefly, $FHScs$) if its complement $(\tilde{I},\Delta)^C$ is $FHSos$.

Definition 2.12 *[3] Let* $(\mathfrak{Z},\mathfrak{R},\tau)$ *be a* $FHSts$ *over* $\mathfrak{Z}$ *and* $(\tilde{I},\Delta)$ *be a* $FHSs$ *in* $\mathfrak{Z}$. *Then*
1. the fuzzy hypersoft interior (briefly, $FHSint$) of $(\tilde{I},\Delta)$ is defined as $FHSint(\tilde{I},\Delta) = \cup \{(\tilde{J},\Delta): (\tilde{J},\Delta) \subseteq (\tilde{I},\Delta)$ where $(\tilde{J},\Delta)$ is $FHSos\}$.
2. the fuzzy hypersoft closure (briefly, $FHScl$) of $(\tilde{I},\Delta)$ is defined as $FHScl(\tilde{I},\Delta) = \cap \{(\tilde{J},\Delta): (\tilde{J},\Delta) \supseteq (\tilde{I},\Delta)$ where $(\tilde{J},\Delta)$ is $FHScs\}$.

Definition 2.13 *[4] Let* $(\mathfrak{Z},\mathfrak{R},\tau)$ *be a* $FHSts$ *over* $\mathfrak{Z}$ *and* $(\tilde{I},\Delta)$ *be a* $FHSs$ *in* $\mathfrak{Z}$. *Then,* $(\tilde{I},\Delta)$ *is called the fuzzy hypersoft semiopen set (briefly,* $FHSSos$*) if* $(\tilde{I},\Delta) \subseteq FHScl(int(\tilde{I},\Delta))$. *A* $FHSs$ $(\tilde{I},\Delta)$ *is called a fuzzy hypersoft semiclosed set (briefly,* $FHSScs$*) if its complement* $(\tilde{I},\Delta)^c$ *is a* $FHSSos$.

Definition 2.14 *[2] Let* $(\mathfrak{Z},\mathfrak{L})$ *and* $(\mathfrak{Y},\mathfrak{M})$ *be classes of* $FHSs$*'s over* $\mathfrak{Z}$ *and* $\mathfrak{Y}$ *with attributes* $\mathfrak{L}$ *and* $\mathfrak{M}$ *respectively. Let* $\omega: \mathfrak{Z} \to \mathfrak{Y}$ *and* $\nu: \mathfrak{L} \to \mathfrak{M}$ *be mappings. Then a* FHS *mappings* $\mathfrak{h} = (\omega,\nu): (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$ *is defined as follows, for a* $FHSs$ $(\tilde{I},\Delta)_A$ *in* $(\mathfrak{Z},\mathfrak{L})$, $f(\tilde{I},\Delta)_A$ *is a* $FHSs$ *in* $(\mathfrak{Y},\mathfrak{M})$ *obtained as follows, for* $\beta \in \nu(\mathfrak{L}) \subseteq \mathfrak{M}$ *and* $\mathfrak{y} \in \mathfrak{y}$, $\mathfrak{h}(\tilde{I},\Delta)_A(\beta)(\mathfrak{y}) = \bigcup_{\alpha\in\nu^{-1}(\beta)\cap\ A, s\in\omega^{-1}(\mathfrak{y})} (\alpha)\mu_s \mathfrak{h}(\tilde{I},\Delta)_A$ *is called a fuzzy hypersoft image of a* $FHSs$ $(\tilde{I},\Delta)$. *Hence* $((\tilde{I},\Delta)_A, \mathfrak{h}(\tilde{I},\Delta)_A) \in \mathfrak{h}$, *where* $(\tilde{I},\Delta)_A \subseteq (\mathfrak{Z},\mathfrak{L}), \mathfrak{h}(\tilde{I},\Delta)_A \subseteq (\mathfrak{Y},\mathfrak{M})$.

Definition 2.15 *[2] If* $\mathfrak{h}: (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$ *be a* FHS *mapping, then* FHS *class* $(\mathfrak{Z},\mathfrak{L})$ *is called the domainof* $\mathfrak{h}$ *and the* FHS *class* $(\tilde{J},\Delta) \in (\mathfrak{Y},\mathfrak{M}): (\tilde{J},\Delta) = \mathfrak{h}(\tilde{I},\Delta))$ *for some* $(\tilde{I},\Delta) \in (\mathfrak{Z},\mathfrak{L})$ *is called the range of* $\mathfrak{h}$. *The* FHS *class* $(\mathfrak{Y},\mathfrak{M})$ *is called co-domain of* $\mathfrak{h}$.

Definition 2.16 *[2] If* $\mathfrak{h}: (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$ *be a* FHS *mapping and* $(\tilde{J},\Delta)_B$, *a* $FHSs$ *in* $(\mathfrak{Y},\mathfrak{Z})$ *where* $\omega: \mathfrak{Z} \to \mathfrak{Y}$, $\nu: \mathfrak{L} \to \mathfrak{M}$ *and* $B \subseteq \mathfrak{M}$. *Then* $\mathfrak{h}^{-1}(\tilde{J},\Delta)_B$ *is a* $FHSs$ *in* $(\mathfrak{Z},\mathfrak{L})$ *defined as follows, for* $\alpha \in \nu^{-1}(B) \subseteq \mathfrak{L}$ *and* $\mathfrak{z} \in \mathfrak{Z}$, $\mathfrak{h}^{-1}(\tilde{J},\Delta)_B(\alpha)(\mathfrak{z}) = (\nu(\alpha))\mu_p(\mathfrak{z})\mathfrak{h}^{-1}(\tilde{J},\Delta)_B$ *is called a* FHS *inverse image of* $(\tilde{J},\Delta)_B$.

Definition 2.17 *[2] Let* $\mathfrak{h} = (\omega,\nu)$ *be a* FHS *mapping of a* FHS *class* $(\mathfrak{Z},\mathfrak{L})$ *into a* FHS *class* $(\mathfrak{Y},\mathfrak{M})$. *Then*
1. $\mathfrak{h}$ is said to be a one-one (or injection) FHS mapping if for both $\omega: \mathfrak{Z} \to \mathfrak{Y}$ and $\nu: \mathfrak{L} \to \mathfrak{M}$ are one-one.
2. $\mathfrak{h}$ is said to be a onto (or surjection) FHS mapping if for both $\omega: \mathfrak{Z} \to \mathfrak{Y}$ and $\nu: \mathfrak{L} \to \mathfrak{M}$ are onto.

If $\mathfrak{h}$ is both one-one and onto, then $\mathfrak{h}$ is called a one-one onto (or bijective) correspondance of FHS mapping.

Definition 2.18 *[2] If* $\mathfrak{h} = (\omega,\nu): (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$ *and* $g = (\mathfrak{m},\mathfrak{y}): (\mathfrak{Y},\mathfrak{M}) \to (\mathfrak{X},\mathfrak{Y})$ *are two* FHS *mappings, then their composite* $\mathfrak{g} \circ \mathfrak{h}$ *is a* FHS *mapping of* $(\mathfrak{Z},\mathfrak{L})$ *into* $(\mathfrak{X},\mathfrak{Y})$ *such that for every* $(\tilde{I},\Delta)_A \in (\mathfrak{Z},\mathfrak{L})$, $(\mathfrak{g} \circ \mathfrak{h})(\tilde{I},\Delta)_A = \mathfrak{g}(\mathfrak{h}(\tilde{I},\Delta)_A)$. *For* $\beta \in \mathfrak{y}(\mathfrak{M}) \subseteq \mathfrak{Y}$ *and* $\mathfrak{x} \in \mathfrak{X}$, *it is defined as* $\mathfrak{g}(\mathfrak{h}(\tilde{I},\Delta)_A(\beta)(\mathfrak{x}) = \bigcup_{\alpha\in\mathfrak{y}^{-1}(\beta)\cap\ \mathfrak{h}(A), s\in\mathfrak{z}^{-1}(\mathfrak{x})} (\alpha)\mu_s$.

Definition 2.19 *[2] Let* $\mathfrak{h} = (\omega,\nu)$ *be a* FHS *mapping where* $\omega: \mathfrak{Z} \to \mathfrak{Z}$ *and* $\nu: \mathfrak{L} \to \mathfrak{L}$. *Then* $\mathfrak{h}$ *is said to be a* FHS *identity mapping if for both* ω *and* ν *are identity mappings.*

Definition 2.20 *[2] A one-one onto* FHS *mapping* $\mathfrak{h} = (\omega,\nu): (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$ *is called* FHS *invertable mapping. Its* FHS *inverse mapping is denoted by* $\mathfrak{h}^{-1} = (\omega^{-1},\nu^{-1}): (\mathfrak{Z},\mathfrak{L}) \to (\mathfrak{Y},\mathfrak{M})$.

3 FUZZY HYPERSOFT CONTRA M CONTINUOUS MAPS

In this section, fuzzy hypersoft contra M continuous maps are introduced and its related properties are discusssed.

Definition 3.1 *Let* $(\mathfrak{Z},\mathfrak{L},\tau)$ *be a* $FHSts$ *over* $\mathfrak{Z}$ *and* $(\tilde{I},\Delta)$ *be a* $FHSs$ *on* $\mathfrak{Z}$. *Then the fuzzy hypersoft*
1. θ-interior (briefly, $FHS\theta int$) of $(\tilde{I},\Delta)$ is defined by
$$FHS\theta int(\tilde{I},\Delta) = \cup \quad \{FHSint(\tilde{J},\Delta): (\tilde{J},\Delta) \subseteq (\tilde{I},\Delta) \text{ and } (\tilde{J},\Delta) \text{ is a } FHScs \text{ in } \mathfrak{Z}\}$$
2. θ-closure (briefly, $FHS\theta cl$) of $(\tilde{I},\Delta)$ is defined by
$$FHS\theta cl(\tilde{I},\Delta) = \cap \quad \{FHSHScl(\tilde{J},\Delta): (\tilde{I},\Delta) \subseteq (\tilde{J},\Delta) \text{ and } (\tilde{J},\Delta) \text{ is a } FHSos \text{ in } \mathfrak{Z}\}$$

Definition 3.2 *Let* $(\mathfrak{Z},\mathfrak{R},\tau)$ *be a* $FHSts$ *over* $\mathfrak{Z}$. *A* $FHSs$ $(\tilde{I},\Delta)$ *is said to be a neutrosophic hypersoft*

1. θ-open set (briefly, $FHS\theta os$) if $(\tilde{I},\Delta) = FHS\theta int(\tilde{I},\Delta)$
2. θ-pre open set (briefly, $FHS\theta\mathcal{P}os$) if $(\tilde{I},\Delta) \subseteq FHSint(FHS\theta cl(\tilde{I},\Delta))$
3. θ-semi open set (briefly, $FHS\theta\mathcal{S}os$) if $(\tilde{I},\Delta) \subseteq FHScl(FHS\theta int(\tilde{I},\Delta))$
4. M- open set(briefly, $FHSMos$) if $(\tilde{I},\Delta) \subseteq FHScl(FHS\theta int(\tilde{I},\Delta)) \cup N_sHSint(FHS\delta cl(\tilde{I},\Delta))$

The complement of $FHS\theta os$ (resp. $FHS\theta\mathcal{P}os$, $FHS\theta\mathcal{S}os$ & $FHSMos$) is called a $FHS\theta$ (resp. $FHS\theta$ pre , $FHS\theta$ semi & $FHSM$) closed set (briefly, $FHS\theta cs$ (resp. $FHS\mathcal{P}cs$, $FHS\theta\mathcal{P}cs$, $FHS\theta\mathcal{S}cs$ & $FHSMcs$)) in $\mathfrak{Z}$.

The family of all $FHS\theta os$ (resp. $FHS\theta cs$, $IHS\theta\mathcal{P}os$, $FHS\theta\mathcal{P}cs$, $FHS\theta\mathcal{S}os$, $FHS\theta\mathcal{S}cs$ &$FHSMcs$) of $\mathfrak{Z}$ is denoted by $FHS\theta OS(\mathfrak{Z})$ (resp. $FHS\theta CS(\mathfrak{Z})$, $FHS\theta\mathcal{P}OS(\mathfrak{Z})$, $IHS\theta\mathcal{P}CS(\mathfrak{Z})$, $FHS\theta\mathcal{S}OS(\mathfrak{Z})$, $FHS\theta\mathcal{S}CS(\mathfrak{Z})$, $FHSMOS(\mathfrak{Z})$ & $FHSMCS(\mathfrak{Z})$).

Definition 3.3 *Let $(\mathfrak{Z},\mathfrak{R},\tau)$ be a $FHSts$ over $\mathfrak{Z}$ and $(\tilde{I},\Delta)$ be a $FHSs$ on $\mathfrak{Z}$. Then the FHS*

1. M-interior (briefly, $FHSMint$) of $(\tilde{I},\Delta)$ is defined by
$$FHSMint(\tilde{I},\Delta)=\cup \quad \{(\tilde{J},\Delta): (\tilde{J},\Delta) \subseteq (\tilde{I},\Delta) \text{ \& } (\tilde{J},\Delta) \text{ is a } FHSMos \text{ in } \mathfrak{Z}\}$$
2. M-closure (briefly, $FHSMcl$) of $(\tilde{I},\Delta)$ is defined by
$$FHSMcl(\tilde{I},\Delta)=\cap \quad \{(\tilde{J},\Delta): (\tilde{I},\Delta) \subseteq (\tilde{J},\Delta) \text{ \& } (\tilde{J},\Delta) \text{ is a } FHSMcs \text{ in } \mathfrak{Z}\}$$

Definition 3.4 *Consider any two FHSts $(\mathfrak{Z},\mathfrak{L},\tau)$ and $(\mathfrak{Y},\mathfrak{M},\sigma)$. A map $\mathfrak{h}: (\mathfrak{Z},\mathfrak{L},\tau) \to (\mathfrak{Y},\mathfrak{M},\sigma)$ is said to be FHS*

1. θ continuous (briefly, $FHS\theta Cts$) if the inverse image of each $FHSos$ in $(\mathfrak{Y},\mathfrak{M},\sigma)$ is a $FHS\theta os$ in $(\mathfrak{Z},\mathfrak{L},\tau)$
2. $\theta\mathcal{S}$ continuous (briefly, $FHS\theta\mathcal{S}Cts$) if the inverse image of each $FHSos$ in $(\mathfrak{Y},\mathfrak{M},\sigma)$ is a $FHS\theta\mathcal{S}os$ in $(\mathfrak{Z},\mathfrak{L},\tau)$
3. M continuous (briefly, $FHSMCts$) if the inverse image of each $FHSos$ in $(\mathfrak{Y},\mathfrak{M},\sigma)$ is a $FHSMos$ in $(\mathfrak{Z},\mathfrak{L},\tau)$

Definition 3.5 *A mapping $\mathfrak{h}: (\mathfrak{Z},\mathfrak{L},\tau) \to (\mathfrak{Y},\mathfrak{M},\sigma)$ is said to be fuzzy hypersoft contra M- continuous (resp. continuous, θ continuous, θ semi continuous , θ pre-continuous, δ continuous, δ- semi continuous, δ-pre continuous &e-continuous) (in short $FHScontraMCts$) (resp. $FHScontra\theta Cts$, $FHScontra\theta\mathcal{S}Ct$, $FHScontra\theta\mathcal{P}Cts$, $FHScontra\delta Cts$, $FHScontra\delta\mathcal{S}Cts$, $FHScontra\delta\mathcal{P}Cts$ & $FHScontrae - Cts$) if the image of each FHS open set of $(\mathfrak{Y},\mathfrak{M},\sigma)$ is $FHSMcs$ (resp. $FHS\theta os$, $FHS\theta\mathcal{S}cs, FHS\theta\mathcal{P}cs$, $FHS\delta cs$, $FHS\delta\mathcal{S}cs, FHS\delta\mathcal{P}cs$ & $FHSecs$) in $(\mathfrak{Z},\mathfrak{L},\tau)$*

Example 3.1 *Let $\mathfrak{Z} = \{\mathfrak{z}_1,\mathfrak{z}_2\}$ and $\mathfrak{Y} = \{\mathfrak{y}_1,\mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ and $\mathfrak{M} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$ respectively. The attributes are given as:*

$\mathfrak{R}_1 = \{a_1,a_2,a_3\}, \mathfrak{R}_2 = \{b_1,b_2\}$
$\mathfrak{R}_{1'} = \{c_1,c_2,c_3\}, \mathfrak{R}_{2'} = \{d_1,d_2\}$.

,, Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{M})$ be the classes of FHS sets. Let the $FHSs$'s

$(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_1),(\tilde{I}_3,\Delta_3),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3),(\tilde{I}_7,\Delta_3)$ and $(\tilde{I}_8,\Delta_3)$, over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1) = \begin{Bmatrix} \langle (a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle, \\ \langle (a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2) = \begin{Bmatrix} \langle (a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle, \\ \langle (a_1,b_1),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_2) = \begin{Bmatrix} \langle (a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle, \\ \langle (a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle \end{Bmatrix}$$

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$$(\tilde{I}_4, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_5, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_6, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_7, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},Q)}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3), (\tilde{I}_7, \Delta_3)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{J}_1, \Delta_2)$ and $(\tilde{J}_2, \Delta_2)$ over the universe $\mathfrak{Y}$ be

$$(\tilde{J}_1, \Delta_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.7}, \frac{\mathfrak{y}_2}{0.8}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{y}_1}{0.6}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{J}_2, \Delta_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.3}, \frac{\mathfrak{y}_2}{0.2}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{y}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},Q)}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_2), (\tilde{J}_2, \Delta_2)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{M})$ be a FHS mapping as follows:

$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$

$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$

$$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_2) = (\tilde{I}_2, \Delta_2)^c, \mathfrak{h}^{-1}(\tilde{J}_2, \Delta_2) = (\tilde{I}_3, \Delta_2)^c,$$

$\therefore \mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{Z})$ is $FHScontraMCts$.

Proposition 3.1 *The statements hold but the converse is not.*

1. Each $FHScontra\theta Cts$ is a $FHScontra\theta\mathcal{S}Cts$.
2. Each $FHScontra\theta Cts$ is a $FHScontraCts$.
3. Each $FHScontra\theta\mathcal{S}Cts$ is a $FHScontraMCts$.
4. Each $FHScontra\delta Cts$ is a $FHScontraCts$.
5. Each $FHScontra\delta Cts$ is a $FHScontra\delta\mathcal{P}Cts$.
6. Each $FHScontra\delta Cts$ is a $FHScontra\delta\mathcal{S}Cts$.
7. Each $FHScontra\delta\mathcal{P}Cts$ is a $FHScontraMCts$.
8. Each $FHScontra\delta\mathcal{S}Cts$ is a $FHScontraeCts$.
9. Each $FHScontraMCts$ is a $FHScontraeCts$.

Proof.

Consider the map $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{Z}, \sigma)$

1. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\theta Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\theta cs$ in $\mathfrak{Z}$. Since all $FHS\theta cs$ are $FHS\theta\mathcal{S}cs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\theta\mathcal{S}cs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontra\theta\mathcal{S}Cts$.
2. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\theta Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\theta cs$ in $\mathfrak{Z}$. Since all $FHS\theta cs$ are $FHScs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHScs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraCts$.
3. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\theta\mathcal{S}Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\theta\mathcal{S}cs$ in $\mathfrak{Z}$. Since all $FHS\theta\mathcal{S}cs$ are $FHSMcs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHSMcs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraMCts$.
4. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\delta Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta cs$ in $\mathfrak{Z}$. Since all $FHS\delta cs$ are $FHScs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHScs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraCts$.
5. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\delta Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta cs$ in $\mathfrak{Z}$. Since all $FHS\delta cs$ are $FHS\delta\mathcal{P}cs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta\mathcal{P}cs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontra\delta\mathcal{P}Cts$.
6. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\delta Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta cs$ in $\mathfrak{Z}$. Since all $FHS\delta cs$ are $FHS\delta\mathcal{S}cs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta\mathcal{S}cs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontra\delta\mathcal{S}Cts$.
7. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\delta\mathcal{P}Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta\mathcal{P}cs$ in $\mathfrak{Z}$. Since all $FHS\delta\mathcal{P}cs$ are $FHSMcs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHSMcs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraMCts$.
8. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontra\delta\mathcal{S}Cts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHS\delta\mathcal{S}cs$ in $\mathfrak{Z}$. Since all $FHS\delta\mathcal{S}cs$ are $FHSecs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHSecs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraeCts$.
9. Let $(\tilde{J}, \Delta)$ be a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontraMCts$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHSMcs$ in $\mathfrak{Z}$. Since all $FHSMcs$ are $FHSecs$, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is $FHSecs$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is a $FHScontraeCts$.

Example 3.2 *Let $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2\}$ and $\mathfrak{Y} = \{\mathfrak{y}_1, \mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ and $\mathfrak{Z} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$, respectively. The attributes are given as:*

$\mathfrak{R}_1 = \{a_1, a_2, a_3\}, \mathfrak{R}_2 = \{b_1, b_2\}$

$\mathfrak{R}_{1'} = \{c_1, c_2, c_3\}, \mathfrak{R}_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{Z}, \mathfrak{L})$ and $(\mathfrak{Y}, \mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3)$ and $(\tilde{I}_7, \Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1, \Delta_1) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\}\rangle \end{matrix}\right\}$$

$$(\tilde{I}_2, \Delta_2) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\}\rangle \end{matrix}\right\}$$

$$(\tilde{I}_3, \Delta_2) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\}\rangle \end{matrix}\right\}$$

$$(\tilde{I}_4, \Delta_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\}\rangle \end{matrix}\right\}$$

$$(\tilde{I}_5, \Delta_3) = \left\{\begin{matrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\}\rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\}\rangle \end{matrix}\right\}$$

$$(\tilde{I}_6,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_7,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})},\tilde{1}_{(\mathfrak{Z},\mathfrak{R})},(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3),(\tilde{I}_7,\Delta_3)\}$ is $FHSts$.
Let the $FHSs$ $(\tilde{J}_1,\Delta_3)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1,\Delta_3)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{y}_1}{0.3},\frac{\mathfrak{y}_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{y}_1}{0.5},\frac{\mathfrak{y}_2}{0.3}\}\rangle,\\ \langle(c_2,d_2),\{\frac{\mathfrak{y}_1}{0.4},\frac{\mathfrak{y}_2}{0.5}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})},\tilde{1}_{(\mathfrak{Y},\mathfrak{R})},(\tilde{J}_1,\Delta_3)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{Z},\mathfrak{L})\to(\mathfrak{Y},\mathfrak{Z})$ be a FHS mapping as follows:
$\omega(\mathfrak{z}_1)=\mathfrak{y}_2,\omega(\mathfrak{z}_2)=\mathfrak{y}_1,$
$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$
$\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_3)=(\tilde{I}_6,\Delta_3)^c$

$\mathfrak{h}$ is $FHScontra\theta SCts$ but not $FHScontra\theta Cts$ because $(\tilde{J}_1,\Delta_3)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_3)=(\tilde{I}_6,\Delta_3)^c$ is a $FHS\theta Scs$ but not $FHS\theta cs$ in $\mathfrak{Z}$

Example 3.3 *Let* $\mathfrak{Z}=\{\mathfrak{z}_1,\mathfrak{z}_2\}$ *and* $\mathfrak{Y}=\{\mathfrak{y}_1,\mathfrak{y}_2\}$ *be the* FHS *initial universes and the attributes be* $\mathfrak{L}=\mathfrak{R}_1\times\mathfrak{R}_2$ *and* $\mathfrak{Z}=\mathfrak{R}_{1'}\times\mathfrak{R}_{2'}$ *respectively. The attributes are given as:*
$\mathfrak{R}_1=\{a_1,a_2,a_3\},\mathfrak{R}_2=\{b_1,b_2\}$
$\mathfrak{R}_{1'}=\{c_1,c_2,c_3\},\mathfrak{R}_{2'}=\{d_1,d_2\}.$

Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3)$ and $(\tilde{I}_7,\Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_4,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_5,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_6,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_7,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})},\tilde{1}_{(\mathfrak{Z},\mathfrak{R})},(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3),(\tilde{I}_7,\Delta_3)\}$ is $FHSts$.
Let the $FHSs\ (\tilde{J}_1,\Delta_1)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1,\Delta_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{y}_1}{0.4},\frac{\mathfrak{y}_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{y}_1}{0.5},\frac{\mathfrak{y}_2}{0.3}\}\rangle,\\ \end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})},\tilde{1}_{(\mathfrak{Y},\mathfrak{R})},(\tilde{J}_1,\Delta_1)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{Z},\mathfrak{L})\to(\mathfrak{Y},\mathfrak{R})$ be a FHS mapping as follows:
$\omega(\mathfrak{z}_1)=\mathfrak{y}_2,\omega(\mathfrak{z}_2)=\mathfrak{y}_1,$
$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$
$\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_1,\Delta_1)^c$

$\mathfrak{h}$ is $FHScontraCts$ but not $FHScontra\theta Cts$ because $(\tilde{J}_1,\Delta_1)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_1,\Delta_1)^c$ is a $FHScs$ but not $FHS\theta cs$ in $\mathfrak{Z}$.

Example 3.4 *Let* $\mathfrak{Z}=\{\mathfrak{z}_1,\mathfrak{z}_2\}$ *and* $\mathfrak{Y}=\{\mathfrak{y}_1,\mathfrak{y}_2\}$ *be the* FHS *initial universes and the attributes be* $\mathfrak{L}=\mathfrak{R}_1\times\mathfrak{R}_2$ *and* $\mathfrak{Z}=\mathfrak{R}_{1'}\times\mathfrak{R}_{2'}$, *respectively. The attributes are given as:*
$\mathfrak{R}_1=\{a_1,a_2,a_3\},\mathfrak{R}_2=\{b_1,b_2\}$
$\mathfrak{R}_{1'}=\{c_1,c_2,c_3\},\mathfrak{R}_{2'}=\{d_1,d_2\}$.

Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3)$ and $(\tilde{I}_7,\Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_4, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_5, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_6, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_7, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3), (\tilde{I}_7, \Delta_3)\}$ is $FHSts$.
Let the $FHSs$ $(\tilde{J}_1, \Delta_1)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1, \Delta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.4}, \frac{\mathfrak{y}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.5}, \frac{\mathfrak{y}_2}{0.3}\} \rangle, \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{Z})$ be a FHS mapping as follows:
$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$
$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$
$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_1) = (\tilde{I}_1, \Delta_1)^c$

$\mathfrak{h}$ is $FHScontraMCts$ but not $FHScontra\theta SCts$ because $(\tilde{J}_1, \Delta_1)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_1) = (\tilde{I}_1, \Delta_1)^c$ is a $FHSMcs$ but not $FHS\theta Scs$ in $\mathfrak{Z}$.

Example 3.5 *Let $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2\}$ and $\mathfrak{Y} = \{\mathfrak{y}_1, \mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ and $\mathfrak{Z} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$, respectively. The attributes are given as:*
$\mathfrak{R}_1 = \{a_1, a_2, a_3\}, \mathfrak{R}_2 = \{b_1, b_2\}$
$\mathfrak{R}_{1'} = \{c_1, c_2, c_3\}, \mathfrak{R}_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{Z}, \mathfrak{L})$ and $(\mathfrak{Y}, \mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3)$ and $(\tilde{I}_7, \Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1, \Delta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_2, \Delta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

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$$(\tilde{I}_3, \Delta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_4, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_5, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_6, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_7, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3), (\tilde{I}_7, \Delta_3)\}$ is $FHSts$.

Let the $FHSs$ $(\tilde{J}_1, \Delta_3)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1, \Delta_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.3}, \frac{\mathfrak{y}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.5}, \frac{\mathfrak{y}_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{y}_1}{0.4}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{Z})$ be a FHS mapping as follows:

$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$

$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$

$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_3) = (\tilde{I}_6, \Delta_3)^c$

$\mathfrak{h}$ is $FHScontraCts$ but not $FHScontra\delta Cts$ because $(\tilde{J}_1, \Delta_3)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_3) = (\tilde{I}_6, \Delta_3)^c$ is a $FHScs$ but not $FHS\delta cs$ in $\mathfrak{Z}$.

Example 3.6 *Let $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2\}$ and $\mathfrak{Y} = \{\mathfrak{y}_1, \mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ and $\mathfrak{Z} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$, respectively. The attributes are given as:*

$\mathfrak{R}_1 = \{a_1, a_2, a_3\}, \mathfrak{R}_2 = \{b_1, b_2\}$

$\mathfrak{R}_{1'} = \{c_1, c_2, c_3\}, \mathfrak{R}_{2'} = \{d_1, d_2\}.$

Let $(\mathfrak{Z}, \mathfrak{L})$ and $(\mathfrak{Y}, \mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3)$ and $(\tilde{I}_7, \Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1, \Delta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_4,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_5,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_6,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_7,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})},\tilde{1}_{(\mathfrak{Z},\mathfrak{R})},(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3),(\tilde{I}_7,\Delta_3)\}$ is $FHSts$.
Let the $FHSs\ (\tilde{J}_1,\Delta_3)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1,\Delta_3)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{y}_1}{0.4},\frac{\mathfrak{y}_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{y}_1}{0.5},\frac{\mathfrak{y}_2}{0.3}\}\rangle,\\ \langle(c_2,d_2),\{\frac{\mathfrak{y}_1}{0.4},\frac{\mathfrak{y}_2}{0.5}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})},\tilde{1}_{(\mathfrak{Y},\mathfrak{R})},(\tilde{J}_1,\Delta_3)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{Z},\mathfrak{L})\to(\mathfrak{Y},\mathfrak{Z})$ be a FHS mapping as follows:

$\omega(\mathfrak{z}_1)=\mathfrak{y}_2,\omega(\mathfrak{z}_2)=\mathfrak{y}_1,$

$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$

$\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_3)=(\tilde{I}_7,\Delta_3)^c$

$\mathfrak{h}$ is $FHScontra\delta\mathcal{P}Cts$ but not $FHScontra\delta Cts$ because $(\tilde{J}_1,\Delta_3)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_3)=(\tilde{I}_7,\Delta_3)^c$ is a $FHS\delta\mathcal{P}cs$ but not $FHS\delta cs$.

Example 3.7 *Let* $\mathfrak{Z}=\{\mathfrak{z}_1,\mathfrak{z}_2\}$ *and* $\mathfrak{Y}=\{\mathfrak{y}_1,\mathfrak{y}_2\}$ *be the* FHS *initial universes and the attributes be* $\mathfrak{L}=\mathfrak{R}_1\times\mathfrak{R}_2$ *and* $\mathfrak{Z}=\mathfrak{R}_{1'}\times\mathfrak{R}_{2'}$*, respectively. The attributes are given as:*

$\mathfrak{R}_1=\{a_1,a_2,a_3\},\mathfrak{R}_2=\{b_1,b_2\}$

$\mathfrak{R}_{1'}=\{c_1,c_2,c_3\},\mathfrak{R}_{2'}=\{d_1,d_2\}.$

Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3)$ and $(\tilde{I}_7,\Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1, \Delta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_2, \Delta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_3, \Delta_2) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_4, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_5, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_6, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_7, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_2), (\tilde{I}_4, \Delta_3), (\tilde{I}_5, \Delta_3), (\tilde{I}_6, \Delta_3), (\tilde{I}_7, \Delta_3)\}$ is $FHSts$.

Let the $FHSs$ $(\tilde{J}_1, \Delta_3)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1, \Delta_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.3}, \frac{\mathfrak{y}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.5}, \frac{\mathfrak{y}_2}{0.3}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{y}_1}{0.4}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \rightarrow (\mathfrak{Y}, \mathfrak{Z})$ be a FHS mapping as follows:

$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$

$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$

$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_3) = (\tilde{I}_6, \Delta_3)^c$

$\mathfrak{h}$ is $FHScontra\delta SCts$ but not $FHScontra\delta Cts$ because $(\tilde{J}_1, \Delta_3)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_3) = (\tilde{I}_6, \Delta_3)^c$ is a $FHS\delta Scs$ but not $FHS\delta cs$.

Example 3.8 *Let* $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2\}$ *and* $\mathfrak{Y} = \{\mathfrak{y}_1, \mathfrak{y}_2\}$ *be the* FHS *initial universes and the attributes be* $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ *and* $\mathfrak{Z} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$*, respectively. The attributes are given as:*

$\mathfrak{R}_1 = \{a_1, a_2, a_3\}, \mathfrak{R}_2 = \{b_1, b_2\}$

$\mathfrak{R}_{1'} = \{c_1, c_2, c_3\}, \mathfrak{R}_{2'} = \{d_1, d_2\}.$

Let $(\mathfrak{Z}, \mathfrak{L})$ and $(\mathfrak{Y}, \mathfrak{Z})$ be the classes of FHS sets. Let the $FHSs$'s

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$(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\wedge_2),(\tilde{I}_3,\wedge_3),(\tilde{I}_4,\wedge_3),(\tilde{I}_5,\wedge_1)$ and $(\tilde{I}_6,\wedge_2)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_4,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_5,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.4}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.3},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})},\tilde{1}_{(\mathfrak{Z},\mathfrak{R})},(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_3),(\tilde{I}_4,\Delta_3)\}$ is $FHSts$.

Let the $FHSs$ $(\tilde{J}_1,\Delta_1)$ be defined as

$$(\tilde{J}_1,\Delta_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{y}_1}{0.6},\frac{\mathfrak{y}_2}{0.8}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{y}_1}{0.5},\frac{\mathfrak{y}_2}{0.7}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})},\tilde{1}_{(\mathfrak{Y},\mathfrak{R})},(\tilde{J}_1,\Delta_1)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{Z},\mathfrak{L})\to(\mathfrak{Y},\mathfrak{Z})$ be a FHS mapping as follows:

$\omega(\mathfrak{z}_1)=\mathfrak{y}_2,\omega(\mathfrak{z}_2)=\mathfrak{y}_1,$

$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$

$\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_5,\Delta_1)^c$

$\mathfrak{h}$ is $FHScontraMCts$ but not $FHScontra\delta\mathcal{P}Cts$ because $(\tilde{J}_1,\Delta_1)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_5,\Delta_1)^c$ is a $FHSMcs$ but not $FHS\delta\mathcal{P}cs$ in $\mathfrak{Z}$.

Example 3.9 *Let* $\mathfrak{Z}=\{\mathfrak{z}_1,\mathfrak{z}_2\}$ *and* $\mathfrak{Y}=\{\mathfrak{y}_1,\mathfrak{y}_2\}$ *be the* FHS *initial universes and the attributes be* $\mathfrak{L}=\mathfrak{R}_1\times\mathfrak{R}_2$ *and* $\mathfrak{M}=\mathfrak{R}_{1'}\times\mathfrak{R}_{2'}$ *respectively. The attributes are given as:*

$\mathfrak{R}_1=\{a_1,a_2,a_3\},\mathfrak{R}_2=\{b_1,b_2\}$

$\mathfrak{R}_{1'}=\{c_1,c_2,c_3\},\mathfrak{R}_{2'}=\{d_1,d_2\}.$

Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{M})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3)$ and $(\tilde{I}_7,\Delta_3)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_3,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$$(\tilde{I}_4,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$
$$(\tilde{I}_5,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.4}\}\rangle\end{Bmatrix}$$
$$(\tilde{I}_6,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.7}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$
$$(\tilde{I}_7,\Delta_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.6}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})},\tilde{1}_{(\mathfrak{Z},\mathfrak{R})},(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_2),(\tilde{I}_4,\Delta_3),(\tilde{I}_5,\Delta_3),(\tilde{I}_6,\Delta_3),(\tilde{I}_7,\Delta_3)\}$ is $FHSts$.
Let the $FHSs$ $(\tilde{J}_1,\Delta_1)$ over the universe $\mathfrak{Y}$ be defined as

$$(\tilde{J}_1,\Delta_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{z}_1}{0.4},\frac{\mathfrak{z}_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.3}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})},\tilde{1}_{(\mathfrak{Y},\mathfrak{R})},(\tilde{J}_1,\Delta_1)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{Z},\mathfrak{R})\to(\mathfrak{Y},\mathfrak{M})$ be a FHS mapping as follows:
$\omega(\mathfrak{z}_1)=\mathfrak{y}_2,\omega(\mathfrak{z}_2)=\mathfrak{y}_1,$
$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$
$\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_1,\Delta_1)^c$

$\mathfrak{h}$ is $FHScontraeCts$ but not $FHScontra\delta\mathcal{S}Cts$ because $(\tilde{J}_1,\Delta_1)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1,\Delta_1)=(\tilde{I}_1,\Delta_1)^c$ is a $FHSecs$ but not $FHS\delta\mathcal{P}cs$ in $\mathfrak{Z}$.

Example 3.10 *Let $\mathfrak{Z}=\{\mathfrak{z}_1,\mathfrak{z}_2\}$ and $\mathfrak{Y}=\{\mathfrak{y}_1,\mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L}=\mathfrak{R}_1\times\mathfrak{R}_2$ and $\mathfrak{M}=\mathfrak{R}_{1'}\times\mathfrak{R}_{2'}$ respectively. The attributes are given as:*
$\mathfrak{R}_1=\{a_1,a_2,a_3\},\mathfrak{R}_2=\{b_1,b_2\}$
$\mathfrak{R}_{1'}=\{c_1,c_2,c_3\},\mathfrak{R}_{2'}=\{d_1,d_2\}.$

Let $(\mathfrak{Z},\mathfrak{L})$ and $(\mathfrak{Y},\mathfrak{M})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1,\Delta_1),(\tilde{I}_2,\Delta_2),(\tilde{I}_3,\Delta_3),(\tilde{I}_4,\Delta_3)$ and $(\tilde{I}_5,\Delta_1)$ over the universe $\mathfrak{Z}$ be

$$(\tilde{I}_1,\Delta_1)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.8},\frac{\mathfrak{z}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{z}_1}{0.7},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$
$$(\tilde{I}_2,\Delta_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{z}_1}{0.2},\frac{\mathfrak{z}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{z}_1}{0.5},\frac{\mathfrak{z}_2}{0.5}\}\rangle\end{Bmatrix}$$

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$$(\tilde{I}_3, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_4, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.7}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_5, \Delta_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.4}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.3}, \frac{\mathfrak{z}_2}{0.5}\} \rangle, \\ \langle (a_1, b_2), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1), (\tilde{I}_2, \Delta_2), (\tilde{I}_3, \Delta_3), (\tilde{I}_4, \Delta_3)\}$ is $FHSts$.

Let the $FHSs$ $(\tilde{J}_1, \Delta_3)$ be defined as

$$(\tilde{J}_1, \Delta_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.6}, \frac{\mathfrak{n}_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.5}, \frac{\mathfrak{n}_2}{0.}\} \rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{y}_1}{0.5}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{M})$ be a FHS mapping as follows:

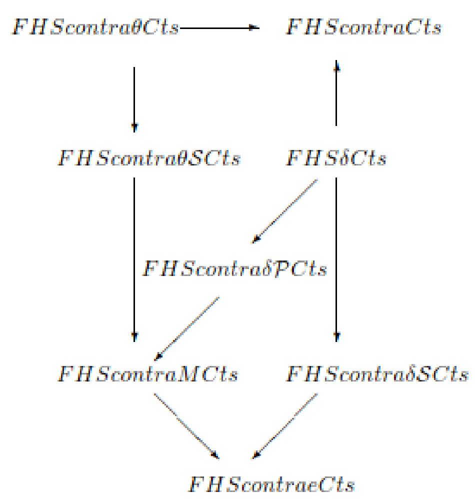
$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$

$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$

$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_3) = (\tilde{I}_5, \Delta_3)^{[}c]$

$\mathfrak{h}$ is $FHScontraeCts$ but not $FHScontraMCts$ because $(\tilde{J}_1, \Delta_1)$ is $FHSos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_1) = (\tilde{I}_1, \Delta_1)^c$ is a $FHSecs$ but not $FHSMcs$ in $\mathfrak{Z}$.

Remark 3.1 *From the results discussed above, the following diagram is obtained.*



Theorem 3.1 *A map* $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ *is* $FHScontraMCts$ *iff the inverse image of every* $FHScs$ *in* $\mathfrak{Y}$ *is* $FHSMos$ *in* $\mathfrak{Z}$.

Proof. consider a $FHScs(\tilde{J}, \wedge)$ in $\mathfrak{Y}$. Then $(\tilde{J}, \Delta)^c$ is $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHScontraMCts$, $\mathfrak{h}^{-1}((\tilde{J}, \Delta)^c)$ is $FHSMcs$ in $\mathfrak{Z}$. As $\mathfrak{h}^{-1}((\tilde{J}, \Delta)^c) = (\mathfrak{h}^{-1}(\tilde{J}, \Delta))^c, \mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is a $FHSMos$ in $\mathfrak{Z}$.

Conversely, Consider $FHScs(\tilde{J}, \Delta)$ in $\mathfrak{Y}$. So $(\tilde{J}, \Delta)^c$ is a $FHSos$ in $\mathfrak{Y}$. By hypothesis, $\mathfrak{h}^{-1}((\tilde{J}, \Delta)^c)$ is $FHSMcs$ in $\mathfrak{Z}$. As $\mathfrak{h}^{-1}((\tilde{J}, \Delta)^c) = (\mathfrak{h}^{-1}(\tilde{J}, \Delta))^c, (\mathfrak{h}^{-1}(\tilde{J}, \Delta))^c$ is $FHSMcs$ in $\mathfrak{Z}$. Hence $(\mathfrak{h}^{-1}(\tilde{J}, \Delta))$ is a $FHSMos$ in $\mathfrak{Z}$. Thus $\mathfrak{h}$ is $FHScontraMCts$.

Theorem 3.2 *Let* $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ *be a* $FHScontraMCts$ *map and* $\mathfrak{g}: (\mathfrak{Y}, M, \sigma) \to (P, N, \rho)$ *be a* $FHSCts$, *then* $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ *is a* $FHScontra\theta Cts$.

Proof. Let $(\widetilde{K}, \Delta)$ be a $FHSos$ in $\mathfrak{X}$. By hypothesis $\mathfrak{g}^{-1}(\widetilde{K}, \Delta)$ is a $FHSos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is a $FHScontraMCts$ map $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\widetilde{K}, \Delta))$ is a $FHSMcs$ in $\mathfrak{Z}$. Thus $\mathfrak{g} \circ \mathfrak{h}$ is a $FHScontraMCts$ map.

Theorem 3.3 *Let* $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ *be a* $FHScontraMCts$ *map. Then the succeeding conditions are hold:*

1. $\mathfrak{h}(FHSMcl(\tilde{I}, \Delta)) \geq FHSint(\mathfrak{h}(\tilde{I}, \Delta))$, for all $FHScs(\tilde{I}, \Delta)$ in $\mathfrak{M}$.
2. $FHSMcl(\mathfrak{h}^{-1}(\tilde{J}, \Delta)) \geq \mathfrak{h}^{-1}(FHSint(\tilde{J}, \Delta))$, for all $FHScs(\tilde{J}, \Delta)$ in $\mathfrak{Y}$.

Proof. (i) As $FHSMcl(\mathfrak{h}(\tilde{I}, \Delta))$ is a $FHSMcs$ in $\mathfrak{Y}$ and $\mathfrak{h}$ is $FHScontraMCts$, we have $\mathfrak{h}^{-1}(FHSMcl(\mathfrak{h}(\tilde{I}, \Delta)))$ is a $FHSMos$ in $\mathfrak{Z}$. Now, as $(\tilde{I}, \Delta) \geq \mathfrak{h}^{-1}(FHSint(\mathfrak{h}(\tilde{I}, \Delta))), FHSMcl(\tilde{I}, \Delta) \geq \mathfrak{h}^{-1}(FHSint(\mathfrak{h}(\tilde{I}, \Delta)))$. Therefore, $\mathfrak{h}(FHSM\ cl(\widetilde{H}, \wedge)) \geq FHSint(\mathfrak{h}(\tilde{I}, \Delta))$.

(ii) By replacing $(\tilde{I}, \Delta)$ with $(\tilde{J}, \Delta)$ in (i), we get $\mathfrak{h}(FHSMcl(\mathfrak{h}^{-1}(\tilde{J}, \Delta))) \geq FHSint(\mathfrak{h}(\mathfrak{h}^{-1}(\tilde{J}, \Delta))) \geq FHSint(\tilde{J}, \Delta)$. Hence, $FHSMcl(\mathfrak{h}^{-1}(\tilde{J}, \Delta)) \geq \mathfrak{h}^{-1}(FHSint(\tilde{J}, \Delta))$.

4 FUZZY HYPERSOFT CONTRA *M*-IRRESOLUTE MAPS

The Fuzzy hypersoft contra M- irresolute maps are introduced and some of its properties are discussed in this section.

Definition 4.1 *A map* $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ *is called a* $FHScontraM$*- irresolute (in short,* $FHScontraMIrr$*) map if* $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ *is a* $FHSMcs$ *in* $(\mathfrak{Z}, \mathfrak{L}, \tau)$ *for each* $FHS\theta os\ (\tilde{J}, \Delta)$ *of* $(\mathfrak{Y}, \mathfrak{M}, \sigma)$.

Theorem 4.1 *Let* $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ *be a* $FHScontraMIrr$ *map. Then* $\mathfrak{h}$ *is a* $FHScontraMCts$ *map. But the converse need not be true.*

Proof. Assume $\mathfrak{h}$ is a $FHScontraMIrr$ map. Consider a FHSos $(\tilde{J}, \Delta)$ in $\mathfrak{Y}$. As each $FHSos$ is a $FHSMos, (\tilde{J}, \Delta)$ is a $FHSMos$ in $\mathfrak{Y}$. By hypothesis, $\mathfrak{h}^{-1}(\tilde{J}, \Delta)$ is a $FHSMcs$ in $\mathfrak{M}$. Hence, $\mathfrak{h}$ is a

$FHScontraMCts$ map.

Example 4.1 *Let $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2\}$ and $\mathfrak{y} = \{\mathfrak{y}_1, \mathfrak{y}_2\}$ be the FHS initial universes and the attributes be $\mathfrak{L} = \mathfrak{R}_1 \times \mathfrak{R}_2$ and $\mathfrak{M} = \mathfrak{R}_{1'} \times \mathfrak{R}_{2'}$ respectively. The attributes are given as:*
$\mathfrak{R}_1 = \{a_1, a_2, a_3\}, \mathfrak{R}_2 = \{b_1, b_2\}$
$\mathfrak{R}_{1'} = \{c_1, c_2, c_3\}, \mathfrak{R}_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{Z}, \mathfrak{L})$ and $(\mathfrak{Y}, \mathfrak{M})$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{I}_1, \Delta_1)$ and $(\tilde{I}_2, \Delta_1)$ over the universe $\mathfrak{M}$ be

$$(\tilde{I}_1, \Delta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.2}, \frac{\mathfrak{z}_2}{0.3}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{I}_2, \Delta_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{z}_1}{0.8}, \frac{\mathfrak{z}_2}{0.7}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{z}_1}{0.5}, \frac{\mathfrak{z}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{Z},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Z},\mathfrak{R})}, (\tilde{I}_1, \Delta_1)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{J}_1, \Delta_1)$ and $(\tilde{J}_2, \Delta_1)$ over the universe $\mathfrak{Y}$ be

$$(\tilde{J}_1, \Delta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.7}, \frac{\mathfrak{y}_2}{0.8}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.6}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{J}_2, \Delta_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{y}_1}{0.3}, \frac{\mathfrak{y}_2}{0.2}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{y}_1}{0.4}, \frac{\mathfrak{y}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{Y},\mathfrak{R})}, \tilde{1}_{(\mathfrak{Y},\mathfrak{R})}, (\tilde{J}_1, \Delta_1), (\tilde{J}_2, \Delta_1)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{M})$ be a FHS mapping as follows:
$\omega(\mathfrak{z}_1) = \mathfrak{y}_2, \omega(\mathfrak{z}_2) = \mathfrak{y}_1,$
$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$

$$\mathfrak{h}^{-1}(\tilde{J}_1, \Delta_1) = (\tilde{I}_1, \Delta_1)^c, \mathfrak{h}^{-1}(\tilde{J}_2, \Delta_2) = (\tilde{I}_2, \Delta_2)^c,$$

$\therefore$ $\mathfrak{h} = (\omega, \nu): (\mathfrak{Z}, \mathfrak{L}) \to (\mathfrak{Y}, \mathfrak{M})$ is $FHScontraMCts$ but not $FHScontraMIrr$, because the set $(\tilde{J}_2, \Delta_2)$ is a $FHSMos$ in $\mathfrak{Y}$ but $\mathfrak{h}^{-1}(\tilde{J}_2, \Delta_2) = (\tilde{I}_2, \Delta_2)^c$ is not $FHSMcs$ in $\mathfrak{Z}$

Theorem 4.2 *Let $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ be a $FHScontraMIrr$ map and $\mathfrak{g}: (\mathfrak{Y}, \mathfrak{M}, \sigma) \to (\mathfrak{X}, \mathfrak{N}, \rho)$ be $FHSMCts$ map. Then $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{X}, \mathfrak{N}, \rho)$ is a $FHScontraMCts$ map.*

Proof. Consider a $FHSos$ $(\widetilde{K}, \Delta)$ in $\mathfrak{X}$. Then $\mathfrak{g}^{-1}(\widetilde{K}, \Delta)$ is a $FHSMos$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is a $FHScontraMIrr$ map, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\widetilde{K}, \Delta))$ is a $FHSMos$ in $\mathfrak{M}$. Hence $\mathfrak{g} \circ \mathfrak{h}$ $FHScontraMCts$ map.

Theorem 4.3 *Let $\mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{Y}, \mathfrak{M}, \sigma)$ and $\mathfrak{g}: (\mathfrak{Y}, \mathfrak{M}, \sigma) \to (\mathfrak{X}, \mathfrak{N}, \rho)$ be mappings. Then $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{Z}, \mathfrak{L}, \tau) \to (\mathfrak{X}, \mathfrak{N}, \rho)$ is [(i)]*

1. $FHScontra\theta Cts$, if $\mathfrak{h}$ is $FHSMIrr$ and $\mathfrak{g}$ is $FHScontraMCts$.
2. $FHSMIrr$, if $\mathfrak{h}$ is $FHSMIrr$ and $\mathfrak{g}$ is $FHScontraMIrr$.

Proof. 1. Consider a $FHSos(\widetilde{K}, \Delta)$ in $\mathfrak{X}$. Since $\mathfrak{g}$ is $FHScontraMCts$, $\mathfrak{g}(\widetilde{K}, \Delta)$ is a $FHSMcs$ in $\mathfrak{Y}$. As $\mathfrak{h}$ is $FHSMIrr$, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\widetilde{K}, \Delta))$ is a $FHSMcs$ in $\mathfrak{Z}$. Therefore, $\mathfrak{g} \circ \mathfrak{h}$ is a $FHSMCts$ map.

2. Consider a $FHSMos(\widetilde{K}, \Delta)$ in $\mathfrak{X}$. Since $\mathfrak{g}$ is $FHSMIrr$, $\mathfrak{g}^{-1}(\widetilde{K}, \Delta)$ is a $FHSMos$ in $\mathfrak{Y}$.

As $\mathfrak{h}$ is $FHScontraMIrr$, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\widetilde{K},\Delta))$ is a $FHSMcs$ in $\mathfrak{Z}$. Therefore, $\mathfrak{g}\circ\mathfrak{h}$ is a $FHScontraMIrr$ map.

5 CONCLUSION

In this paper, $FHScontraMCts$ map is defined using $FHScontraM-$ open set and its properties are analysed with examples. Then $FHScontraCts$ and some new kinds of contra continuous maps are compared with $FHScontraMCts$ maps. In addition, these maps are extended to $FHScontraM$-irresolute maps.

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