

Optimal Inventory Decisions Under Partial Trade Credit And Preservation Technology Investment With Carbon Emissions Considerations

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Abstract: This study pioneers a groundbreaking approach to sustainable inventory management by developing a novel two-warehouse inventory model that simultaneously optimizes adjustments to the selling price and investments in preservation technology can help mitigate losses and optimize profitability. under the complex interplay of partial trade credit, inflation, and stringent carbon emission regulations. Through a meticulous analysis of the intricate dynamics between these factors, our model provides actionable insights that empower businesses to strike a harmonious balance between economic profitability and Eco-conscious initiatives sustainability. The study's key findings have far-reaching implications for industries navigating the dual challenges of inventory management and environmental stewardship by providing insight into on the critical role of preservation technology also in sustainable inventory practices, this study offers a paradigm shift in strategic decision-making, enabling businesses to make informed choices that drive both economic growth and environmental responsibility.

This pioneering research provides insights into significantly towards the existing framework on sustainable stock management, Offering a comprehensive framework for businesses to optimize their inventory management practices while minimizing their environmental footprint. The insights gleaned from this study can inform the development of sustainable supply chain strategies, enabling companies to enhance their competitiveness while reducing their environmental impact.

Ultimately, this study's innovative approach to sustainable inventory management has the potential to transform the way businesses approach inventory management, encouraging a shift towards more environmentally responsible and economically viable practices that drive long-term sustainability and growth.

Keywords: Deterioration, Partial trade credit, Preservation technology, Inflation, Carbon Tax.

1. INTRODUCTION

The burgeoning imperative of sustainability has precipitated a paradigm shift in stock management, necessitating incorporating economic, ecological, and in social impact. In response to this shift, our study pioneers a novel two-warehouse inventory model that synergistically maximizes profitability through pricing and preservation technology investment, while navigating the complexities of partial trade credit, inflation, and stringent carbon emission regulations. By incorporating preservation technology investment as a decision variable, our model provides a more nuanced understanding of its impact on inventory management, enabling businesses to make informed decisions that balance economic profitability with environmental sustainability.

This investigation contributes significantly in the existing inventory optimization research by providing a holistic framework that acknowledges the intricate interplay between economic and environmental factors. Unlike extant research, which often treats these factors in isolation, our model offers a comprehensive approach that considers the impact of partial trade credit, inflation, and carbon emission regulations on inventory management. Our research reveals far-reaching implications on industries grappling with the dual challenges of inventory management and environmental stewardship, offering a paradigm shift in strategic decision-making.

The introduced inventory model is formulated as mathematical optimization problem, which is solved using advanced analytical techniques. By optimizing selling price and preservation technology investment, our model enables businesses to maximize profitability while minimizing environmental impact. The study's results provide valuable insights into the development of sustainable inventory management practices reducing environmental impact while boosting profitability. Ultimately, this research expands the current knowledge of literature on sustainable inventory management, providing a foundation for future research and practical applications. This study focuses on the specific case where the condition in which the credit period (M) is taken to be less than the time to deplete inventory in rented warehouse (T_1), i.e., $0 < M < T_1 < T$. This scenario is justified due to the complexity introduced by the stock-dependent demand function ($a - bp + cI(t)$). Considering inventory levels beyond T_1 or T

would lead to additional complexities, such as negative inventory levels or unrealistic demand patterns. By limiting the analysis to the $0 < M < T_1 < T$ case, this research provides a more nuanced understanding of the interactions between trade credit, inventory management, and sustainability. This scenario is also practically relevant, as businesses often aim to deplete inventory in rented warehouses before own warehouses. The insights gained from this analysis can inform inventory management strategies and contribute to more sustainable supply chain practices. Thirdly, it integrates carbon emission regulations, providing a more comprehensive understanding of the environmental implications of inventory management decisions. Finally, it optimizes selling price and preservation technology investment in a two-warehouse setting, offering practical insights for industries with perishable goods. By bridging these gaps, this study makes a significant contribution towards inventory management yielding a more realistic as well sustainable framework for businesses to operate in. The findings of this research have far-reaching implications in support of industries in balancing economic profitability with sustainable development, and offer a valuable resource for policymakers and practitioners alike.

2. LITERATURE REVIEW

The realm of inventory management has undergone significant transformations over the years, driven by the need to optimize inventory operations and reduce costs. The concept of exponentially decaying inventory, introduced by Ghare and Schrader (1963), laid the foundation for subsequent research in this area. One of the key factors that has been extensively explored in inventory management is permissible delay in payments

Goyal (1985) and Aggarwal and Jaggi (2001) examined the effect of Credit term flexibility on inventory decisions, highlighting about importance Connected to trade credit in inventory management. Their research demonstrated that permissible delay in payments can have a significant impact on inventory levels and costs.

Another important aspect of inventory management is partial backlogging. Chang et al. (2003) developed inventory models that account for partial backlogging and trade credit, providing valuable insights into the complex relationships between inventory levels, demand, and backlogging. Their research showed that partial backlogging can have a significant impact on inventory costs and levels. Preservation technology investment is another factor that has been shown to significantly reduce deterioration rates. Jaggi et al. (2013) and Mishra et al. (2014) demonstrated the importance of investing in preservation technologies to minimize losses due to deterioration. Jaggi et al. (2017) developed a dual-warehouse Inventory control mechanism for short-shelf-life goods with imperfect quality with payment delay. The model Analyses the role of defective items and trade credit on Stock decisions, aiming to optimize profit per unit time. Mishra and Singh (2017) propose an inventory model for deteriorating items, incorporating preservation technology investment and partial backlogging. Their study demonstrates that investing in preservation technology can significantly reduce total costs by minimizing deterioration. The model highlights the importance of optimal inventory policies, preservation technology investment, and partial backlogging in managing deteriorating items.

In recent years, two-warehouse inventory models have gained significant attention. These models consider the complexities of managing inventory across multiple storage facilities. Researchers Khan et al. (2019) proposed a dual-warehouse inventory system for perishable goods, incorporating partial backlogging and prepayment options. The model considers demand dependent on selling price and allows Partial backlogging of shortages with a fixed rate. The study highlights the effect of prepayment on optimal inventory decisions.

Yang et al. (2020) and Khan et al. (2022) investigated the influence of Spoilage reduction technologies and inflation on inventory decisions in a two-warehouse setting. Their investigations highlighted the importance of considering preservation technology and inflation in inventory management decisions. Sustainability has also become a key consideration in inventory management. Bazan et al. (2020) examined collaborative supply chain management encouraging low-emission practices, while Hua et al. (2021) investigated carbon pricing and its impact on inventory decisions. Liu et al. (2021) highlighted the importance of considering environmental costs in inventory management decisions. These studies demonstrate the growing importance of sustainability in inventory management. Ahmed and John (2023) examined warehouse management issues, focusing on inventory control, space utilization, personnel management, and technology integration. The study highlights the importance of effective strategies for

managing inventory, space, and staff, as well as leveraging technology like automation and data analytics to improve warehouse efficiency and reduce costs.

The integration of artificial intelligence in inventory management has also shown promise. Dhaliwal et al. (2023) demonstrated the potential AI-driven solutions in demand forecasting, while Naik (2023) showed its application in inventory optimization. These studies highlight the potential of artificial intelligence to improve stock management efficiency. Pal et al. (2024) investigated a dual-warehouse inventory system with non-instantaneous deterioration, credit policy, and inflation. They developed a mathematical model to optimize optimal order quantity, pricing strategy, and cycle duration. This study highlights the importance of considering to manage credit risk in an inflationary environment in inventory management decisions. Patel and Desai (2024) explored the impact of advanced technologies on inventory management, revealing benefits like improved efficiency and accuracy in managing stock levels and orders.

Our study builds on this foundation, examining the impact of carbon tax optimizing inventory decisions with two storage facilities with preservation technology, inflation, and trade credit. By exploring the interplay between these factors, our research aims to yield useful information for businesses and decision-makers looking for effective solutions to optimize inventory operations while reducing their environmental footprint. This study will contribute to the evolving discourse on sustainable inventory management practices and offer in-depth insights of the complex relationships in inventory management, sustainability along with environmental costs.

Table 1: Literature Review Table

Authors	Demand pattern	Deterioration	Preservation technology	Shortages	Trade credit	Inflation	Warehouse use	Carbon Tax
Mishra & Singh (2017)	Constant	Time dependent	consider	Partial backordering	consider	Not consider	Single warehouse	Not consider
Bazan et al. (2020)	Price dependent	Constant	consider	Not consider	Not consider	Not considered	Not specified	Not consider
Yang & Chang (2020)	Constant	Time dependent	Not consider	Not consider	Not consider	Not consider	Two warehouse	Not consider
Hua et al. (2021)	Price dependent	Constant	Not consider	Not consider	Not consider	Not consider	Two warehouse	Not consider
Li et al. (2019)	Price dependent	Non instantaneous	consider	Not consider	Not consider	consider	Not specified	Not consider
Khan et al. (2022)	Price dependent	Time dependent	consider	Not consider	consider	consider	Two warehouse	consider
Current paper	Time-price and stock dependent	Constant	consider	No backlogging	consider	consider	Two warehouses	

3. Assumptions and Notations

With the following considerations of this model-

- The model focuses on a single product inventory.
- Instantaneous replenishment.
- The owned warehouse's capacity is limited to W , while the rented warehouse has no capacity constraints.
- RW inventory depleted before OW to minimize holding costs $h_r > h_w$.
- RW's better service reduces deterioration rate compared to OW that is $\theta_2 > \theta_1$.

- Demand is taken price and stock dependent.
- $P(\xi) = e^{(-\lambda\xi)}$ where ξ is the preservation technology investment and λ is the efficiency parameter that reducing OW's deterioration rate through preservation tech while inflation is also considered in this model.
- Stockout are not permitted.
- On $(1 - \delta)$ trade credit is taken, δ amount is paid on delivery rest $(1 - \delta)$ is taken on trade credit.

The following notations are considered for this model-

Notations	Descriptions
$I_0(t)$	OW's current stock level
$I_r(t)$	RW's current stock quantity
A	Fixed ordering cost in rupees.
h_r	Holding cost per unit per unit time in RW (₹/unit/time)
$= g + ft$	
h_0	Holding cost per unit per unit time in OW (₹/unit/time)
$= g_1$	
$+ f_1t$	
θ_1	Deterioration rate in RW $0 < \theta_1 < 1$
θ_2	Deterioration rate in OW $0 < \theta_1 < 1, \theta_2 > \theta_1$
T	Length of replenishment cycle in days
T_1	RW's stock duration ($0 < T_1 < T$) in days.
T	OW's stock duration ($0 < T_1 < T$)
Q	Peak stock level in OW.
r	Rate of inflation
c_p	Unit purchase price.
p	Unit selling price.
k	CO2 emission per unit time.
$h = G + Ft$	Carbon output increase by per unit holding cost
e	Carbon output increased by ordering
c_T	The Emissions charge per unit carbon emission
I_p	Interest rate paid by retain
I_e	Interest rate earn by retain
$1 - \delta$	Inventory on which trade credit is taken paid partially in partial trade credit
Decision variables	
ξ	Preservation technology cost in rupees
p	Selling price in rupees

4. Mathematical modelling

Proposed inventory framework for a single Perishable goods with an initial purchase quantity of Q units. The items are stored in dual warehouses: "Owned storage facility (OW) and rented storage space (RW), each having different storage conditions and capacities. OW has storage capacity constraint and is used to store W units of the item, while the left out $Q-W$ units are stored in the Rented Warehouse. The stocks levels in both warehouses decrease over time as a result of customer demand and decay, but at different rates. The demand rate is effected by both the selling price and the current inventory level, making it a stock and price-dependent demand function, represented as $D(t) = a - bp + cI(t)$, where a , b , and c are constants, p is the selling price, and $I(t)$ is the total inventory level at time t .

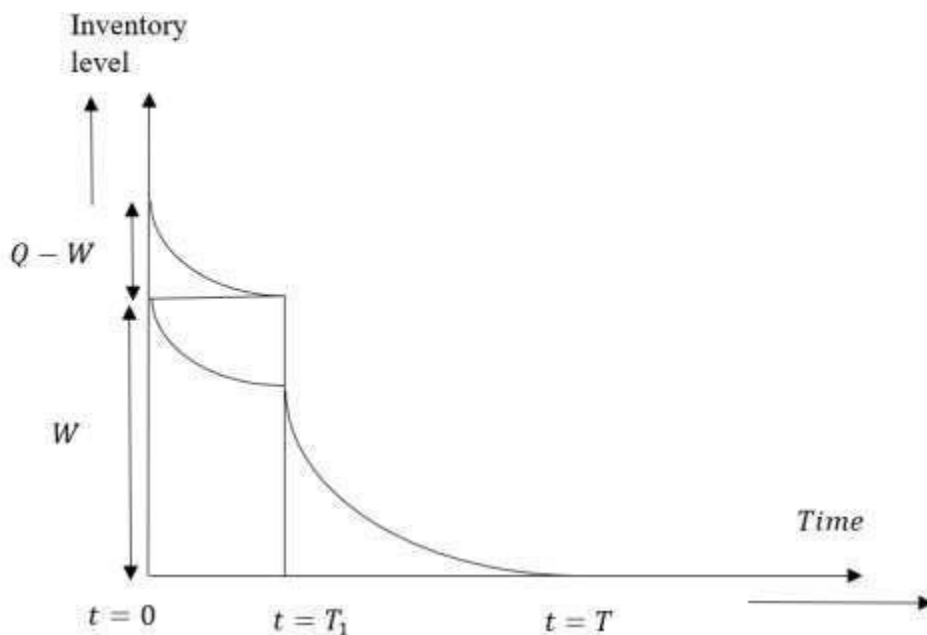


Fig 1: Inventory Position w.r.t. in both warehouses

The stock level in RW declines due to customer market demand and product deterioration. The inventory level in RW can be modelled by the differential equation:

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -D(t), \quad 0 < t < T_1$$

$$\frac{dI_o(t)}{dt} + \theta I_o(t) = -(a - bp + cI(t)), \quad 0 < t < T$$

Inventory Level in OW ($0 \leq t \leq T_1$)

The inventory level in the Own Warehouse (OW) during the period $[0, T_1]$ can be represented by the following differential equation, which accounts for the decrease in Stock levels solely due to deterioration:

$$\frac{dI_o(t)}{dt} + \theta P(\xi) I_o(t) = 0, \quad 0 < t < T_1$$

This differential equation captures the impact of deterioration on the inventory level in the Own Warehouse in the course of specified time period.

Inventory Level in OW ($T_1 \leq t \leq T$)

during the period $[T_1, T]$, the inventory level in the Own Warehouse (OW) decreases due to both deterioration and customer demand. Considering the impact of preservation technology, the differential equation representing the inventory level in OW can be formulated as:

$$\frac{dI_o(t)}{dt} + \theta P(\xi) I_o(t) = -(a - bp + cI(t)), \quad T_1 < t < T$$

With boundary condition $I_r(0) = Q - W, I_r(T_1) = 0, I_o(T_1) = W, I_o(T) = 0$

The solution of these equations –

$$I_r(t) = \frac{a - bp}{\theta_1 + c} [e^{(\theta_1 + c)(T_1 - t)} - 1]$$

$$I_0(t) = \frac{a - bp}{\theta_2 p(\xi) + c} [e^{(\theta_2 p(\xi) + c)(T - t)} - 1]$$

$$Q = \frac{a - bp}{(\theta_1 + c)} [e^{(\theta_1 + c)T_1} - 1] + \frac{a - bp}{\theta_2 p(\xi) + c} e^{\theta_2 p(\xi)T_1} [e^{(\theta_2 p(\xi) + c)(T - T_1)} - 1]$$

This differential equation captures the combined effects of deterioration (mitigated by preservation technology) and customer demand on the inventory level in the Own Warehouse during the specified time period.

Holding cost in RW-

$$HC = (g + ft) \int_0^{T_1} I_r(t) e^{-rt} dt$$

$$HC = (g + ft) \int_0^{T_1} \frac{a - bp}{\theta_1 + c} [e^{(\theta_1 + c)(T_1 - t)} - 1] e^{-rt} dt$$

$$= \frac{a - bp}{\theta_1 + c} \left[\frac{ge^{(\theta_1 + c)T_1} - (g + fT_1)e^{-rT_1}}{\theta_1 + c + r} + \frac{fe^{(\theta_1 + c)T_1} - fe^{-rT_1}}{(\theta_1 + c + r)^2} + \frac{(g + fT_1)e^{-rT_1} - g}{r} + \frac{fe^{-rT_1} - f}{r^2} \right]$$

Holding cost in OW-

$$HC = (g_1 + f_1 t) \left[\int_0^{T_1} I_0(t) e^{-rt} dt + \int_{T_1}^T I_0(t) e^{-rt} dt \right]$$

$$= W \left[(g_1 + f_1 T) \left(\frac{-e^{-rT_1 - \theta_2 P(\xi)T_1}}{r + \theta_2 P(\xi)} - f_1 \left(\frac{e^{-rT_1 - \theta_2 P(\xi)T_1}}{(r + \theta_2 P(\xi))^2} \right) + \frac{g_1}{r + \theta_2 P(\xi)} + \frac{f_1}{(r + \theta_2 P(\xi))^2} \right) \right.$$

$$+ \frac{a - bp}{\theta_2 P(\xi) + c} \left[(g_1 + f_1 T) \left[\frac{e^{-rT}}{-\theta_2 P(\xi) + c + r} - e^{-rT} \right] \right.$$

$$- f_1 \left[\frac{e^{-rT}}{(r + \theta_2 P(\xi))^2} - \frac{e^{-rT}}{r^2} \right] - (g_1 + f_1 T_1) \left[\frac{e^{(\theta_2 p(\xi) + c)(T - T_1) - rT_1}}{-(\theta_2 P(\xi) + c + r)} + \frac{e^{-rT_1}}{r} \right]$$

$$\left. + f_1 \left[\frac{e^{(\theta_2 p(\xi) + c)(T - T_1) - rT_1}}{(\theta_2 P(\xi) + c + r)^2} - \frac{e^{-rT_1}}{r^2} \right] \right]$$

Deterioration cost in RW-

$$DC = d \int_0^{T_1} e^{-rt} \frac{a - bp}{\theta_1 + c} [e^{(\theta_1 + c)(T_1 - t)} - 1] dt$$

$$= \frac{d(a - bp)}{(\theta_1 + c)} \left[\frac{e^{(\theta_1 + c)T_1} - e^{-rT_1}}{(\theta_1 + c + r)} + \frac{e^{-rT_1} - 1}{r} \right]$$

Deterioration cost in OW-

$$DC = d\theta_2 P(\xi) \left[\int_0^{T_1} e^{-rt} I_0(t) dt + \int_{T_1}^T e^{-rt} I_0(t) dt \right]$$

$$d\theta_2 P(\xi) \left[\frac{W(1 - e^{-rT_1 - \theta_2 P(\xi)T_1})}{r + \theta_2 P(\xi)} + \frac{a - bp}{\theta_2 P(\xi) + c} \left[\frac{e^{-rT} - e^{-rT_1}}{r} + \frac{e^{\theta_2 P(\xi) + c)(T - T_1) - rT_1} - e^{-rT_1}}{\theta_2 P(\xi) + c + r} \right] \right]$$

Preservation technology cost in OW = ξT

Sales revenue = pQ

Purchasing cost = cQ

Additionally, to comprehensively assess the environmental impact of inventory systems, it's essential to consider the carbon footprint associated with various aspects, including delivery, Warehouse operations and stable carbon emissions per order. The total Carbon output per inventory cycle can be calculated by accounting for emissions from transportation, storage, and order processing, providing a more detailed understanding of the inventory system's environmental sustainability.

$$\frac{(a-bp)e}{(\theta_1+c)} \left[M \left[\frac{e^{-rM}}{r} - \frac{e^{\theta_1+c}(T_1-M)-rM}{(\theta_1+c+r)} \right] + \frac{e^{\theta_1+c}T_1 - e^{\theta_1+c}(T_1-M)-rM}{(\theta_1+c+r)^2} + \frac{e^{-rM}-1}{r^2} \right] - ci_{pp} (1 - \delta) \left[\frac{a-bp}{\theta_1+c} \left[\frac{e^{(\theta_1+c)(T_1-M)-rM} - e^{-rT_1}}{(\theta_1+c+r)} + \frac{e^{-rT_1} - e^{-rM}}{r} \right] + \frac{a-bp}{(\theta_2p(\xi)+c+r)} \left[\frac{e^{(\theta_2p(\xi)+c)(T-M)-rM} - e}{(\theta_2p(\xi)+c+r)} + \frac{e^{-rT} - e^{-rM}}{r} \right] \right]$$

5. Solution methodology.

We used tool Mathematica 12 to evaluate the total profit.

6. Numerical Analysis

Numerical examples have a significant impact in understanding the inventory models. They provide clarity, validate theories, aid in decision-making, and enhance problem-solving skills. Whether in an educational setting or a business environment, numerical examples are essential for understanding and optimizing inventory management practices. We have used mathematica-12 software to validate and solve numerical section for this model.

Table 2- Represents numerical values for case-I

Parameters	Values	Parameters	Values
a	100	f_1	0.01
b	0.1	λ	0.04
C_p	100	I_c	0.13 %
c	1	I_e	0.15 %
r	0.001	F_1	.04
g	1 rupees	G_1	.02
f	0.04 %	θ_1	.002
d	4 days	θ_2	.004
g_1	2 rupess	C_t	.05
W	3000	δ	0.5
A	300	G	.01
e	.05	F	.01
k	1.7	M	3
T	10		
T_1	8		

Optimal Solution for case -I

Parameters	Values
ξ (Preservation technology cost)	38.3839 Rupees
p (selling Price)	550.791 Rupees
Total profit	6.47114×10^6

In a groundbreaking development, our study pioneers a novel inventory management model that seamlessly integrates deterioration, inflation, trade credit, and carbon emissions. By harnessing the power of optimization, we unlock a trifecta of benefits: a preservation technology investment of 38.38 rupees, a strategic selling price of 550.79 rupees, and a staggering total profit of 6.47114×10^6 rupees. This innovative approach not only maximizes profitability but also champions sustainability, reducing carbon footprints and waste. As a beacon for businesses, our research illuminates the path to informed decision-making, empowering organizations to thrive in an ever-evolving market landscape.

7. Graphical Representation

The concave graph illustrates the relationship between preservation technology investment (ξ), selling price (p), and total profit (TP). The graph shows that total profit initially increases with preservation technology investment, reaches a maximum, and then decreases, indicating an optimal investment level (ξ^*) that maximizes total profit (TP*). This relationship highlights the importance of balancing preservation technology costs with benefits to achieve maximum profitability.

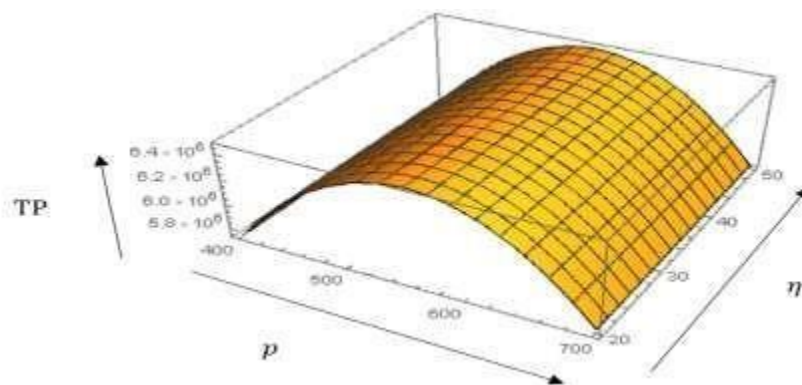


Fig 1: Concave Graph of Preservation Technology Investment vs. Total Profit

8. Sensitivity Analysis

Parameters	% Change	p	ξ	Total Profit
a	+10%	600.79	26.70	7.99304×10^6
	+5%	575.792	33.66	7.21203×10^6
	-5%	525.79	41.76	5.77036×10^6
	-10%	500.79	44.22	5.1097×10^6
b	+10%	459.88	49.74	4.52717×10^6
	+5%	479.361	47.36	4.80437×10^6
	-5%	524.475	40.22	5.44761×10^6
	-10%	550.792	34.73	5.82353×10^6
c_p	+10%	555.632	49.84	6.3322×10^6
	+5%	553.212	44.80	6.40151×10^6
	-5%	548.37	29.51	6.54114×10^6
	-10%	545.955	14.9015	6.61153×10^6
r	+10%	550.79	37.908	6.4704×10^6
	+5%	550.79	38.1469	6.4708×10^6
	-5%	550.79	38.38	6.44117×10^6
	-10%	550.79	38.43	6.4712×10^6
g	+10%	550.838	38.36	6.46979×10^6
	+5%	550.851	38.37	6.47046×10^6
	-5%	550.768	38.39	6.47181×10^6
	-10%	550.744	38.39	6.47249×10^6
d	+10%	550.978	38.97	6.46574×10^6
	+5%	550.885	38.76	6.47884×10^6
	-5%	550.698	37.99	6.47384×10^6
	-10%	550.604	37.59	6.47654×10^6
λ	+10%	550.79	37.07	6.47114×10^6
	+5%	550.79	37.72	6.47114×10^6
	-5%	550.792	40.02	6.47113×10^6
	-10%	550.793	41.50	6.47112×10^6
c_T	+10%	550.791	38.37	6.47113×10^6
	+5%	550.791	38.37	6.47113×10^6
	-5%	550.791	38.38	6.47114×10^6
	-10%	550.791	38.39	6.47114×10^6
w	+10%	550.791	37.48	6.47065×10^6
	+5%	550.791	37.94	6.47089×10^6
	-5%	550.791	38.81	6.47138×10^6

	-10%	550.791	39.24	6.47162×10^6
A	+10%	550.791	38.38	6.47113×10^6
	+5%	550.791	38.38	6.47113×10^6
	-5%	550.791	38.38	6.47114×10^6
	-10%	550.791	38.38	6.47114×10^6
e'	+10%	550.791	38.38	6.47113×10^6
	+5%	550.791	38.38	6.47113×10^6
	-5%	550.791	38.38	6.47114×10^6
	-10%	550.791	38.38	6.47114×10^6
I_p				
	10%	550.914	49.38	6.46754×10^6
	5%	550.853	44.49	6.46934×10^6
	-5%	550.731	30.20	6.47294×10^6
δ	-10%	550.672	17.60	6.47474×10^6
	10%	550.95	17.40	6.42998×10^6
	5%	550.87	30.14	6.45055×10^6
	-5%	550.71	44.53	6.49172×10^6
k	-10%	550.62	49.45	6.51232×10^6
	10%	550.791	38.38	6.47114×10^6
	5%	550.791	38.38	6.47114×10^6
	-5%	550.791	38.38	6.47114×10^6
I_e	-10%	550.791	38.38	6.47114×10^6
	10%	550.507	38.47	6.5159×10^6
	5%	550.648	38.42	6.49352×10^6
	-5%	550.944	38.33	6.44726×10^6
	-10%	551.079	38.29	6.42638×10^6

9. Observations

1. The sensitivity analysis shows that as parameter 'a' increase, the selling price and profit also increase, whereas the investment in preservation technology decreases, indicating that firms can leverage higher demand potential to boost profitability while optimizing pricing and preservation strategies.
2. The sensitivity analysis with respect to parameter 'b' in the demand function $D = a - bp + cI(t)$ reveals that as 'b' increases, the optimal selling price decreases, preservation technology cost increases, and total profit decreases. This suggests that higher price sensitivity (captured by 'b') necessitates lower prices to stimulate demand, while increased investment in preservation technology is required to mitigate potential losses, ultimately resulting in reduced profitability.
3. As purchasing price (C_p) increases, the optimal investment in preservation technology also increases due to the need to mitigate inventory losses and maximize profit under inflationary pressures and carbon tax implications. Conversely, the total profit decreases, indicating a trade-off between preservation costs and profitability. This insight highlights the importance of carefully managing purchasing costs in the presence of stock-dependent demand, trade credit constraints ($M < T1$), and carbon emission regulations.
4. As the inflation rate increases, the optimal selling price remains relatively stable, while investment in preservation technology slightly decreases, and total profit decreases, indicating that inflationary pressures negatively impact profitability despite adjustments in preservation technology investment.
5. As holding cost increases, selling price decreases, preservation technology investment decreases slightly, and total profit decreases, indicating that higher holding costs negatively impact profitability and lead to strategic adjustments.
6. As carbon tax increases, selling price and total profit remain relatively stable, indicating that the model's optimal strategy is insensitive to changes in carbon tax rates, likely due to effective management of preservation technology and inventory.

7. As the efficiency parameter (λ) of preservation technology increases, selling price remains relatively stable, preservation technology cost decreases, and total profit slightly increases, indicating that improved preservation technology efficiency leads to cost savings and marginally higher profitability.
8. As deterioration cost increases, selling price and preservation technology investment decrease slightly, while total profit decreases, suggesting that higher deterioration costs erode profitability despite minor adjustments in selling price and preservation investment.
9. As own warehouse storing capacity increases, selling price remains stable, preservation technology investment rises to potentially maintain product quality, but total profit declines, suggesting that expanded storage capacity leads to higher costs that outweigh benefits, ultimately negatively impacting profitability.
10. As ordering cost (A) increases, selling price and preservation technology cost remain relatively stable, while total profit decreases slightly, indicating that higher ordering costs directly impact profitability, but don't trigger adjustments in pricing or preservation investment strategies.
11. As the amount of carbon emissions from ordering increases, selling price and preservation technology investment - remain constant, while profit decreases slightly, suggesting that increased carbon emissions lead to higher costs or penalties that negatively impact profitability without altering optimal pricing and preservation strategies.
12. As interest payable increases selling price and preservation technology cost investment both increase, yet total profit decreases, indicating that higher interest costs lead to increased expenses that aren't fully offset by price hikes, ultimately eroding profitability.
13. In a partial trade credit scenario, as the advance payment amount increases, selling price rises, preservation technology investment declines, and total profit decreases, suggesting that higher upfront payments lead to strategic adjustments, but ultimately result in reduced profitability, potentially due to decreased investment in preservation technology.
14. As carbon emission amount increases, selling price, preservation technology investment, and total profit remain unchanged, indicating that carbon emissions don't directly impact decision variables or profitability in this scenario. As interest earned increases, selling price decreases, preservation technology investment cost rises, and total profit increases, suggesting that higher interest earnings lead to strategic adjustments, including reduced prices and increased preservation investment, ultimately resulting in higher profitability.

10. CONCLUSION:

This study comprehensively examines a two-warehouse inventory model that integrates preservation technology, carbon emissions, and trade credit, providing valuable insights into the intricate relationships between inventory management, sustainability, and profitability. The findings of this research have significant implications for businesses and policymakers seeking to balance economic and environmental objectives.

The study's results demonstrate that optimizing preservation technology investment and selling price can enable retailers to maximize profits while minimizing environmental impact. This is particularly relevant for retailers of energy-efficient products, such as LED bulbs, which are susceptible to fluctuations in demand and supply chain dynamics.

The research highlights the importance of strategic adjustments in selling price and preservation technology investment in responding to changing costs, environmental concerns, and trade credit scenarios. These adjustments can have a significant impact on overall profitability, and businesses that adopt sustainable inventory management practices can reduce their environmental footprint while improving their bottom line.

This study contributes to the growing body of literature on environmentally responsible supply chain management, offering valuable insights for businesses and policymakers seeking to balance economic and environmental objectives. The findings of this research can inform the development of sustainable inventory management practices that minimize environmental impact while maximizing profitability.

Overall, this study provides a comprehensive framework for managing inventory in a sustainable and economically viable manner, and its findings have significant implications for businesses and policymakers seeking to achieve a more sustainable future. By advancing our understanding of sustainable inventory management, this study provides a foundation for future research and practical applications, ultimately

contributing to more environmentally responsible and economically viable business practices. Future research directions may include:

- Exploring stochastic demand patterns and supply chain disruptions
- Developing multi-product inventory models with preservation technology
- Investigating the impact of emerging technologies on sustainable inventory management

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