International Journal of Environmental Sciences
ISSN: 2229-7359
Vol. 11 No. 23s, 2025
https://theaspd.com/index.php

A Comprehensive Review on Optimal PMU Deployment: Challenges, Developments, and Future Directions in Wide-Area Power System Monitoring

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Abstract:
Phasor Measurement Unit (PMU) technology has emerged as an essential component in modern power systems due to its superior resolution and real-time monitoring capabilities, which far surpass those of conventional estimation tools used in wide-area surveillance. PMUs provide synchronized measurements of voltage and current phasors, which are crucial for accurate state estimation and for enhancing system reliability to prevent widespread blackouts. However, the adoption of PMUs is often constrained by their high cost and complex installation requirements. A key advantage of PMUs lies in their ability to observe adjacent buses, enabling full system observability through strategic, optimal placement of a limited number of units often fewer than the total number of buses. Over the past two decades, this optimization challenge has been the focus of extensive research. Recognizing the significance of this area, this paper presents a comprehensive review of the advancements achieved to date and critically examines the shortcomings of existing literature. It highlights unresolved issues in the domain of optimal PMU placement and aims to offer fresh insights that will guide future research, ultimately helping to bridge the gaps and drive innovation in power system monitoring.
KeyWords: *Synchronized Phasor Measurement, Optimal PMU Placement, Measurement-Based Monitoring, Wide-Area Monitoring Systems (WAMS), Power System Reliability*

1. INTRODUCTION

The power system is composed of various electrical components that facilitate the conversion of multiple forms of energy into electrical energy, as well as its efficient transmission, distribution, and end-use. Key electrical parameters within the system include voltage, current, phasors, and frequency. Among these, frequency remains consistent across the system, while voltage, current, and phasors vary at different locations. The seamless functioning of all subsystems within the power network is essential, and achieving this requires continuous and accurate monitoring of these electrical parameters. Such monitoring is fundamental to ensuring optimal performance, system reliability, operational security, contingency planning, and effective restoration during disturbances. A power system monitoring framework typically performs three core functions: determining the network topology, ensuring system observability, and conducting state estimation. Traditionally, these parameters particularly voltage and current have been monitored using Supervisory Control and Data Acquisition (SCADA) systems.

The SCADA system is inadequate for monitoring modern power systems due to its limited resolution and inability to provide critical parameters such as phasor measurements, frequency, and the rate of change of frequency. Its shortcomings were notably highlighted in the investigation report by the North American Electric Reliability Corporation (NERC) on the widespread cascading blackout that occurred on August 14, 2003, in the northeastern United States.

The report revealed that the phase angle between western Michigan and Cleveland exhibited a steady increase over the span of an hour, as illustrated in Figure 1. Had this phase angle data been available to system operators in real time, timely corrective measures could have been implemented to mitigate or even prevent the ensuing blackout.

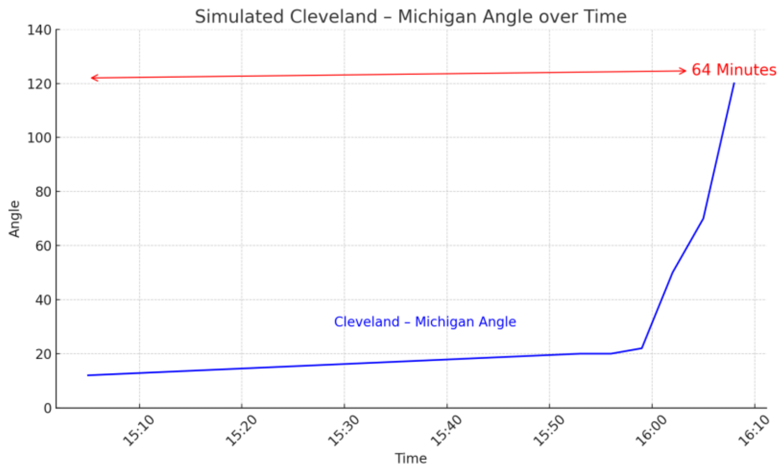

Figure 1. Phase angle divergence between Cleveland and Michigan.

Modern power systems are increasingly integrated with renewable energy sources, intelligent electronic devices (IEDs), and advanced protection and control technologies, necessitating the implementation of fast, reliable, and resilient monitoring systems to ensure safe and efficient operation. In addition to monitoring the magnitudes of voltage and current, capturing synchronized phasor information has become equally critical. The foundational concept of phasor monitoring in power systems was first introduced in 1980 by Professor Phadke at Virginia Tech, USA. As the name suggests, the PMU is a unit designed to measure phasors; the first fully functional PMU was built in 1988 and demonstrated in a practical application by Macrodyne Corporation. After this, in 1991, commercial production of PMUs began by Macrodyne, in collaboration with Virginia Tech, marking a major development in real-time monitoring of power systems. Phasor Measurement Units (PMUs) find countless critical applications involving power systems, such as smart grid implementations, integration of renewable energy, transition from steady-state to dynamic control, real-time operational control, frequency regulation, state estimation, protection schemes with adaptive features, Energy Management Systems (EMS) that function autonomously, the detection of thermal overload, assessment of stability, system restoration, and post-disturbance analysis. Considered synonymous, a WAMS defines a large-scale monitoring system ensuring secure and efficient operations of emerging power systems, which strongly depends on the integration of PMUs. The following section provides a concise comparison of the SCADA system and PMU technology concerning WAMS.

1.1. SCADA versus PMU

While judging WAMS, one observes SCADA limitations concerning sampling rate, time synchronization, and phase angle data recording in the modern power networks. These limitations have turned PMUs into the heart of WAMS, for their high-speed data acquisition and synchronized phasor-data-giving capabilities. SCADA systems typically communicate by employing the IEC 60870-5 protocol, whereas PMUs communicate by utilizing the National Standard of the People's Republic of China/Recommended (GB/T) 26865.2-2011) protocol. A comparative review of SCADA versus PMU technologies in the context of WAMS is given in Table 1.

Attribute	SCADA	PMU
Universal time synchronization	NA	Available
Local estimation of phase angles	NA	Available
Reporting rate	Once in (4 to 6) s	(10/12/15/20/30/60 frames)/s
Data flow latency	High	Negligible
State view of power system	Steady	Dynamic
Total input/output channels	100+ Analog and digital	10 Phasors, 32+ analog and digital
Communication method	Serial communication	Network communication

1.2 Existing Surveys in OPP

Over the past two decades, significant research has been conducted in the domain of Optimal PMU Placement (OPP), with various review articles offering distinct perspectives and insights into the subject. Among the earliest notable contributions, In references provided a review focusing exclusively on meta-heuristic and deterministic techniques employed in OPP. Based on the literature available up to that time, integer linear programming was identified as the most versatile and widely applicable method across diverse scenarios. Subsequently, Manousakis et al. presented a comprehensive analysis of various problem formulations and synthesized existing research findings, while also highlighting emerging trends in the field. In 2014, Amini far et al. offered a systematic review encompassing the architectural design of PMUs, their optimal placement strategies, practical applications, and the overarching role of Wide-Area Measurement Systems (WAMS). In 2016, In reference [20] purported to review both mathematical and artificial intelligence-based techniques for Optimal PMU Placement (OPP); however, this claim is inaccurate, and a more precise classification is provided in our current work. In reference [8] conducted a systematic review of PMUs as sensing devices and offered a concise overview of various formulations and methodologies employed in OPP. In reference [2] examined OPP within the context of smart grid environments, yet their review encompassed only a limited number of publications directly related to OPP. Furthermore, misclassifications of metaheuristic techniques present in their review are rectified in our study. In reference [21] focused on select literature addressing OPP for achieving network observability. In reference [22] analyzed different aspects of OPP problem formulations and highlighted certain methodological shortcomings. While their effort is commendable, the challenges they identify as future research directions such as global optimality, Zero Injection Buses (ZIBs), N−1 contingency analysis, communication channel limitations, and numerical observability have already been addressed in prior research and are well-documented in the literature. Our work revisits these aspects and instead identifies the genuine, unresolved challenges that warrant further investigation. In reference [23] provided a review of selected publications on Optimal PMU Placement (OPP), highlighting the advantages and limitations of various approaches. Building upon the rapid advancements in OPP research, the present study addresses the gaps and limitations of previous reviews by introducing a comprehensive flowchart that encapsulates the full spectrum of optimization techniques employed in the literature. We have systematically categorized these techniques into two primary groups: conventional and non-conventional methods. The most widely adopted techniques are discussed in detail with supporting tables, while less frequently used methods are also briefly outlined. Additionally, we have emphasized key objective functions, associated constraints, and prominent PMU installation schemes to offer readers a well-rounded understanding accessible even through a quick review of the content. The paper also explores various computational tools utilized for solving OPP problems. In conclusion, we identify existing research gaps and propose future directions to guide ongoing and prospective studies in this evolving field. This paper makes several significant contributions to the study of optimal Phasor Measurement Unit (PMU) placement. First, it presents a comprehensive and up-to-date review of state-of-the-art methodologies, covering a broad spectrum of approaches proposed in recent years. The strengths and limitations of existing research are critically analyzed, including a detailed assessment of previously published review articles in this area. An organized taxonomy of optimization methods is introduced for a systematic classification of different algorithms, organizing the entire body of literature examined in this work under this classification. Albeit popular methods such as linear programming, genetic algorithms, or particle swarm optimization are overviewed and discussed by giving comparative analyses of performances in a concise table form. Then, the paper sheds light on testbed systems, optimization solvers, along with the good and bad of techniques found in literature. Finally, an extensive collection of references from the last two decades has been given with critical research gaps that remain open, thereby suggesting dire research areas for further work in the subject of optimal PMU placement. The paper begins with a taxonomy of optimization techniques used to solve the Optimal PMU Placement problem, and the formulation of the objective functions and observability constraints (Section 2). Sections 3 and 4 discuss the classical and non-classical optimization techniques used for the OPP problem, respectively. Section 5 discusses investigated testbed systems and optimization solvers, and it compares the advantages and disadvantages of different algorithms. Further, it identifies some essential topics for future research directions. The discussions are concluded in Section 6.

2. Optimal PMU Placement

In reality, many problems are solved based on a mathematical formulation when solving which gives the best alternative. Therefore, the theory of optimization provides algorithms that can produce solutions to well-defined mathematical models with the help of computers and software [24, 25]. In more general terms, there are two major areas into which optimization algorithms fall: classical or conventional methods on one hand and modern or non-conventional algorithms on the other. Classical optimization techniques include linear programming (LP), nonlinear programming (NLP), dynamic programming

(DP), and combinatorial optimization, among many others. Non-traditional methods, on the other hand, comprise modern heuristics, namely particle swarm optimization, evolutionary algorithms, genetic algorithms, simulated annealing, tabu search methods [26], just to mention a few. The OPP problem is the process of selecting the best locations to install PMUs within a power system. However, equipping every bus with PMU installation would create highly redundant measurement data and would not be economically feasible. The cost of PMU devices, their installation, phasor data concentrators (PDCs), communication infrastructure, possible upgrades to legacy substations, and data management on a large scale are few major factors to make decisions of installing PMUs in all buses economically impractical. According to Ohm's Law, evidently, one can infer that a PMU installed at one bus also permits observability of its adjacent buses. This key principle allows for the full or near-full observability of the system both numerically and topologically by a strategically minimal number of PMUs, especially when technically optimized over large-scale networks, as in reference [27]. These optimization methods can consider constraints and operational contingencies regarding PMU placement, including the presence of limited communication infrastructure, integration with conventional measurements, fault conditions, and equipment loading. Modern optimization techniques have demonstrated higher efficiency than traditional approaches in dealing with such complexities in reference [18]. Such techniques for solving the OPP problem include linear programming, particle swarm optimization, genetic algorithms, and greedy search algorithms among others. Figure 2 depicts an overview of the methods used in the above optimizations applied to OPP. There may be objectives, constraints, contingencies, and installation strategies involved in formulating the Optimal PMU Placement (OPP) problem. Among these, the most commonly encountered formulation is the binary integer linear programming model. The whole problem setup generally hones in on defining the objective function and constraints.

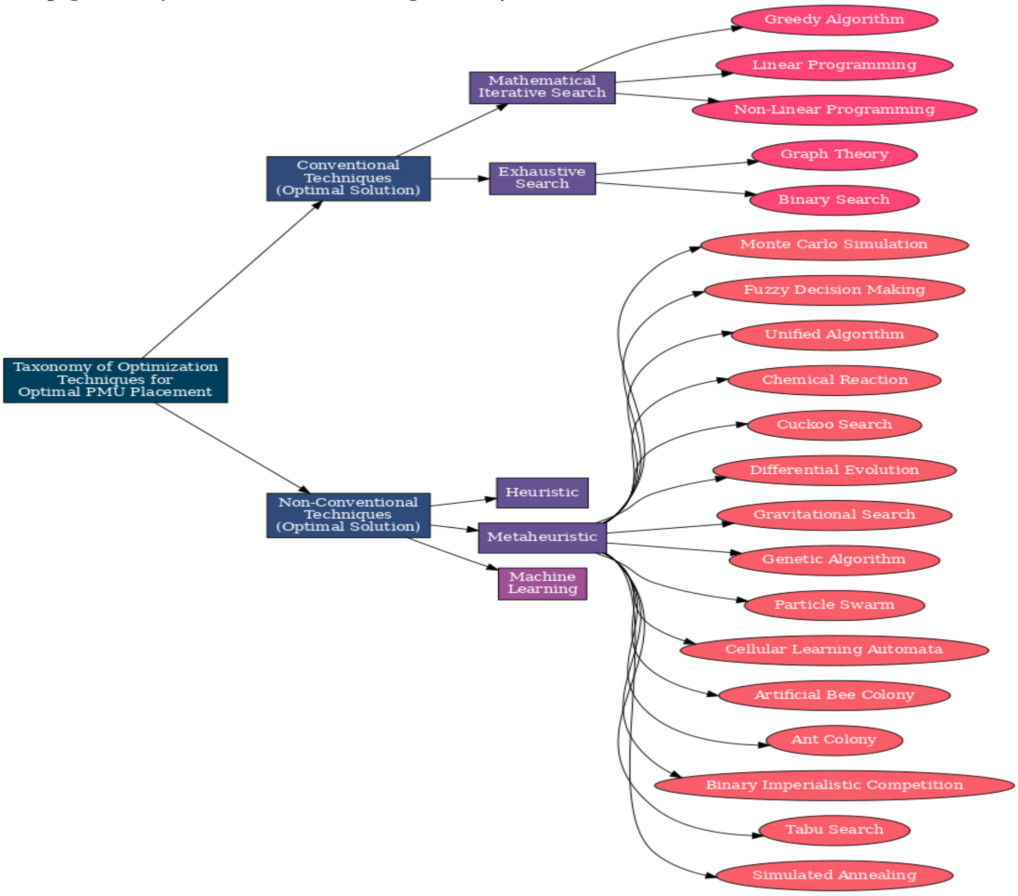

Figure 2. Taxonomy of optimization techniques used in optimal PMU placement.

2.1. Fundamental Statement of Objective Function

In the existing literature, two primary objective functions are predominantly utilized. The first aims to minimize the total number of PMUs installed, thereby reducing the overall cost. The second focuses on maximizing measurement redundancy by integrating system observability directly into the objective function. The formulation for minimizing the number of PMUs is represented in Equation (1), as shown below:

$$F_1 = \min \sum_{i=1}^{m} P_i X\, C_i \quad \text{..(1)}$$

In Equation (1), the variable P is the PMU device to be installed on a bus i, m is the total number of buses, C is the cost of device that can be a constant if all the devices are of the same price, and Pi is a binary decision variable.

$$A_{i,j} = \begin{cases} 1 & \text{In case where device P is located on bus i} \\ 0 & \text{In case where device P is not located on bus i} \end{cases}$$

In Equation (2), the objective function is designed to enhance measurement redundancy by embedding observability within the optimization criterion. Here, O_i represents the number of times bus i is observed through the deployment of installed PMUs.

$F_2 = \max \sum_{i=1}^{m} O\ i$..(2)

As a multi-objective problem, both of the objective functions can be combined easily by changing the sign of function F2 and treating it as a minimization function. The combined formulation is given in Equation (3), as stated below:

$$F_3 = \sum_{i=1}^{m} P_i - \frac{1}{N \times \max(O_i^{allPMU} + 1)} \sum_{i=1}^{m} O_i - 0.35$$

........(3)

Here, N denotes the total number of buses in the power system, and Oi – All PMU represents the number of times bus i is observed when PMUs are installed on all buses. The weighting coefficient in the second term of Equation (3) ensures that the objective function F1 is prioritized over F2 in the context of the OPP formulation. For a more detailed exploration of various objective functions and both linear and non-linear constraints, readers are referred to the review article or reference [22] on Optimal PMU Placement.

2.2. Fundamental Statement of Observability Constraint

In power system monitoring, observability is defined as the ability to gather sufficient measurement data to accurately estimate the system's state. State estimation involves determining the optimal current operating condition of the power system using measurement inputs and network topology information. The key state variables required for this process are the bus voltage magnitudes and phase angles. Once these variables are known, other parameters such as line currents (both magnitude and phasor), as well as active and reactive power flows in lines and loads can be readily calculated in reference [28–30]. Traditionally, state estimation relies on measurements of voltage magnitudes and angles, power flows, and power injections, all of which have non-linear relationships with the system states. As a result, conventional state estimators are non-linear, computationally intensive, and typically solved through iterative algorithms. The weighted least squares (WLS) method, represented in Equation (4), is the most widely used approach. However, the introduction of Synchrophasor measurements from PMUs enables a linear estimation process, allowing for non-iterative, real-time solutions in reference [31].

$z = h(x) + e$ ....(4)

In Equation (4), z represents the measurement vector composed of conventional measurement data; h(x) denotes the measurement function, which characterizes the non-linear relationship between the measurement vector and the state vector x; and e is the error vector accounting for measurement inaccuracies.

Power system observability is primarily assessed using two major approaches: numerical observability and topological observability. Numerical observability involves the evaluation of complex Jacobian matrices, which makes the process computationally intensive and less commonly applied in practice. In contrast, topological observability is more widely used due to its simplicity, relying on the concept of obtaining a full-rank spanning tree within the network in reference [27]. Observability methods can generally be categorized into topological, numerical, and hybrid approaches (which combine both). Among these, topological observability is the most frequently employed constraint in existing literature. Under standard operating conditions, complete topological observability is represented by Equations (5) and (6), where O denotes the observability vector composed of observability expressions Oi, each corresponding to the i^{th} observability constraint.

$O = Ax$.. (5)

$O \geq u$... (6)

A = [1 0 . . . . 1 1 1 . . . . 0 ∷ . . . . : 1 0 . . . . 1]

The elements are presented in the following manner:

$$A_{i,j} = \{1 \text{ If either i equals j, or buses i and j are directly connected tby a branch. } 0 \text{ otherwise}$$

$x = \left[x_1, x_{2,\ldots x_n}\right]^T$ and $u = [1,1,\ldots 1]^T$

where A is the node incidence matrix or binary connectivity matrix, with size N × N; X is a row vector having size N × 1 with elements xi, i = 1, 2, ..., N, and u is a row vector N × 1 consisting of ones, representing a bus observable by one PMU.

3. OPP Using Conventional Optimization

Conventional approaches employed in optimal PMU placement are broadly categorized into mathematical iterative search techniques and exhaustive search methods. The following sections provide a detailed discussion of research articles that have utilized these conventional techniques.

3.1. Mathematical Iterative Algorithms

Mathematical iterative search involves starting with an initial estimate and generating a sequence of progressively refined solutions, where each subsequent estimate is derived from the preceding one. To solve for a variable x, an initial value x_0 is assumed, which is then used to compute the next estimate x_1. This process continues iteratively until the solution converges typically when the difference between x_{n-1} and x_n becomes negligible, often within three decimal places. The application of mathematical iterative techniques in PMU placement is discussed in the following section.

3.1.1. Greedy Algorithm-Based OPP

As per [32], greedy search algorithms are a whole class of search methods: they seek to find quick-and-feasible solutions rather than being guaranteed to find the best for a particular optimization problem. These algorithms generally provide locally optimal solutions, whereas a global optimum is found somewhere else for the problem structure. The succeeding section discusses the greedy search algorithms in OPP.

(a) Information Theoretic Approach (ITA)

The Information Theoretic Approach (ITA), also referred to as the data analytics method, involves evaluating a set of candidate models to identify the one with the highest likelihood of approximating the true system behavior more accurately than the others. In reference [33], a greedy ITA was applied to the Optimal PMU Placement (OPP) problem, utilizing mutual information (MI) between system states and measurements. MI served as the objective function, effectively capturing both system observability and the reduction of uncertainty..

(b) Posterior Cramér-Rao Bound (PCRB)

The Posterior Cramér-Rao Bound is a conventional method utilized to estimate the performance of estimators, especially when linked with Markovian models. It specifies the lower bounds on the variance of biased estimators. In [34], PCRB-based greedy algorithms were proposed to solve the OPP problem. Since the greedy algorithm always finds the best feasible solution for submodular objective functions and tends to do better for non-submodular functions, the optimization was solved using a greedy approach.

3.1.2. Linear Programming (LP) Based OPP

According to [35], in LP, optimization involves maximization and minimization of a linear objective function subject to a set of linear constraints on decision variables. In LP-based models, the objective function has been taken as cost, whereas, in addition to those constraints, the decision variables must be positive. Linear programming techniques are helpful in tackling NP-complete problems. Linear programming methods in PMU placements are discussed in the next section.

(a) Binary Integer Linear Programming (BILP)

In BILP, one typically solves systems of linear equalities or inequalities, and the decision variables are assumed to be binary, usually having values 0 or 1. This kind of binary formulation fits naturally with problems that need a discrete yes or no kind of decision.

[36] established the BILP method for the optimal placement of PMUs considering conventional injection flow (IF) measurements and tests involving either the failure of one PMU or the failure of more than one PMU. In [37] developed BILP for CNO and measurement redundancy enhancement with respect to PMU and line outage scenarios. This study dealt also with PMU channel limitations and the impact of ZIBs.

In [38], BILP was used for OPP under CNO with N−1 contingency considerations. Their formulation took into account the depth of unobservable buses, sequential multi-stage PMU deployments, monitoring with respect to critical buses, existing PMU locations, unsuitable PMU sites, and critical bus data.

In reference [39] also used BILP to solve the OPP, emphasizing post-disturbance coherency at various levels of observability. To generate scenarios, they introduced diverse probabilistic parameters and employed a subtractive clustering algorithm to identify central buses based on post-disturbance behavior. Each identified central bus was then evaluated through multiple placement scenarios.

In reference [13] applied BILP to achieve full network observability under N−1 contingency conditions. In [40] further extended the BILP framework to address CNO, multi-stage PMU installation, simultaneous monitoring through two adjacent injection measurements, and N−1 contingencies.

In reference [41] proposed a generalized approach to solve the OPP using BILP, considering scenarios such as PMU loss, presence of zero and non-zero injection buses, and integration of power flow measurement devices on transmission lines. Their study also examined PMU failures through linear estimator-based measurements, evaluating computational efficiency and bad data handling.

In reference [42] proposed a strategy for integrating additional PMUs into existing networks to enhance robustness, improve data availability, and mitigate communication interruptions and transmission faults. To address this, they developed a three-stage, scalable synchrophasor availability-constrained placement algorithm based on Integer Linear Programming (ILP) to solve the OPP problem. In reference [43] introduced a generalized OPP formulation that incorporates both Complete Network Observability (CNO) and Incomplete Network Observability (INO), along with redundant PMU placements, also using ILP. Reference [44] has presented a cost-efficient strategy for jointly installing PMUs and conventional measurements (CMs). Initially posed as a non-linear integer programming problem, it was then converted into an equivalent ILP model using Boolean logic. Through this method, fewer PMUs are required, and it proves to be a cost-efficient solution in comparison to conventional measurement methods-I.e., fixed or intrusion measurement points.

Reference [45] approached the OPP efforts in stages to lessen the burden of initial investment. Thus, ILP was used to optimally place PMUs for complete system observability, under contingencies of line or PMU failures, while MCDM techniques were applied to rank PMU placement sites depending on utility-specific preferences. Weighted criteria dictated the approach to the staged installation planning, giving priority to the balance between performance and cost-effectiveness.

The paper [46] proposed an OPP strategy that integrates conventional measurement equipment, while taking into account single-line or PMU contingencies as part of bursting observability. Differently from earlier approaches, this study modelled the problem using circuit equations that represent PMUs, conventional measurements, and network topology to obtain a globally optimal solution. The model also accounts for communication constraints as an important factor in deciding about placements.

An attempt to unify an algorithm for the OPP problem that simultaneously executes bad data detection and observability analysis is given in [47]. In each iteration, the algorithm provides an optimal solution for placing PMUs by either ensuring system-wide observability or by transforming critical buses into non-critical ones, thus improving system robustness and reliability.

Newness of Synchrophasor technology always carries some kind of cost; however, when it comes to substation upgrades, they often weigh far more heavily on the total bill than do the PMUs themselves. Considering this perspective, it is insufficient to minimize just the number of PMUs and equally important to do so for substations requiring upgrades. Reference [48] addressed this problem by tackling it through an Integer Linear Programming (ILP) method with DULRs aiming to minimize the total system cost. They considered the simultaneous optimization of communication infrastructure, cybersecurity, labor, and device costs to render a more holistic and cost-effective solution.

In order to consider Zero Injection Buses, conventional measurements, or CMs, were formulated in [49] by logically combining the bus observability functions. The approach also highlighted some issues with ZIBs and CMs that had not been identified before. In [50], the OPP problem was addressed by means of Integer Linear Programming (ILP), considering two kinds of contingencies: one associated with voltage stability and the other with severe islanding scenarios. Both contingencies were combined in a single multi-objective function to minimize the number of PMUs and maximize system observability. In the same way, an ILP-based OPP strategy, considering power injection measurements and not, was proposed in [51].

[52] deals with the simultaneous placement of traditional PMUs and dual-use line relays to optimize the number of substations. The study proposed the General Optimal Substation Coverage (GOSC) algorithm to ensure redundancy for measurements of critical system elements as well as enable the estimation of transformer tap ratios. They demonstrated that the GOSC algorithm offers strong techno-economic aspects. Reference [53] used a Lyapunov exponent-based method to guarantee complete system observability, allowing real-time stability monitoring and system assessment; their methodology focused first on maximizing redundancy of critical buses, then on identifying buses' contributions to system stability and finally on classifying critically based on the Lyapunov exponent)-based approach. Reference [54] also applied Integer Linear Programming (ILP) for optimization of redundant measurements, the solution of which was coupled with a heuristic to select optimal PMU locations.

With the concept of branch-level placement of PMUs for monitoring current and voltage phasors, the optimal placement of PMUs was found to completely observe the network using Integer Linear Programming (ILP) technique in [55]. In reference [56] was another OPP method developed to ensure observability under both normal operating conditions and cases of controlled islanding of the power system. Their method is intended to maximize the measurement redundancies and minimize the number of PMUs simultaneously through a weighted function, considering the occurrences of zero injection buses, PMU failures, and line outages, such that the system stays observe at all times.

Security vulnerabilities require periodic patching for the PMUs, thus sometimes taking them offline temporarily. Redundant placement of PMUs is necessary to ensure observability of the whole system during maintenance. The challenge of scheduling a patching mechanism such that all PMUs were patched in the minimum number of rounds while maintaining continuous observability was looked into in reference [57]. Integer Linear Programming was proposed to solve this problem, but since it could not process large-scale networks, a greedy heuristic algorithm was developed for that particular case.

In reference [58], a two-stage OPP strategy using ILP was proposed: the first phase accounted for CNO, whereas the second accounted for $N-1$ contingencies along with transmission line switching scenarios.

In reference [59], an OPP method was presented that considered the steady-state availability of Synchrophasor data at each bus to meet a predetermined reliability threshold under communication constraints. A Markov model was developed for the evaluation of Synchrophasor availability.

In reference [60], PMU placement was based on estimation theory criteria. This approach incorporated PMU measurements of current and voltage along with conventional state estimation within a Bayesian framework. Convex optimization relaxation techniques were employed to reduce computational complexity, providing a numerically optimal solution while avoiding the exhaustive combinatorial search typically required.

In reference [61] proposed an optimal PMU placement strategy that enhances the power system's resilience against cyberattacks by redundantly allocating PMUs on vulnerable buses. The problem was modeled using a binary formulation and solved through Binary Integer Linear Programming (BILP). Similarly, In reference [62] developed a BILP-based approach that improves redundancy while accounting for Zero Injection Buses (ZIBs), conventional measurements, contingency scenarios, and PMU channel limitations. The redundancy at each bus was optimized by introducing weighted auxiliary variables. In reference [63] focused on ensuring complete network observability and fault detection on transmission lines, incorporating the channel capacities of different PMU manufacturers into their optimization model. In reference [64] introduced a two-stage approach first maximizing measurement redundancy based on three key attributes: Degree of the Vertex (DOV), Average Neighborhood Degree of the Vertex (ANDOV), and Bus Observability Index (BOI), followed by minimizing the number of PMUs in the second stage. [65] used a BILP framework to simultaneously maximize redundancy and minimize the number of PMUs, considering contingencies, communication constraints, and the role of ZIBs, referred to as pure transit nodes. In reference [66] presented an ILP-based PMU placement model that included network impedance characteristics (both series and shunt), contingency analysis, ZIB effects, channel limitations, and the small-signal stability of the power system.

(b) Mixed-Integer Linear Programming (MILP)

Mixed-Integer Linear Programming (MILP) extends the traditional Integer Linear Programming (ILP) framework by allowing at least one decision variable to take a continuous (non-integer) value. In other words, not all variables in the formulation are restricted to discrete values. The application of MILP in solving the Optimal PMU Placement (OPP) problem is discussed in the following section.

In reference [67] introduced an Optimal PMU Placement (OPP) approach that incorporates a predefined probability of observability, accounting for equipment outages and the stochastic nature of system components. The optimization was performed using Mixed-Integer Linear Programming (MILP), and due to financial and physical limitations, the placement strategy was extended into a multi-year planning horizon. By integrating probabilistic constraints, the proposed model ensured an acceptable level of outage protection from a probabilistic perspective. In a related study in reference [68], an analytical framework for OPP was developed, taking into account cost–benefit analysis, long-term economic considerations, and existing technical challenges.

A MILP was formulated in [69] to solve a special OPP problem that develops a new redundant observability scheme. This MILP approach aimed at augmenting observability redundancy without increasing the number of PMUs with respect to current methods. Reference 70] treated the contingency-constrained OPP based on n−k redundancy criteria via robust optimization. The security criterion they proposed guaranteed network observability under any contingency comprising at most k = 2 PMU failures. Ref. [71] extended the OPP framework to AC/DC hybrid systems through MILP so as to guarantee that systems are observable notwithstanding the absence of direct phasor measurements in HVDC networks. It aims at minimizing the installation cost of PMUs with respect to observability of AC and DC transmission systems and different prices of PMU devices; it is also found that DC lines make the OPP solution worse by increasing the number of PMUs required. The paper [72] treated the problem of minimizing PMU installation cost and observation for gross error detection for Complete Network Observability (CNO) using MILP formulations, allowing flexibility on the weights of the objectives according to the user's budget constraints.

In [73], an optimal allocation of PMU placements and communication links was presented, including using zero injections buses to minimize the total installation cost of the wide-area monitoring system. Incorporating the data transmission bandwidth as an important factor to the optimization was one of the main contributions of this work.

(c) Equivalent Integer Linear Programming (EILP)

The theory of Extended Integer Linear Programming (EILP) posits that every integer programming problem belongs to a class of infinitely many equivalent problems, meaning that solving one member of the class enables the solution of all others within it. In reference [74] applied EILP to the Optimal PMU Placement (OPP) problem, resulting in a fully linear state estimation model that overcomes the limitations associated with SCADA-based estimations. The EILP framework allows for the straightforward inclusion of additional constraints such as N−1 contingency scenarios, communication channel limitations, and the presence of Zero Injection Buses (ZIBs), enhancing its flexibility and applicability.

3.1.3. Non-Linear Programming (NLP) Based OPP

Nonlinear Programming (NLP) is an optimization technique in which either the objective function or one or more constraints exhibit nonlinearity. In reference [75] addressed the Optimal PMU Placement (OPP) problem as a quadratic minimization task using continuous decision variables, considering the nonlinear boundaries associated with observability. They employed an unconstrained nonlinear weighted least squares method to derive the optimal solution. In reference [76] tackled the OPP problem using integer quadratic programming, ensuring measurement redundancy to achieve Complete Network Observability (CNO), even under N−1 contingency scenarios. Their formulation also allowed the integration of existing measurement technologies. Additionally, In reference [77] proposed an OPP strategy based on calculating the empirical observability Gramian across the power system's operational region to evaluate the system's state observability for a given placement. The optimization aimed to maximize the determinant of the empirical observability Gramian and was solved using a nomad solver.

In reference [78] formulated the Optimal PMU Placement (OPP) problem using a Binary Semi-Definite Programming (BSDP) model, which was subsequently solved through Binary Integer Linear Programming (BILP). This model accommodated any combination of existing PMUs, SCADA measurements, and AC or DC data, while ensuring a globally optimal solution. Notably, it required fewer PMUs compared to other methodologies. In [79] also employed BSDP to address the OPP problem, incorporating considerations such as zero injection buses, Synchrophasor channel limitations, and conventional measurements within the power network. In reference [80] advanced the approach by using Integer Semi-Definite Programming for OPP, integrating data from both PMUs and SCADA systems to enhance hybrid state estimation. They introduced a comprehensive evaluation metric to assess the effectiveness of PMU placement based on three critical aspects of state estimation: convergence, observability, and overall performance.

In reference [81] proposed an optimal PMU placement strategy aimed at minimizing the mean squared error between the system's output and the measured data. The formulation accounted for Zero Injection Buses (ZIBs), N−1 contingency scenarios, and communication channel limitations per PMU. Classified under binary nonlinear optimization, the problem was addressed using a specially developed algorithm designed for scalability and efficiency, capable of achieving at least a locally optimal solution. The proposed algorithm was validated through testing on standard benchmark power system networks.

3.2. Exhaustive Search

Exhaustive search optimization involves evaluating every possible solution within the search space. In our view, this approach is well-suited for solving the Optimal PMU Placement (OPP) problem, as OPP is typically addressed in an offline setting, where increased computation time can be justified by the potential cost savings from reducing the number of PMUs. The primary limitation of this method lies in its computational intensity, particularly for large-scale power networks. However, this trade-off is acceptable in offline planning scenarios. Various exhaustive search techniques applied to PMU placement are discussed below.

3.2.1. Graph Theory-Based OPP

Graph theory, which analyzes relationships through nodes and edges, is commonly used to model pairwise interactions among elements. In reference [82] applied graph theory in conjunction with the Analytical Hierarchy Process (AHP) to facilitate multi-criteria decision-making for Optimal PMU Placement (OPP). Their formulation addressed Complete Network Observability (CNO), maximized measurement redundancy, accounted for Zero Injection Buses (ZIBs), and incorporated N−1 contingency scenarios, resulting in enhanced outcomes and improved redundancy. Similarly, In reference [83] employed a graph-theoretical approach to determine the optimal PMU locations and identify the minimum set of critical measurements required for achieving CNO. A key innovation in their work was the development of a decentralized monitoring framework.

3.2.2. Binary Search-Based OPP

Also referred to as the half-interval search, the binary search method works by dividing a sorted interval into two halves and determining which half contains the target value by comparing it with the midpoint. In reference [84] applied the binary search technique to the Optimal PMU Placement (OPP) problem, addressing both Complete Network Observability (CNO) and N−1 contingency scenarios. This method was introduced as a solution to mitigate the limitations associated with genetic algorithms and integer programming approaches.

4. OPP Using Non-Conventional Optimization

Non-conventional techniques can be broadly categorized into heuristic, metaheuristic, and machine learning-based approaches. Heuristic methods are tailored to specific problems and operate using a predefined set of rules to navigate the search space. In contrast, metaheuristic techniques are problem-independent and designed to guide or adapt underlying heuristics, enabling the generation of high-quality solutions efficiently across a wide variety of optimization challenges.

4.1. Heuristic Techniques

A heuristic technique addresses a problem using practical approaches or shortcuts that aim to produce feasible solutions within a limited time frame, though the results may not always be optimal. The application of heuristic methods for PMU placement is explored in the following discussion.

4.1.1. Unified Algorithm Based OPP

It is a consolidated approach that represents the integration of adaptive learning rate optimization techniques such as AMSGrad (Adaptive Mean Square Gradient), Adam (Adaptive Moment Estimation), AMSGWDC (AMSGrad with Weighted Gradient and Dynamic Learning Rate Bound), GWDC (Adam with Weighted Gradient and Dynamic Learning Rate Bound), and AdaBelief (which decrease step sizes whenever there is less confidence about gradient observations). It selectively uses some components of these optimization frameworks to efficiently determine an optimal solution. The reference [85] used a unified algorithm for PMU placement taking into account zero-injection (ZI) measurement reliability. The proposed methodology reduces the number of PMUs required and improves the ZI measurement reliability to guarantee a complete observability of the system.

4.1.2. Fuzzy Decision Based OPP

Fuzzy decision-making methodologies are used, sometimes for problems involving single or multiple criteria, whenever there is uncertainty or scarcity of data in information availability. Aghaei et al. [86], in turn, proposed an OPP framework set in a multi-objective probabilistic environment. Their model aims at minimizing simultaneously the total number of PMUs in the network and maximizing redundancy in N − 1 contingencies taking into consideration Zero Injection Buses (ZIBs).

4.1.3. Monte Carlo Simulation Based OPP

Given the occurrence of extended or complicate datasets, the Monte Carlo method is used to generate random samples to approximate an optimal solution. Reference [87] gave an Optimal PMU Placement (OPP) strategy concerning a channel-based framework wherein explicit costs associated with Synchrophasors and their communication channels undergo optimization. Channel allocation is therefore made selectively based on economic justification. To ensure system security, redundant deployment of Synchrophasors and their channels occurs at critical buses and branches. These instruments are also assigned to monitor vulnerable grid sections, especially those prone to voltage instability, allowing for timely intervention to avoid voltage collapse. The Monte Carlo simulation would also be used to discover potential contingencies within the placement problem. In an alternative route, Reference [88] formulates the OPP problem based on the vulnerability index and related parameter. This includes data generation through Monte Carlo sampling, network segmentation with a genetic algorithm, and quadratic programming for deciding the PMU configuration limits, thus creating a dynamic evaluation on vulnerability based on PMU placements at bus nodes.

4.2. Metaheuristic Techniques

Metaheuristic algorithms navigate the solution space, looking for a near-optimal solution, by means of mechanisms ranging from simple local operators to adaptive machine learning. The following section studies the deployment of metaheuristics within the context of the Optimal PMU Placement (OPP) problem.

4.2.1. Chemical Reaction Based OPP

under a budget constraint, applying an algorithm modeled after chemical kinetics where reactants proceed, via several steps of reaction, toward the formation of products. In an enlarging model, it becomes that much harder and computationally demanding to conclude whether to reposition existing PMUs or to install new ones, so the classic mathematical programming method would not yield solutions within real time. The CRO was thereby applied as an efficient heuristic-solving means of the OPP problem.

4.2.2. Cuckoo Search Based OPP

An algorithm inspired by this concept is the one that mimics the brood parasitism behavior exhibited by cuckoo birds, with Lévy flight-based random walks added to enhance search efficiency. In [90], a binary-enhanced version called the Modified Binary Cuckoo Optimization Algorithm (MBCOA) was implemented for the Synchrophasor placement problem. This technique was placed under topological solution methods. The comparison showed that MBCOA produced better results, especially when it came to the number of required iterations as compared to other optimization techniques.

4.2.3. Differential Evolution Based OPP

This approach utilizes principles of evolutionary algorithms to traverse the search space efficiently. In [16], the authors introduced a novel multi-objective differential evolution approach for the optimal placement of PMUs, incorporating considerations such as communication network design, phasor data concentrators (PDCs), optical fiber connectivity, existing control modules (CMs), and $N - 1$ single-line outage contingencies. Their method can also consider cases in which PMUs and fiber-optic routes have already been deployed.

4.2.4. Gravitational Search-Based OPP

A nature-inspired computational method based on Newtonian gravity and motion principles has been applied for optimal PMU placement in a power system. In reference [91], an Optimal PMU Placement (OPP) framework based on the GSA is proposed that maximizes system observability. The Dijkstra Algorithm was used to obtain the efficient communication paths from the Phasor Measurement Units (PMUs) to the Phasor Data Concentrator (PDC). Extending this, reference [92] proposed a binary version of GSA to guarantee full network observability. Their formulation solves a multi-objective optimization problem whereby it tried to minimize the total number of PMUs while maximizing redundancy in observability. The results showed a reduction or maintainance of PMU counts and a better or equal measure of observability performance of the network.

4.2.5. Genetic Algorithm (GA) Based OPP

The genetic algorithm, a widely recognized metaheuristic optimization method, draws its conceptual foundation from Darwinian evolution, particularly the mechanism of natural selection and genetic crossover. In addressing the Optimal PMU Placement (OPP) problem under the constraint of single PMU failure, in reference introduced a framework that incorporated system security considerations and realistic scenarios involving synchrophasor pre-allocation. Satish Kumar et al. explored a hybrid approach combining Integer Linear Programming (ILP) with genetic algorithms to achieve full system observability, optimizing computational efficiency by leveraging a root vector-based observability test instead of conventional matrix triangularization. Marin et al. applied genetic algorithms to formulate the OPP for Complete Network Observability (CNO), revealing a correlation between the quantity of PMUs and the current phasors each unit could measure, ultimately minimizing both the number of PMUs and the measured phasors. Further extending the application, Kumar et al. utilized genetic algorithms to address OPP with a focus on component reliability and system observability enhancement. Their methodology generated multiple optimal solutions for CNO, from which the most appropriate configuration was selected using a proposed reliability index that guided PMU placement based on observability criteria. Additionally, the Analytic Hierarchy Process (AHP) was employed to facilitate phased synchrophasor deployment. In reference contributed to the field by suggesting a co-optimization model for PMU deployment alongside communication infrastructure (CI), aimed at minimizing propagation delays in wide-area monitoring systems. Their study accounted for both link reliability and regional topological diversity, concluding that microwave-based CI offered superior performance in terms of cost and reliability, as demonstrated through analysis of India's eastern power grid.

(a) Binary Genetic Algorithm (BGA)
In Binary Genetic Algorithms (BGA), individuals within the population are encoded using binary strings, where each gene is represented by either a 0 or a 1. These binary genes are then mapped to real-valued numbers within the normalized range [0,1] to facilitate interpretation in continuous optimization spaces. In reference [100] introduced a multistage Optimal PMU Placement (OPP) strategy leveraging BGA, where priority is given to selecting the most critical buses in the initial stages, thereby enhancing the efficiency and applicability of the placement process in practical power system scenarios.

(b) Non-Dominated Sorting Genetic Algorithm (NSGA)
The Non-dominated Sorting Genetic Algorithm (NSGA) is widely utilized for tackling multi-objective optimization problems due to its effectiveness. However, it has faced criticism, particularly concerning its high computational demands and absence of elitism in solution selection. To address these challenges, in reference [101] introduced the Optimized Pareto Procedure (OPP), which aims to enhance the generation of Pareto-optimal solutions. Unlike approaches that focus on identifying a singular key solution, OPP constructs a comprehensive Pareto-optimal front, making it especially suitable for applications involving extensive multi-objective search spaces.

(c) Immunity Genetic Algorithm (IGA)
The Immune Genetic Algorithm (IGA), drawing inspiration from the biological immune system's defense mechanisms, integrates genetic algorithm principles to efficiently refine the initial pool of antibody solutions. In their study, in reference [102] introduced an Optimal PMU Placement (OPP) framework leveraging IGA. By embedding an immune-based operator within the standard genetic algorithm structure, the overall performance of the optimization process was significantly enhanced. Central to this approach was the incorporation of prior domain knowledge particularly derived from system observability and overlooked network conditions conceptualized as a vaccine. The strategic application of this vaccine led to notably faster convergence. Furthermore, in reference [103] expanded upon this concept by developing an OPP strategy applicable to both static and dynamic PMU deployment scenarios, all under the $N-1$ contingency criterion. A novel inclusion of the $N-1$ reliability index in the objective function enabled the model to account for measurement channel effects and redundancy. The primary goal was to minimize the total number of PMUs, while secondary aims involved reducing voltage and current measurement requirements. The study compared two expansion strategies: one allowing for existing PMU repositioning and another preserving current placements while only adding new units. Results indicated the latter approach offers greater feasibility in real-world applications.

(d) Cellular Genetic Algorithm (CGA)
The Clustering Genetic Algorithm (CGA) is a hybrid optimization approach that integrates principles from both evolutionary algorithms and traditional genetic algorithms. Unlike standard GA techniques where mating occurs randomly, CGA restricts crossover operations to occur only between individuals and their nearest neighbors. This approach encompasses three main components: selection, variation, and replacement. In their work, In reference [104] introduced an Optimal PMU Placement (OPP) strategy utilizing CGA, taking into account key factors such as channel availability, fundamental system observability, the resilience of the metering infrastructure, and $N-1$ contingency conditions. Their

findings demonstrate that accounting for PMU channel availability can significantly reduce the cost associated with optimal metering schemes.

4.2.6. Particle Swarm Optimization (PSO) Based OPP

Particle Swarm Optimization (PSO) is one of the evolutionary computation methods drawing inspiration from the coordinated movement patterns of bird flocks and fish schools. A population-based algorithm, PSO starts by randomly creating a bunch of candidate solutions and then iteratively works on refining these as generations proceed to converge onto an optimal solution [105,106]. In BPSO, the binary version of the basic PSO, the algorithm adapts the search to discrete space while maintaining some key elements of the continuous model, such as velocity and momentum.

Reference [107] describes an Optimal PMU Placement (OPP) approach to state estimation in systems with Conventional Non-observability (CNO) using a modified PSO technique coupled with the weighted least squares method. Reference [108] upgraded this work by introducing a more economically realistic model that took into account hidden costs associated with PMU deployment costs, which are often ignored yet can substantially affect the installation budgets. They treated the acquired practical expenses as an essential part of this total cost.

On the other hand, [107] proposed an OIE-based method for minimizing the number of substations needed to keep the system observable. Their PSO method took real-world constraints into account and underscored the fact that the main cost in the establishment of substations lies in deployment logistics rather than in the equipment itself. In [110] extended BPSO applying v-shaped sigmoid transfer functions and mutation strategies so the algorithm could tackle complex OPP scenarios with CNO, Zero Injection Buses (ZIB), N − 1 contingencies, and PMUs channel constraints. Their work also carried a strong focus on measurement redundancy.
In [111], they have proposed the Exponential Binary PSO Algorithm, which aims at enhancing the search performance by adhering to a nonlinear inertia weight mechanism, thus speeding up the convergence rate as well as exploration.

4.2.7. Cellular Learning Automata (CLA) Based OPP

CA are discrete spatially distributed dynamical systems usually used in the study or simulation of some physical phenomena. An extension of multi-objective OPP framework was presented in [112] considering enhanced measurement redundancy, critical measurements (CMs), zero-injections at buses (ZIBs), and N−1-contingency conditions under constrained network observability (CNO). A CLA approach with newly defined local transition rules was used to find a solution to this very difficult placement problem.

4.2.8. Artificial Bee Colony Based (ABC) OPP

The Artificial Bee Colony algorithm is bio-inspired in that it imitates the foraging behavior of honeybees. Loukais, in [113], proposed a multi-objective model for PMU placement (OPP) concerning both control network observability (CNO) and enhanced voltage stability. To resolve the conflicting objectives therein, the author applied an ABC algorithm in its binary form, with a fuzzy logic component. PMU placement was given extra importance in weak buses as determined by fast voltage stability index, striving to minimize outages for the system.

4.2.9. Ant Colony (AC) Based OPP

The Ant Colony Optimization indeed hinges on collective foraging behavior of ants, which are arguably most efficient in finding shortest paths within a graph. Being highly robust in nature, it adapts to dynamically shifting network topologies, unlike, say, simulated annealing or genetic algorithms. An OPP problem was addressed using a two-phase strategy [114]. In the first phase, the ant colony algorithm is applied to find the minimum number of PMUs and their optimal placement while considering channel capacity constraints and N − 1 contingency scenarios to ensure complete network observability (CNO). Then, the second phase concentrates on a refinement approach that aims to minimize the number of PMUs to comply with channel capacity constraints without disrupting CNO.

4.2.10. Binary Imperialistic Competition Algorithm (BICA) Based OPP

The Imperialist Competitive Algorithm (ICA), initially propounded by in references [115], is based on models of socio-political dynamics. A binary version of ICA (BICA) was developed and implemented in reference [116] to solve the Optimal Phasor Measurement Unit Placement (OPP) problem while acknowledging the existing synchrophasor installations and optical fiber communication infrastructure within the power grid. On the other hand, reference [117] employs BICA towards solving the OPP problem, taking into account the constraints posed by Communication Network Optimization (CNO).

4.2.11. Tabu Search (TS) Based OPP

Tabu search provides an extremely productive way of tackling optimization in case of complex models with many parameters usually yielding better-quality solutions. In spite of complex-day implementation, TS being once established displays an increasing potential to be used in various optimization schemes. Korres et al. in their study [118] developed an Optimal PMU Placement (OPP) frame based on a recursive type of the Tabu Search (RTS) for the Complete Network Observability (CNO) problem. They experimented with two important parameters of TS: tabu list length and number of iterations, while bore at the performance of three different initialization techniques. The work finally drew some quantitative results on system observability.

4.2.12. Simulated Annealing Based OPP

Simulated annealing is employed to generate candidate structural configurations for energy landscape analysis, allowing the algorithm to escape local barriers and discover minimal energy zones. Thus, a large degree of freedom in selecting initial conditions is generated. In Reference [119], a tree search was combined with simulated annealing in a hybrid technique to solve the Optimal PMU Placement (OPP) problem; it was observed that the size of unobservable areas gradually decreases with increasing numbers of candidate node observabilities (CNOs). The methodology was used not only to solve the OPP but also to locate the best sites in which to place new facilities as well.

4.3. Machine Learning (ML) Based OPP

Machine learning establishes adroitness in computer systems to learn patterns and improve upon performance based on such data without being programmed for any specific task. A machine learning technique would offer solutions that are effective and fast in the optimization world, provided there is access to continuous real-time streams of data. Among these approaches would be to apply logistic regression models as well as train with artificial neural networks for greater decision-making accuracy.

In Reference 120,[Unnumbered] employed Bayesian network modeling for the assessment of the generalized reliability of Phasor Measurement Units (PMUs) by analyzing the operational features and interdependence of internal components and submodules of PMUs. The reliability of the system was calculated by taking into account the functioning or failure status within the individual components of a PMU. On this basis, a reliability allocation procedure was developed to improve system reliability by strengthening the critical subcomponents. To confirm the viability of these improvements, key reliability measures such as Observability Reliability-OR, Loss of Data Expectation-LODE, and Loss of Situational Awareness-LOSA were introduced. Meanwhile, an important aspect of single-line outage detection was handled in Reference 121, representing bus node voltage phasors as being due to changes in network topology. In an idealized situation, with PMUs installed at all buses, a regularized optimization procedure was then implemented with the aim of identifying the smallest set of buses for which PMUs could still detect most line outages well. Simulation results showed that placing PMUs on only 25% of buses could give classification performances nearly equivalent to those of the fully instrumented case, thus rating the efficiency of the optimization-based deployment strategy.

5. DISCUSSION AND FUTURE WORK

5.1. Testbed Systems

In studies addressing the Optimal PMU Placement (OPP) problem, researchers have predominantly utilized standard IEEE benchmark systems such as the 14-bus, 30-bus, 57-bus, and 118-bus networks for result verification and comparative analysis. Among the more extensive systems employed for evaluating large-scale OPP frameworks, the Polish power grid and systems from major energy corporations represent the largest test cases to date, comprising 2,746 and 2,285 buses, respectively.

5.2. Optimization Solvers

To address the challenge of optimal PMU placement, researchers have employed a variety of optimization solvers that enhance planning and resource allocation under constrained conditions. These computational tools play a crucial role in formulating efficient strategies for the deployment of limited resources in power systems. Among the widely adopted platforms in existing studies are **MATLAB, TOMLAB, IBM's CPLEX, ILOG, VPLEX, GAMS,** and **GUROBI**, each offering unique capabilities suited for solving complex optimization problems in this domain.

5.3. Pros and Cons

Various optimization methods have been used to solve the OPP problem, each with its specific strengths and limitations. Linear programming (LP) is typically chosen when the problem is formulated as a linear program, such as with Complete Network Observability (CNO) constraints. Real-world examples, however, introduce nonlinearities that cannot be addressed by standard LP. Integer Linear Programming (ILP) though extremely fast computationally, has difficulty when nonlinear constraints are involved. Hence, linearization approaches are taken in formulating those problems to convert nonlinear parts into linear. The Particle Swarm Optimization (PSO) has gained popularity among others due to the ease of implementation, fewer parameters to tune, and efficient control over the solutions obtained. However, the computational cost increases dramatically with the growth of the solution space, and thus PSO becomes ineffective for large-scale systems.

A contrasting view suggests the use of GA in Pareto front identification, where the methods can provide more solutions than single-point methods, though the downside being its longer execution time. The greedy algorithm is an efficient method that quickly finds local optima but cannot generally find the global optimum. Simulated Annealing achieves full system observability and dynamic features of the power grid very well while being highly computation-intensive, thereby warranting the implementation of time-reduction techniques. Artificial Neural Networks offer a flexible approach to modeling a solution based on its network structure; however, these systems can become too complex and therefore hinder performance and scalability. Comparatively, ILP is found to be better than conventional methods, whereas metaheuristic algorithms like PSO and GA outperform heuristic and machine learning approaches in solving the OPP problem.

5.4. Future Scope

Several critical aspects within the PMU placement problem remain insufficiently explored and warrant deeper analytical investigation.

5.4.1. Application Based Planning

In recent studies, the focus of researchers has expanded beyond achieving full network observability in the Optimal PMU Placement (OPP) problem. Emphasis is now being placed on identifying PMU locations that are optimal from a practical application standpoint. Key areas gaining increasing attention include OPP strategies integrated with controlled islanding, enhanced fault tolerance, small-signal stability assessment, and voltage stability monitoring.

5.4.2. Node-Breaker Model

Most existing studies on PMU (Phasor Measurement Unit) placement have predominantly utilized simplified representations such as the bus-bus or bus-branch models, primarily applied to standard IEEE test systems, with limited validation on real-world power networks. However, actual power systems are more accurately represented by the node-breaker model. Unlike the simplified models, the node-breaker framework enables more precise monitoring and control, especially under contingency conditions, and eliminates the need for network reduction facilitating integrated state estimation and accurate topology identification without relying on approximations In Reference [122–126]. Despite its advantages, the adoption of the node-breaker model for PMU placement remains largely underexplored. A key challenge lies in the requirement for detailed, often proprietary, system information, which limits its application in practical deployment scenarios.

5.4.3. Hybrid Optimization Algorithm

A hybrid algorithm integrates multiple methodologies aimed at addressing the same computational challenge, either by selecting the most suitable approach based on input characteristics or by dynamically alternating between them during execution. Within the context of the Optimal PMU Placement (OPP) problem, the integration of machine learning, artificial intelligence, and conventional optimization methods remains a relatively underexplored direction, offering significant potential for future investigation. Furthermore, we assert that exhaustive search strategies merit additional attention. Since OPP is inherently an offline optimization task, greater computational effort can be justified if it leads to reduced deployment costs for PMUs, as elaborated in Section 3.2.

5.4.4. Performance Evaluators

In the Optimal PMU Placement (OPP) problem, a variety of techniques yield multiple solution sets depending on the chosen constraints and considered critical nodes or observability criteria (CNO). However, the existing methods lack a comprehensive set of performance metrics for assessing these solutions. There is a clear need for further research to develop and integrate diverse performance indices that can effectively evaluate and compare the quality of the obtained solution sets.

5.4.5. μPMU Placement

The exploration of Optimal PMU Placement (OPP) within distribution systems has received relatively limited attention in existing literature. Unlike transmission networks, distribution systems exhibit smaller variations in the magnitudes and phase angles of electrical parameters, leading to distinct operational challenges. These include the absence of standardized protocols, complications arising from distributed generation such as islanding, reverse power flow, and issues like inadvertent tripping. To address these challenges, micro-phasor measurement units (μPMUs) have been introduced, specifically engineered for the unique demands of distribution networks. However, enhancing the resolution of μPMUs to suit distribution-level applications significantly increases their cost compared to conventional PMUs used in transmission systems. Despite this, μPMUs hold promising potential across various domains including real-time operation, system control, reliability enhancement, and strategic planning, as emphasized in recent studies. Nonetheless, the advancement of μPMU technology for wide-area monitoring systems (WAMS) is constrained by the lack of robust communication infrastructure and standardized frameworks.

6. CONCLUSIONS

One of the main issues in modern power system monitoring is enabling transition from traditional power grids to advanced intelligent networks while keeping hopeful observability of the system through PMUs. Full or almost-full observability with the least possible number of PMUs is the objective with primary concern, which basically leads to the formulation of the Optimal PMU Placement (OPP) problem. This research merges a wide variety of optimization approaches with their corresponding objective functions, which range from merely cost minimization of PMU installation to maximization of the observable network area. Also, it considers a wide array of relevant constraints that are taken into account in PMU placement, including full coverage of the system, the possibility of incorporating conventional measurements, zero-injection buses, handling of contingencies, consideration of communication infrastructure, controlled islanding, or attributes such as an increase in PMU cost, decreasing number of channels per PMU, multi-stage installation, and the existence of already installed PMUs.

Benchmarking test systems such as IEEE 14-bus, 30-bus, 57-bus, and 118-bus models are generally used to evaluate the solution of OPP. The implementation phase sees the employment of different solvers such as MATLAB, TOMLAB, IBM ILOG, VPLEX, GAMS, and GUROBI, with ILP being one of the most used techniques. In the discussion section, the critics views will be addressed concerning the pros and cons of different optimization techniques used in the OPP. Hence, the paper aims to present a more guided approach of the research field, emphasizing research areas that lack attention such as application-driven planning, node-breaker modeling for a more practical representation, hybrid optimization approaches, and standardization of performance metrics. These gaps call for attention and further research to be made in achieving effective and scalable solutions for future power systems.

REFERENCES :

[1] Saly, S.; Signer, K.; Sullivan, A. Power System Monitoring. IFAC Proc. Vol. 1980, 13, 189–198.
[2] Sefid, M.; Rihan, M. Optimal PMU placement in a smart grid: An updated review. Int. J. Smart Grid Clean Energy 2019, 8, 59–69.
[3] Dusabimana, E.; Yoon, S.G. A survey on the micro-phasor measurement unit in distribution networks. Electronics 2020, 9, 305.

[4] Madani, V.; Parashar, M.; Giri, J.; Durbha, S.; Rahmatian, F.; Day, D.; Adamiak, M.; Sheble, G. PMU placement considerations – A roadmap for optimal PMU placement. In Proceedings of the 2011 IEEE/PES Power Systems Conference and Exposition, Phoenix, AZ, USA, 20–23 March 2024; pp. 1–7.
[5] Virginia, D.; Kyle, P.; Gardner, M.; Energy, D.; Bradberry, T. Using Synchrophasor Data for Phase Angle Monitoring NASPI Control Room Solutions Task Team Paper. pp. 1–54.
[6] Moxley, R.; Anderson, C.; Laboratories, S.E. Uses and Limitations of Advanced Analog Quantities. 2003; pp. 1–7.
[7] Phadke, A.G. Synchronized phasor measurements: A historical overview. Proc. IEEE Power Eng. Soc. Transm. Distrib. Conf. 2002, 1, 476–479.
[8] Mohanta, D.K.; Murthy, C.; Sinha Roy, D. A Brief Review of Phasor Measurement Units as Sensors for Smart Grid. Electr. Power Components Syst. 2025, 44, 411–425.
[9] Hojabri, M.; Dersch, U.; Papaemmanouil, A.; Bosshart, P. A comprehensive survey on phasor measurement unit applications in distribution systems. Energies 2024, 12, 4552.
[10]Liu, Y.; Wu, L.; Li, J. D-PMU based applications for emerging active distribution systems: A review. Electr. Power Syst. Res. 2024, 179, 106063.
[11]Kerdphol, T.; Matsukawa, Y.; Watanabe, M.; Mitani, Y. Application of PMUs to monitor large-scale PV penetration infeed on frequency of 60 Hz Japan power system: A case study from Kyushu Island. Electr. Power Syst. Res. 2020, 185, 106393.
[12]Pal, A.; Chatterjee, P.; Thorp, J.S.; Centeno, V.A. Online Calibration of Voltage Transformers Using Synchrophasor Measurements. IEEE Trans. Power Deliv. 2016, 31, 370–380.
[13]Ahmed, M.; Imran, K. An Optimal PMU Placement Against N-1 Contingency of PMU Using Integer Linear Programming Approach. In Proceedings of the 2019 International Conference on Applied and Engineering Mathematics, Taxila, Pakistan, 27–29 August 2019.
[14]Tao, W.; Ma, M.; Ding, M.; Xie, W.; Fang, C. A priority-based synchronous phasor transmission protocol extension method for the active distribution network. Appl. Sci. 2019, 9, 2135.
[15]Nanda, P.; Panigrahi, C.K.; Dasgupta, A. Phasor Estimation and Modelling Techniques of PMU—A Review. Energy Procedia 2017, 109, 64–77.
[16]Dubey, R.; Popov, M.; Muro, J.d.J.C. Cost-Effective Wide Area Measurement Systems for Smart Power Network. IEEE Power Energy Technol. Syst. J. 2018, 5, 85–93.
[17]Yuill, W.; Edwards, A.; Chowdhury, S.; Chowdhury, S.P. Optimal PMU placement: A comprehensive literature review. In Proceedings of the 2011 IEEE Power and Energy Society General Meeting, Detroit, MI, USA, 24–28 July 2011; pp. 1–8.
[18]Manousakis, N.M.; Korres, G.N.; Georgilakis, P.S. Taxonomy of PMU placement methodologies. IEEE Trans. Power Syst. 2022, 27, 1070–1077.
[19]Synchrophasor Measurement Technology in Power Systems: Panorama and State-of-the-Art. IEEE Access 2014, 2, 1607–1628.
[20]Negash, K.; Khan, B.; Yohannes, E. Artificial Intelligence Versus Conventional Mathematical Techniques: A Review for Optimal Placement of Phasor Measurement Units. Technol. Econ. Smart Grids Sustain. Energy 2016, 1, 1–13.
[21]Baba, M.; Nor, N.B.; Ibrahim, T.B.; Sheikh, M.A. A comprehensive review for optimal placement of phasor measurement unit for network observability. Indones. J. Electr. Eng. Comput. Sci. 2020, 19, 301–308.
[22]Johnson, T.; Moger, T. A critical review of methods for optimal placement of phasor measurement units. Int. Trans. Electr. Energy Syst. 2022, 31, e12698.
[23]Abdulkareem, A.; Ogbe, D.; Somefun, T. Review of different methods for optimal placement of phasor measurement unit on the power system network. Adv. Sci. Technol. Eng. Syst. 2021, 5, 1071–1081.
[24]Kallrath, J.; Molone, E. Brief Review of Mathematical Optimization; Springer: Berlin, Germany, 2009; pp. 351–427.
[25]Adibhatla, S.; Leigh, D.C. A comparison of optimization methods for engineering system optimization. Eng. Optim. 1987, 12, 13–23.
[26]Lee, J.Y.; Hwang, S.N. A high-gain boost converter using voltage-stacking cell. Trans. Korean Inst. Electr. Eng. 2008, 57, 982–984.
[27]Sodhi, R.; Srivastava, S.C.; Singh, S.N. Optimal PMU placement method for complete topological and numerical observability of power system. Electr. Power Syst. Res. 2010, 80, 1154–1159.
[28]Lukomski, R.; Wilkosz, K. Power network observability for state estimation review of the methods. In Proceedings of the 9th International Scientific Conference Electric Power Engineering 2008, EPE 2008, Brno, Czech Republic, 13–15 May 2008; pp. 183–190.
[29]Crainic, E.D.; Do, X.D.; Mukhedkar, D.; Horisberger, H.P. Power network observability: The assessement of the measurement system strength. IEEE Trans. Power Syst. 1990, 5, 1267–1285.
[30]Dehghani, M.; Goel, L.; Li, W. PMU based observability reliability evaluation in electric power systems. Electr. Power Syst. Res. 2014, 116, 347–354.
[31]Chen, X.; Tseng, K.J.; Amaratunga, G. State estimation for distribution systems using micro-synchrophasors. In Proceedings of the Asia-Pacific Power and Energy Engineering Conference, Suzhou, China, 15–17 April 2016.
[32]Resende, M.G.; Ribeiro, C.C. Optimization by GRASP; Springer Science + Business Media: New York, NY, USA, 2016.
[33]Li, Q.; Cui, T.; Weng, Y.; Negi, R.; Franchetti, F.; Ilic, M.D. An information-theoretic approach to PMU placement in electric power systems. IEEE Trans. Smart Grid 2013, 4, 446–456.
[34]Yang, P.; Tan, Z.; Wiesel, A.; Nehorai, A. Placement of PMUs Considering Measurement Phase-Angle Mismatch. IEEE Trans. Power Deliv. 2015, 30, 914–922.
[35]Islam, S.M.S.; Mohammad, H.; Science, D. Linear Programming; Prentice-Hall, Inc.: Englewood Cliffs, NJ, USA, 2018
[36]Abbasy, N.H.; Ismail, H.M. A unified approach for the optimal PMU location for power system state estimation. IEEE Trans. Power Syst. 2009, 24, 806–813.
[37]Enshaee, A.; Hooshmand, R.A.; Fesharaki, F.H. A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies. Electr. Power Syst. Res. 2012, 89, 1–10.
[38]Amare, K.; Centeno, V.A.; Pal, A. Unified PMU Placement Algorithm for Power Systems. In Proceedings of the 2015 North

American Power Symposium (NAPS), Charlotte, NC, USA, 4–6 October 2015; p. 6.
[39]Rezaeian Koochi, M.H.; Esmaeili, S.; Ledwich, G. Locating minimum number of PMUs for pre- and post-disturbance monitoring of power systems. IET Gener. Transm. Distrib. 2019, 13, 127–136.
[40]Mahaei, S.M.; Hagh, M.T. Minimizing the number of PMUs and their optimal placement in power systems. Electr. Power Syst. Res. 2012, 83, 66–72.
[41]Bei, X.; Yoon, Y.J.; Abur, A. Optimal Placement and Utilization of Phasor Measurements for State Estimation; PSERC Publication: Tempe, AZ, USA, 2005; pp. 1–6.
[42]Sarailoo, M.; Wu, N.E. Cost-Effective Upgrade of PMU Networks for Fault-Tolerant Sensing. IEEE Trans. Power Syst. 2018, 33, 3052–3063.
[43]Gou, B. Generalized integer linear programming formulation for optimal PMU placement. IEEE Trans. Power Syst. 2008, 23, 1099–1104.
[44]Kavasseri, R.; Srinivasan, S. Joint placement of phasor and conventional power flow measurements for fault observability of power systems. IET Gener. Transm. Distrib. 2011, 5, 1019.
[45]Sodhi, R.; Srivastava, S.; Singh, S. Multi-criteria decision-making approach for multi-stage optimal placement of phasor measurement units. IET Gener. Transm. Distrib. 2011, 5, 181.
[46]Dobakhshari, A.S.; Member, S. Optimal integration of phasor measurement units in power systems considering conventional measurements. IEEE Trans. Smart Grid 2013, 4, 1113–1121.
[47]Gou, B.; Kavasseri, R.G. Uni fi ed PMU Placement for Observability and Bad Data Detection in State Estimation. IEEE Trans. Power Syst. 2014, 29, 2573–2580.
[48]Pal, A.; Vullikanti, A.K.S.; Ravi, S.S. A PMU Placement Scheme Considering Realistic Costs and Modern Trends in Relaying. IEEE Trans. Power Syst. 2017, 32, 552–561.
[49]Khajeh, K.G.; Bashar, E.; Rad, A.M.; Gharehpetian, G.B. Integrated Model Considering Effects of Zero Injection Buses and Conventional Measurements on Optimal PMU Placement. IEEE Trans. Smart Grid 2017, 8, 1006–1013.
[50]Singh, S.P.; Thakur, A.K.; Singh, S.P. PMU Placement for Maximum Observability of Power System under Different Contingencies. Energy Procedia 2017, 117, 893–900.
[51]Gou, B. Optimal Placement of PMUs by Integer Linear Programming. IEEE Trans. Power Syst. 2008, 23, 1525–1526.
[52]Pal, A.; Mishra, C.; Vullikanti, A.K.S.; Ravi, S. General optimal substation coverage algorithm for phasor measurement unit placement in practical systems. IET Gener. Transm. Distrib. 2017, 11, 347–353.
[53]Rashidi, M.; Farjah, E. Lyapunov exponent-based optimal PMU placement approach with application to transient stability assessment. IET Sci. Meas. Technol. 2016, 10, 492–497.
[54]Rakpenthai, C.; Premrudeepreechacharn, S.; Uatrongjit, S.; Watson, N.R. An optimal PMU placement method against measurement loss and branch outage. IEEE Trans. Power Deliv. 2007, 22, 101–107.
[55]Emami, R.; Abur, A. Robust measurement design by placing synchronized phasor measurements on network branches. IEEE Trans. Power Syst. 2010, 25, 38–43.
[56]Huang, L.; Sun, Y.; Xu, J.; Gao, W.; Zhang, J.; Wu, Z. Optimal PMU placement considering controlled islanding of power system.
[57]IEEE Trans. Power Syst. 2014, 29, 742–755. MICHELOGIANNAKIS G, SANCHEZ D, DALLY W J, et al. Evaluating Bufferless Flow Control for On-Chip Networks Proceedings of the 4th ACM/IEEE International Symposium on Networks-on-Chip (NOCS 2010).
[58]Mousavian, S.; Valenzuela, J.; Wang, J. A two-phase investment model for optimal allocation of phasor measurement units considering transmission switching. Electr. Power Syst. Res. 2015, 119, 492–498considering transmission switching. Electr. Power Syst. Res. 2015, 119, 492–498
[59]Huang, J.; Wu, N.E.; Ruschmann, M.C. Data-availability-constrained placement of PMUs and communication links in a power system. IEEE Syst. J. 2014, 8, 483–492.
[60]Kekatos, V.; Giannakis, G.B.; Wollenberg, B. Optimal placement of phasor measurement units via convex relaxation. IEEE Trans. Power Syst. 2012, 27, 1521–1530.
[61]Bae, J. Cost-Effective placement of phasor measurement units to defend against false data injection attacks on power grid. Energies 2020, 13, 3862.
[62]Chen, X.; Wei, F.; Cao, S.; Soh, C.B.; Tseng, K.J. PMU Placement for Measurement Redundancy Distribution Considering Zero Injection Bus and Contingencies. IEEE Syst. J. 2020, 14, 5396–5406.
[63]Rezaeian Koochi, M.H.; Dehghanian, P.; Esmaeili, S. PMU Placement with Channel Limitation for Faulty Line Detection in Transmission Systems. IEEE Trans. Power Deliv. 2020, 35, 819–827.
[64]Hyacinth, L.R.; Gomathi, V. Optimal pmu placement technique to maximize measurement redundancy based on closed neighbourhood search. Energies 2021, 14, 4782.
[65]Baba, M.; Nor, N.B.; Sheikh, M.A.; Baba, A.M.; Irfan, M.; Glowacz, A.; Kozik, J.; Kumar, A. Optimization of phasor measurement unit placement using several proposed case factors for power network monitoring. Energies 2021, 14, 5596.
[66]Elimam, M.; Isbeih, Y.J.; Moursi, M.S.E.; Elbassioni, K.; Hosani, K.H.A. Novel Optimal PMU Placement Approach Based on the Network Parameters for Enhanced System Observability and Wide Area Damping Control Capability. IEEE Trans. Power Syst. 2021, 36, 5345–5358.
[67]Aminifar, F.; Fotuhi-Firuzabad, M.; Shahidehpour, M.; Khodaei, A. Probabilistic multistage PMU placement in electric power systems. IEEE Trans. Power Deliv. 2011, 26, 841–849.
[68]Esmaili, M.; Gharani, K.; Shayanfar, H.A. Redundant observability PMU placement in the presence of flow measurements considering contingencies. IEEE Trans. Power Syst. 2013, 28, 3765–3773.
[69]Nikkhah, S.; Aghaei, J.; Safarinejadian, B.; Norouzi, M.A. Contingency constrained phasor measurement units placement with n-k redundancy criterion: A robust optimisation approach. IET Sci. Meas. Technol. 2018, 12, 151–160.
[70]Aminifar, F.; Fotuhi-firuzabad, M.; Member, S. Observability of Hybrid AC/DC Power Systems With Variable-Cost PMUs. IEEE Trans. Power Deliv. 2014, 29, 345–352.
[71]Ruben, C.; Dhulipala, S.C.; Bretas, A.S.; Guan, Y.; Bretas, N.G. Multi-objective MILP model for PMU allocation

considering enhanced gross error detection: A weighted goal programming framework. Electr. Power Syst. Res. 2020, 182, 106235.
[72]Zhu, X.; Wen, M.H.; Li, V.O.; Leung, K.C. Optimal PMU-Communication Link Placement for Smart Grid Wide-Area Measurement Systems. IEEE Trans. Smart Grid 2019, 10, 4446–4456.
[73]Azizi, S.; Dobakhshari, A.S.; Nezam Sarmadi, S.A.; Ranjbar, A.M. Optimal PMU placement by an equivalent linear formulation for exhaustive search. IEEE Trans. Smart Grid 2012, 3, 174–182.
[74]Manousakis, N.M.; Korres, G.N. AWeighted Least Squares Algorithm for Optimal PMU Placement. IEEE Trans. Power Syst. 2013, 28, 3499–3500.
[75]Chakrabarti, S.; Kyriakides, E.; Eliades, D.G. Placement of synchronized measurements for power system observability. IEEE Trans. Power Deliv. 2009, 24, 12–19.
[76]Qi, J.; Sun, K.; Kang, W. Optimal PMU Placement for Power System Dynamic State Estimation by Using Empirical Observability Gramian. IEEE Trans. Power Syst. 2015, 30, 2041–2054.
[77]Löfberg, J.; Manousakis, N.M.; Korres, G.N.; Xygkis, T.C. Optimal phasor measurement unit placement for numerical observability in the presence of conventional measurements using semi-definite programming. IET Gener. Transm. Distrib. 2015, 9, 2427–2436.
[78]Manousakis, N.M.; Korres, G.N. Optimal PMU Placement for Numerical Observability Considering Fixed Channel Capacity-A Semidefinite Programming Approach. IEEE Trans. Power Syst. 2016, 31, 3328–3329.
[79]Li, X.; Scaglione, A.; Chang, T.H. A framework for phasor measurement placement in hybrid state estimation via Gauss-Newton. IEEE Trans. Power Syst. 2014, 29, 824–832.
[80]Shi, Y.; Tuan, H.D.; Duong, T.Q.; Poor, H.V.; Savkin, A.V. PMU Placement Optimization for Efficient State Estimation in Smart Grid. IEEE J. Sel. Areas Commun. 2020, 38, 71–83.
[81]Ghosh, P.K.; Chatterjee, S.; Saha Roy, B.K. Optimal PMU placement solution: Graph theory and MCDM-based approach. IET Gener. Transm. Distrib. 2017, 11, 3371–3380.
[82]Xie, N.; Torelli, F.; Bompard, E.; Vaccaro, A. A graph theory based methodology for optimal PMUs placement and multiarea power system state estimation. Electr. Power Syst. Res. 2015, 119, 25–33.
[83]Chakrabarti, S.; Kyriakides, E. Optimal placement of phasor measurement units for power system observability. IEEE Trans. Power Syst. 2008, 23, 1433–1440.
[84]Lu, C.; Wang, Z.; Ma, M.; Shen, R.; Yu, Y.; Member, S. An Optimal PMU Placement With Reliable Zero Injection Observation. IEEE Access 2018, 6, 54417–54426.
[85]Aghaei, J.; Baharvandi, A.; Rabiee, A.; Akbari, M.A. Probabilistic PMU Placement in Electric Power Networks: An MILP-Based Multiobjective Model. IEEE Trans. Ind. Inform. 2015, 11, 332–341.
[86]Esmaili, M.; Ghamsari-Yazdel, M. Voltage Stability-Constrained Optimal Simultaneous Placement of PMUs and Channels Enhancing Measurement Reliability and Redundancy. IEEE Power Energy Technol. Syst. J. 2017, 4, 32–39.
[87]Lu, C.; Wang, Z.; Yu, Y. Optimal PMU placement for pessimistic dynamic vulnerability assessment. IET Gener. Transm. Distrib. 2018, 12, 2231–2237.
[88]Wen, M.H.; Xu, J.; Li, V.O. Optimal multistage PMU placement for wide-area monitoring. IEEE Trans. Power Syst. 2013, 28, 4134–4143.
[89]Kazemi Karegar, H.; Dalali, M. Optimal PMU placement for full observability of the power network with maximum redundancy using modified binary cuckoo optimisation algorithm. IET Gener. Transm. Distrib. 2016, 10, 2817–2824.
[90]Singh, S.P.; Singh, S.P. Optimal cost wide area measurement system incorporating communication infrastructure. IET Gener. Transm. Distrib. 2017, 11, 2814–2821.
[91]A Multi-objective PMU Placement Method in Power System via Binary Gravitational Search Algorithm. Electr. Power Components Syst. 2017, 45, 1832–1845.
[92]Alzohairy, A.M. Darwin's Theory Of Evolution. J. Genet. Univ. Zagagig 2014, 1, 1–23.
[93]Immanuel, S.D.; Chakraborty, U.K. Genetic Algorithm: An Approach on Optimization. In Proceedings of the 4th International Conference on Communication and Electronics Systems, ICCES 2019, Coimbatore, India, 17–19 July 2019; pp. 701–708.
[94]Castro, C.A.; Müller, H.H. Genetic algorithm-based phasor measurement unit placement method considering observability and security criteria. IET Gener. Transm. Distrib. 2016, 10, 270–280.
[95]Satish Kumar, K.; Sydulu, M. Optimal PMU Placement Techniques for the Topological Observability of a Partial Network of the Southern Grid of India; IFAC: Prague, Czech Republic, 2014; Volume 3, pp. 1044–1048.
[96]Marin, F.; Garcia-Lagos, F.; Joya, G.; Sandoval, F. Genetic algorithms for optimal placement of phasor measurement units in electrical networks. Electron. Lett. 2003, 39, 1403–1405.
[97]Kumar, S.; Tyagi, B.; Kumar, V.; Chohan, S. Incremental PMU placement considering reliability of power system network using analytical hierarchical process. IET Gener. Transm. Distrib. 2018, 12, 3900–3909.
[98]Appasani, B.; Mohanta, D.K. Co-optimal placement of PMUs and their communication infrastructure for minimization of propagation delay in the WAMS. IEEE Trans. Ind. Inform. 2018, 14, 2120–2132.
[99]Almasabi, S.; Mitra, J. Multistage Optimal PMU Placement Considering Substation Infrastructure. IEEE Trans. Ind. Appl. 2018, 54, 6519–6528.
[100] Milosevic, B.; Begovic, M. Nondominated sorting genetic algorithm for optimal phasor measurement placement. IEEE Trans. Power Syst. 2003, 18, 69–75.
[101] Aminifar, F.; Lucas, C.; Khodaei, A.; Fotuhi-Firuzabad, M. Optimal placement of phasor measurement units using immunity genetic algorithm. IEEE Trans. Power Deliv. 2009, 24, 1014–1020.
[102] Asgari, A.; Firouzjah, K.G. Optimal PMU placement for power system observability considering network expansion and N-1 contingencies. IET Gener. Transm. Distrib. 2018, 12, 4216–4224.
[103] Miljani ´c, Z.; Djurovi ´c, I.; Vujoševi ´c, I. Optimal placement of PMUs with limited number of channels. Electr. Power Syst. Res. 2012, 90, 93–98.
[104] Parsopoulos, K.E.; Vrahatis, M.N. Particle Swarm Optimization and Intelligence; IGI Global: Hershey, PA, USA, 2010;

Volume 270, pp. 373–394.
[105] Clerc, M. Discrete Particle Swarm Optimization, Illustrated by the Traveling Salesman Problem; Springer: Berlin/Heidelberg, Germany 2004; pp. 219–239.
[106] Saleh, A.A.; Adail, A.S.; Wadoud, A.A. Optimal phasor measurement units placement for full observability of power system using improved particle swarm optimisation. IET Gener. Transm. Distrib. 2017, 11, 1794–1800.
[107] Rather, Z.H.; Chen, Z.; Thogersen, P.; Lund, P.; Kirby, B. Realistic approach for phasor measurement unit placement: Consideration of practical hidden costs. IEEE Trans. Power Deliv. 2015, 30, 3–15.
[108] Pal, A.; Jones, K.D.; Mishra, C.; Centeno, V.A. Binary particle swarm optimisation-based optimal substation coverage algorithm for phasor measurement unit installations in practical systems. IET Gener. Transm. Distrib. 2016, 10, 555–562.
[109] Rahman, N.H.A.; Zobaa, A.F. Integrated Mutation Strategy with Modified Binary PSO Algorithm for Optimal PMUs Placement. IEEE Trans. Ind. Inform. 2017, 13, 3124–3133.
[110] Maji, T.K.; Acharjee, P. Multiple Solutions of Optimal PMU Placement Using Exponential Binary PSO Algorithm for Smart Grid Applications. IEEE Trans. Ind. Appl. 2017, 53, 2550–2559.
[111] Mazhari, S.M.; Monsef, H.; Lesani, H.; Fereidunian, A. A multi-objective PMU placement method considering measurement redundancy and observability value under contingencies. IEEE Trans. Power Syst. 2013, 28, 2136–2146.
[112] Arul Jeyaraj, K.; Rajasekaran, V.; Nandha Kumar, S.K.; Chandrasekaran, K. A multi-objective placement of phasor measurement units using fuzzified artificial bee colony algorithm, considering system observability and voltage stability. J. Exp. Theor. Artif. Intell. 2016, 28, 113–136.
[113] Mouwafi, M.T.; El-Sehiemy, R.A.; Abou El-Ela, A.A.; Kinawy, A.M. Optimal placement of phasor measurement units with minimum availability of measuring channels in smart power systems. Electr. Power Syst. Res. 2016, 141, 421–431.
[114] Hosseini, S.; Al Khaled, A. A survey on the Imperialist Competitive Algorithm metaheuristic: Implementation in engineering domain and directions for future research. Appl. Soft Comput. 2014, 24, 1078–1094.
[115] Mohammadi, M.B.; Hooshmand, R.A.; Fesharaki, F.H. A new approach for optimal placement of PMUs and their required communication infrastructure in order to minimize the cost of the WAMS. IEEE Trans. Smart Grid 2016, 7, 84–93.
[116] Mahari, A.; Seyedi, H. Optimal PMU placement for power system observability using BICA, considering measurement redundancy. Electr. Power Syst. Res. 2013, 103, 78–85.
[117] Korres, G.N.; Georgilakis, P.S.; Koutsoukis, N.C.; Manousakis, N.M. Numerical observability method for optimal phasor measurement units placement using recursive Tabu search method. IET Gener. Transm. Distrib. 2013, 7, 347–356.
[118] Nuqui, R.F.; Phadke, A.G. Phasor measurement unit placement techniques for complete and incomplete observability. IEEE Trans. Power Deliv. 2005, 20, 2381–2388.
[119] Ghosh, S.; Ghosh, D.; Kumar Mohanta, D. Multi-Level Reliability Allocation of Phasor Measurement Unit Using Bayesian Networks. Electr. Power Components Syst. 2018, 46, 1756–1768.
[120] Kim, T.; Wright, S.J. PMU placement for line outage identification via multinomial logistic regression. IEEE Trans. Smart Grid 2018, 9, 122–131.
[121] NERC. Node-Breaker Modeling Representation; Technical Report By NERC: North American Electric Reliability Corporation; NERC: Atlanta, GA, USA, 2016.
[122] Ramanathan, R.; Tuck, B. Contingency analysis using node/breaker model for operation studies. In Proceedings of the 2015 IEEE Power & Energy Society General Meeting, Denver, CO, USA, 26–30 July 2015.
[123] Park, B.; Demarco, C.L. Optimal Network Topology for Node-Breaker Representations with AC Power Flow Constraints. IEEE Access 2020, 8, 64347–64355.
[124] Celik, M.K.; Management, D.G. Mixed Node/Breaker and Bus/Branch Models Using CIM Exponentially Increasing Challenges for the Grid; Nexant, Inc.: San Francisco, CA, USA, 2015.
[125] Koenig, M.; Sagareli, S.; Vaiman, M.; Vaiman, M. Node-breaker topology representation of Con Edison's Stations for planning studies. In Proceedings of the 2015 IEEE Power & Energy Society General Meeting, Denver, CO, USA, 26–30 July 2015.
[126] Alam, M.N.; Das, B.; Pant, V. Protection scheme for reconfigurable radial distribution networks in presence of distributed generation. Electr. Power Syst. Res. 2021, 192, 106973.
[127] Lee, L.A.; Centeno, V. Comparison of mPMU and PMU. In Proceedings of the Clemson University Power Systems Conference (PSC), Clemson, SC, USA, 4–7 September 2019.
[128] Von Meier, A.; Arghandeh, R. Every Moment Counts: Synchrophasors for Distribution Networks with Variable Resources; Jones, L.E., Ed.; Academic Press: Cambridge, MA, USA, 2017; pp. 435–444.
[129] Ghalei Monfared Zanjani, M.; Mazlumi, K.; Kamwa, I. Combined analysis of distribution-level PMU data with transmission-level PMU for early detection of long-term voltage instability. IET Gener. Transm. Distrib. 2019, 13, 3634–3641.
[130] Samantaray, S.R.; Kamwa, I.; Joos, G. Phasor measurement unit based wide-area monitoring and information sharing between micro-grids. IET Gener. Transm. Distrib. 2017, 11, 1293–1302.
[131] Von Meier, A.; Stewart, E.; McEachern, A.; Andersen, M.; Mehrmanesh, L. Precision Micro-Synchrophasors for Distribution Systems: A Summary of Applications. IEEE Trans. Smart Grid 2017, 8, 2926–2936.
[132] Arghandeh, R.; Gahr, M.; Von Meier, A.; Cavraro, G.; Ruh, M.; Andersson, G. Topology detection in microgrids with microsynchrophasors. In Proceedings of the 2015 IEEE Power & Energy Society General Meeting, Denver, CO, USA, 26–30 July 2015.