

Mathematical Modeling of Vibration of Non-Homogeneous Orthotropic Trapezoidal Plate with Linear Variation In Density

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Abstract

The objective of present paper is to study the effect of aspect ratio, thermal gradient and non-homogeneity on vibration of non-homogeneous orthotropic trapezoidal plate with thickness varies parabolically in both directions. By using Rayleigh-Ritz method governing differential equations has been attained by taking two term deflection function corresponding to clamped-simply supported clamped-simply supported (C-S-C-S) boundary conditions. The effects of structural parameters such as aspect ratio, thermal gradient and non-homogeneity have also been studied. Results are calculated with great accuracy and compare the present model with the other in literature with the help of tables and graphs.

Keywords: Non homogeneity, aspect ratio, vibration, density, thickness

1. INTRODUCTION

In recent years, vibration of non-homogeneous plate under different condition of temperature attracts the mind of researcher in its direction. Also studying its applications in different fields of engineering. Vibrational effects are natural in machines, therefore every machine have the capacity to changes its characteristics under different temperature conditions. So, before finalizing the model of any system it is necessary for engineers and researchers to have full information about the modes of vibration. In this research paper we have produce a new mathematical model for engineers by studying the modes of vibration for non-homogeneous orthotropic plate with linear vibration in density. The results are validated with previous study in that direction. Gupta and Sharma [1] studied the vibrational effect of trapezoidal plate with varying thickness and density. Gupta and Sharma [2] analysed the vibrational behavior of trapezoidal plate with varying thickness parabolically. Gupta and Sharma [3] studied the thermal effect on trapezoidal plate with linear thickness and density. Gupta and Sharma [4] studied vibrational effect on orthotropic trapezoidal plate with linearly varying thickness. Gupta and Sharma [5] analyzed the vibrational behavior of orthotropic trapezoidal plate with parabolically varying thickness. Kavita et al. [6] investigated the vibrational analysis of non-homogeneous trapezoidal plate of varying thickness and density under thermal gradient with clamped and simply supported conditions. Kavita et al. [7] studied the thermal effect on vibrations of a symmetric non-homogeneous trapezoidal plate, whose thickness varies bilinearly and density varies parabolically. Leissa [8, 9, 10] did his work on vibration of plates using Rayleigh Ritz technique for two modes of frequency and produces so many new model in engineering fields. Tomar and Gupta [11] studied the vibrational effect on orthotropic rectangular plate with linearly varying thickness. Kalita et al. [12] studied the tapered thickness for rotatry isotropic plate. Sharma [13] produced the mathematical model for skew plate. Sharma [14] studied the vibrational effect on parallelogram plate with circular vibrations by developing a mathematical model with the help of soft computing. Kaur et al. [15] studied the tapered non-homogeneous rectangular plate with structural parameters.

2. MATHEMATICAL FORMULATION

2.1. Geometrical Representation

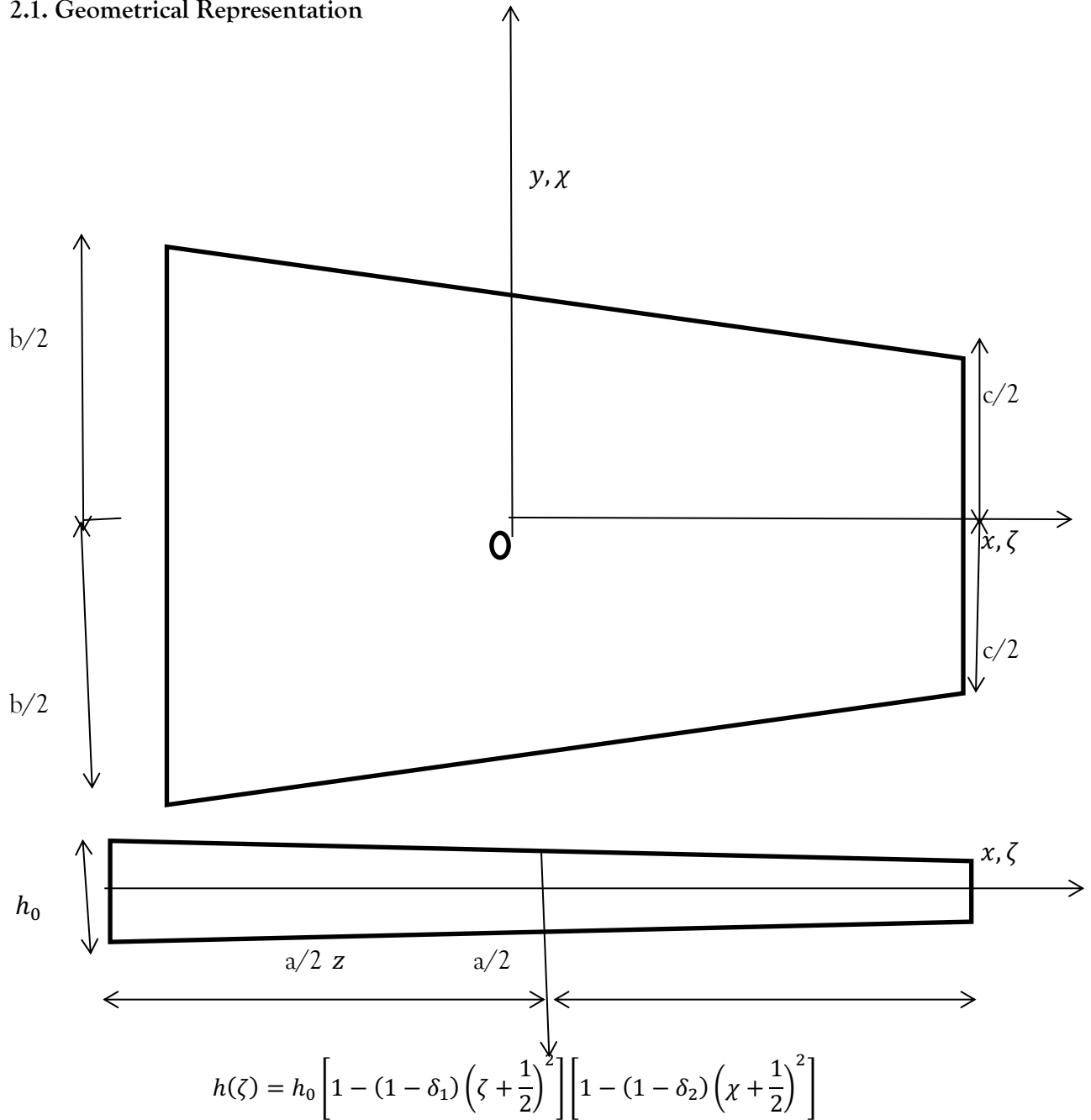


Figure 1. Geometry of orthotropic trapezoidal plate

2.1 Governing Equation Becomes

Expression for maximum strain energy and maximum kinetic energy for orthotropic trapezoidal plate are [1] as

$$P = \frac{ab}{2} \iint \left[\mathfrak{D}_{\zeta} \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + \mathfrak{D}_{\chi} \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + 2\mathfrak{D}_1 \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right) \left(\frac{\partial^2 \psi}{\partial \chi^2} \right) + 4\mathfrak{D}_{\zeta\chi} \left(\frac{\partial^2 \psi}{\partial \zeta \partial \chi} \right)^2 \right] d\zeta d\chi, \quad (1)$$

and

$$K = \frac{ab}{2} \omega^2 h_0 \iint \rho h(\zeta) \psi^2 d\zeta d\chi, \quad (2)$$

where ω is the angular frequency of vibration.

2.2 Deflection function and corresponding boundary condition:

Two term deflection function with boundary condition clamped simply supported-clamped simply supported (C-S-C-S) for vibrational analysis has been considered and can be written as

$$\psi = P_1 \left\{ \left(\zeta + \frac{1}{2} \right) \left(\zeta - \frac{1}{2} \right) \right\}^2 \left\{ \chi - \left(\frac{b-c}{2} \right) \zeta + \left(\frac{b+c}{4} \right) \right\} \cdot \left\{ \chi + \left(\frac{b-c}{2} \right) \zeta - \left(\frac{b+c}{4} \right) \right\} + P_2 \left\{ \left(\zeta + \frac{1}{2} \right) \left(\zeta - \frac{1}{2} \right) \right\}^3 \cdot \left\{ \chi - \left(\frac{b-c}{2} \right) \zeta + \left(\frac{b+c}{4} \right) \right\}^2 \left\{ \chi + \left(\frac{b-c}{2} \right) \zeta - \left(\frac{b+c}{4} \right) \right\}^2, \quad (3)$$

P_1 and P_2 are two unknown constants to be determined.

Thus, for all the non-homogeneous trapezoidal plate eq. (6) satisfied the following conditions clamped simply supported-clamped simply supported (C-S-C-S) for vibrational analysis such as

$$\begin{aligned} \chi &= \frac{c}{4b} - \frac{\zeta}{2} + \frac{1}{4} + \frac{c\zeta}{2b}, \\ \chi &= -\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} - \frac{c\zeta}{2b}, \\ \zeta &= -\frac{1}{2}, \zeta = \frac{1}{2}, \end{aligned} \quad (4)$$

introducing the following non-dimensional variables as $\zeta = \frac{x}{a}$ and $\chi = \frac{y}{b}$.

2.3 Temperature

The general equation of temperature with linear temperature distribution in x-direction is

$$\theta = \theta_0 \left(\frac{1}{2} - \zeta \right) \quad (5)$$

where θ and θ_0 denote the temperature excess above the reference temperature on the plate at any point and at the origin, respectively.

For most orthotropic materials, modulus of elasticity is described as a function of temperature as

$$\begin{aligned} \check{E}_\zeta(\theta) &= \check{E}_1(1 - \gamma\theta), \\ \check{E}_\chi(\theta) &= \check{E}_2(1 - \gamma\theta), \\ \mathcal{G}_{\zeta\chi}(\theta) &= \mathcal{G}_0(1 - \gamma\theta), \end{aligned} \quad (6)$$

where \check{E}_ζ and \check{E}_χ are Young's moduli in x-direction and y-direction, respectively, and $\mathcal{G}_{\zeta\chi}$ is the shear modulus, γ is slope of vibration of moduli with temperature, and \check{E}_1 , \check{E}_2 and \mathcal{G}_0 are the values of moduli at some reference temperature; that is, $\theta = 0$.

Using (3), (4) becomes

$$\begin{aligned} \check{E}_\zeta(\theta) &= \check{E}_1 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right), \\ \check{E}_\chi(\theta) &= \check{E}_2 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right), \\ \mathcal{G}_{\zeta\chi}(\theta) &= \mathcal{G}_0 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right). \end{aligned} \quad (7)$$

where $\delta = \gamma\theta_0$ ($0 \leq \delta < 1$) known as thermal gradient

2.4 Thickness and Density

For tapering in plate thickness has important role as compared to uniform thickness. The general equation of thickness with parabolic variation in both directions is

$$h(\zeta) = h_0 \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right] \quad (8)$$

where h_0 is the maximum plate thickness occurring at the left edge, δh_0 is the minimum plate thickness occurring at the right edge and δ_1, δ_2 are the taper constants.

For non-homogeneity of the plate density is assumed linear in x-direction. The general equation of density with linear variation in x-direction is

$$\rho = \rho_0 \left[1 - (1 - \beta) \left(\zeta + \frac{1}{2} \right) \right] \quad (9)$$

where β is the non-homogeneity constant of the plate, ρ_0 is the mass density at $\zeta = -\frac{1}{2}$.

Also flexural and torsion rigidity is given by [4] as

$$\begin{aligned} D_\zeta &= \frac{\check{E}_\zeta h^3}{12(1 - \nu_\chi \nu_\zeta)}, \\ D_\chi &= \frac{\check{E}_\chi h^3}{12(1 - \nu_\chi \nu_\zeta)}, \\ D_{\zeta\chi} &= \frac{G_{\zeta\chi} h^3}{12}. \end{aligned} \quad (10)$$

Using (5), (10) becomes

$$\begin{aligned} D_\zeta &= \frac{\check{E}_1 h^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right)}{12(1 - \nu_\chi \nu_\zeta)}, \\ D_\chi &= \frac{\check{E}_2 h^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right)}{12(1 - \nu_\chi \nu_\zeta)}, \\ D_{\zeta\chi} &= \frac{G_0 h^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right)}{12}. \end{aligned} \quad (11)$$

Also

$$D_1 = \nu_\chi D_\zeta = \nu_\zeta D_\chi, \quad (12)$$

where h is the thickness of the plate.

After applying boundary conditions (7), (8) and (9) becomes

$$P = \frac{ab}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} + \frac{c\zeta}{2b}} \left[D_\zeta \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + D_\chi \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + 2D_1 \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right) \left(\frac{\partial^2 \psi}{\partial \chi^2} \right) + 4D_{\zeta\chi} \left(\frac{\partial^2 \psi}{\partial \zeta \partial \chi} \right)^2 \right] d\zeta d\chi \quad (13)$$

$$K = \frac{ab}{2} \omega^2 h_0 \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} + \frac{c\zeta}{2b}} [\rho h(\zeta) \psi^2] d\zeta d\chi \quad (14)$$

2.5 Rayleigh Ritz technique

To obtain equation of frequency and vibrational frequency Rayleigh Ritz technique is used according to which

$$\begin{aligned} \delta(P - K) &= 0 \quad \text{implies} \\ \delta(P_1 - \mu^2 K_1) &= 0 \end{aligned} \quad (15)$$

$$P_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} + \frac{c\zeta}{2b}} \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right]^3 \left(1 - \delta \left(\frac{1}{2} - \zeta \right) \right) \left[\left(\frac{\partial^2 \psi}{\partial \zeta^2} \right)^2 + \frac{E_2}{E_1} \left(\frac{\partial^2 \psi}{\partial \chi^2} \right)^2 + 2\nu_\zeta \left(\frac{\partial^2 \psi}{\partial \zeta^2} \right) \left(\frac{\partial^2 \psi}{\partial \chi^2} \right) + 4 \frac{G_0(1 - \nu_\chi \nu_\zeta)}{E_1} \left(\frac{\partial^2 \psi}{\partial \zeta \partial \chi} \right)^2 \right] d\zeta d\chi$$

$$K_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} + \frac{\zeta}{2} - \frac{1}{4} + \frac{c\zeta}{2b}} \left[\rho \left[1 - (1 - \delta_1) \left(\zeta + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \delta_2) \left(\chi + \frac{1}{2} \right)^2 \right] \psi^2 \right] d\zeta d\chi$$

$$\text{Where } \mu^2 = \frac{12 \rho_0 \omega^2 a^5 (1 - \nu \chi \nu \zeta)}{\tilde{E}_1 h_0^2} \quad (16)$$

Equation (15) involves two constants C_1 and C_2 to be evaluated as follows

$$\frac{\partial(P_1 - \mu^2 K_1)}{\partial C_m} = 0 \quad : m=1,2 \quad (17)$$

On simplifying (16) we get

$$T_{m1} C_1 + T_{m2} C_2 = 0 \quad : m=1,2 \quad (18)$$

In this way the frequency equation can be obtained as

$$\begin{vmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{vmatrix} = 0 \quad (19)$$

and results validated with pre-existing literature.

3. RESULTS AND DISCUSSION

With the help of Mathematica Software two values of frequency parameter are calculated for different values of thermal gradient, aspect ratio and non-homogeneity constant respectively.

Table 3.1. In the following table first and second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and taper constant $\delta_1 = \delta_2 = 0.0, 0.6$ and non-homogeneity constant $\beta = 0.0, 0.4$ and fixed values of $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$ are calculated.

δ	$\delta_1 = \delta_2 = 0.0$				$\delta_1 = \delta_2 = 0.6$			
	$\beta = 0.0$		$\beta = 0.4$		$\beta = 0.0$		$\beta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	35.9037	180.673	31.7407	159.776	37.1871	199.76	32.6727	175.225
0.2	32.6292	167.656	28.8452	148.268	34.0926	186.346	29.9531	163.463
0.4	28.9851	153.54	25.6229	135.789	30.6864	171.888	26.9594	150.786
0.6	24.8074	137.988	21.9287	122.9287	26.849	156.098	23.5869	136.942
0.8	19.756	120.446	17.4621	106.535	22.3573	138.52	19.6393	121.531
1.0	12.8182	99.873	11.3282	88.3508	16.6832	118.362	14.6531	103.859

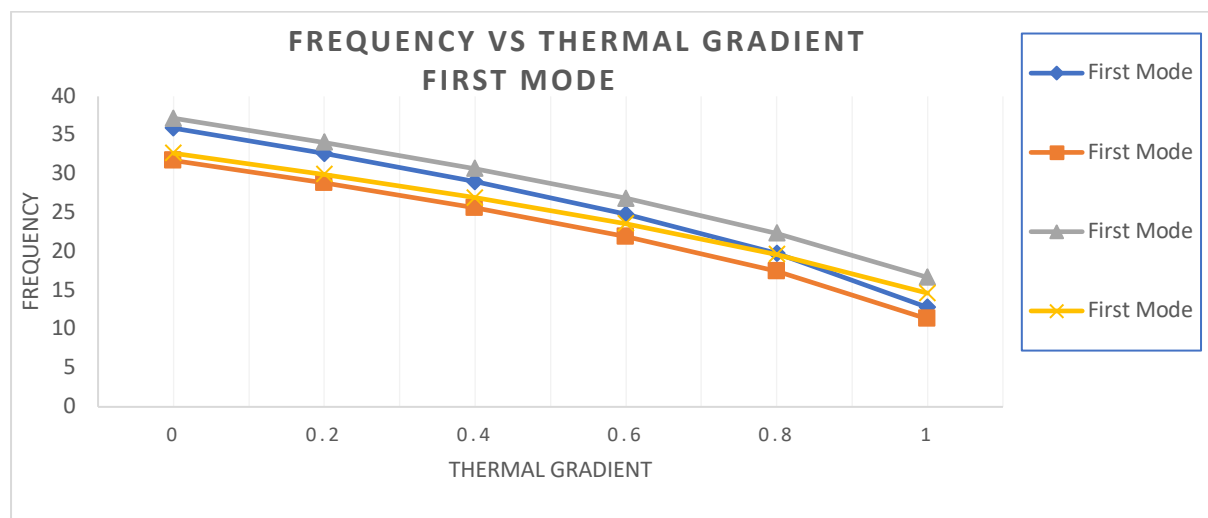


Figure 2. Frequency Vs Thermal Gradient First Mode

Figure 2. Represents the first mode of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, non-homogeneity constant $\beta=0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

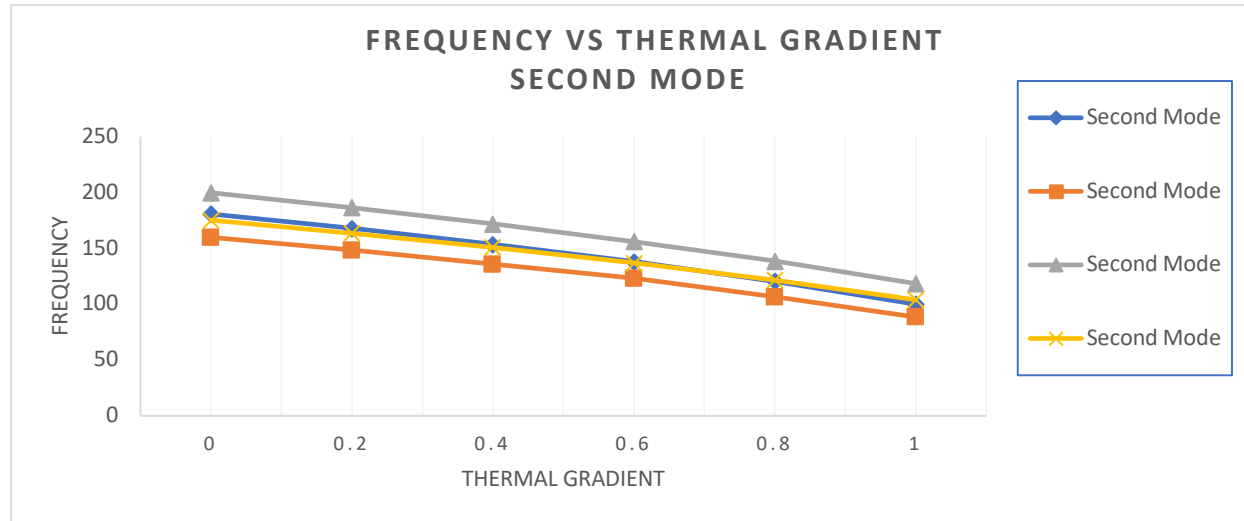


Figure 3. Frequency Vs Thermal Gradient Second Mode

Figure3. Represents the second mode values of frequency parameter μ for orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, non-homogeneity constant $\beta=0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table3.2. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of aspect ratio $\frac{c}{b}$ and constant aspect ratio $\frac{a}{b} = 0.75$, $\beta=0.0$, $\delta_1 = \delta_2=0.0, 0.6$ and $\delta = 0.0$ are calculated.

$\frac{c}{b}$	$\beta = 0.0$							
	$\delta_1 = \delta_2=0.0$ $\delta = 0.0$		$\delta_1 = \delta_2=0.0$ $\delta = 0.4$		$\delta_1 = \delta_2=0.6$ $\delta = 0.0$		$\delta_1 = \delta_2=0.6$ $\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.25	41.4003	168.517	32.9253	139.272	41.6148	176.346	33.4726	147.102
0.50	33.7265	130.69	27.0402	109.179	33.913	138.21	27.662	116.969
0.75	28.2488	102.019	22.895	86.1656	28.8184	109.398	24.0236	94.0792
1.0	24.5683	81.7347	20.1564	69.8007	25.8242	89.5975	22.1197	78.495

When $a/b=0.75$

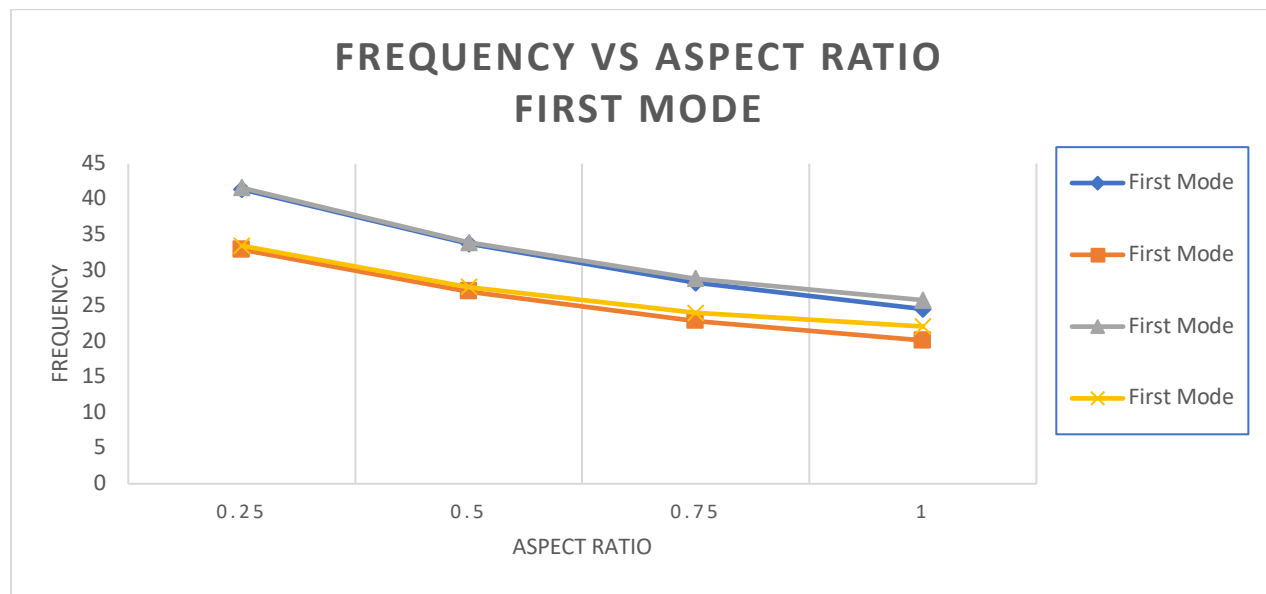


Figure 4. Frequency Vs Aspect Ratio First Mode

Figure 4. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 0.75$, non-homogeneity constant $\beta=0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

When $a/b=0.75$

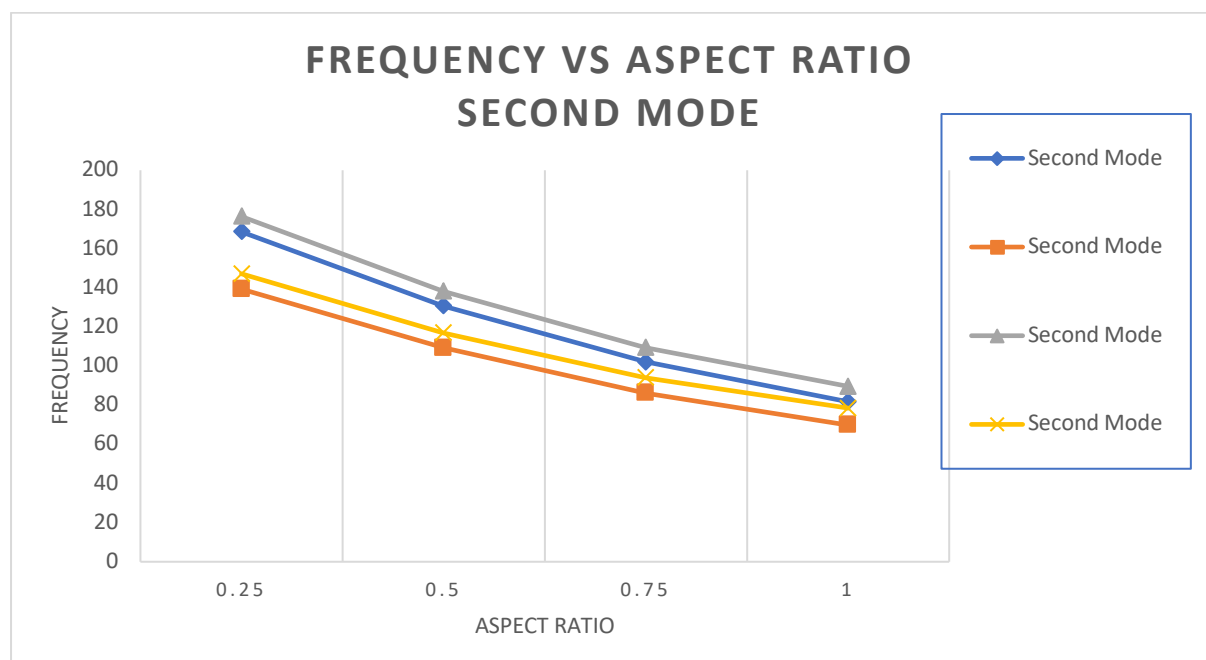


Figure 5. Frequency Vs Aspect Ratio Second Mode

Figure 5. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratios $\frac{a}{b} = 0.75$, non-homogeneity constant $\beta=0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table3.3. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of aspect ratio $\frac{c}{b}$ and constant aspect ratio $\frac{a}{b} = 1.0, \beta=0.0, \delta_1 = \delta_2=0.0$ and $0.6, \delta = 0.0$ are calculated.

$\frac{c}{b}$	$\beta = 0.0$							
	$\delta_1 = \delta_2=0.0$ $\delta = 0.0$		$\delta_1 = \delta_2=0.0$ $\delta = 0.4$		$\delta_1 = \delta_2=0.6$ $\delta = 0.0$		$\delta_1 = \delta_2=0.6$ $\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.25	43.9697	224.113	35.2626	188.29	45.6855	245.024	36.3401	208.309
0.50	35.9037	180.673	28.9851	153.54	37.1871	199.76	30.6864	171.888
0.75	30.1009	144.234	24.5145	123.837	31.4183	160.473	26.3804	139.857
1.0	26.117	115.915	21.4904	100.42	27.8392	129.349	23.9101	114.163

When $a/b=1.0$

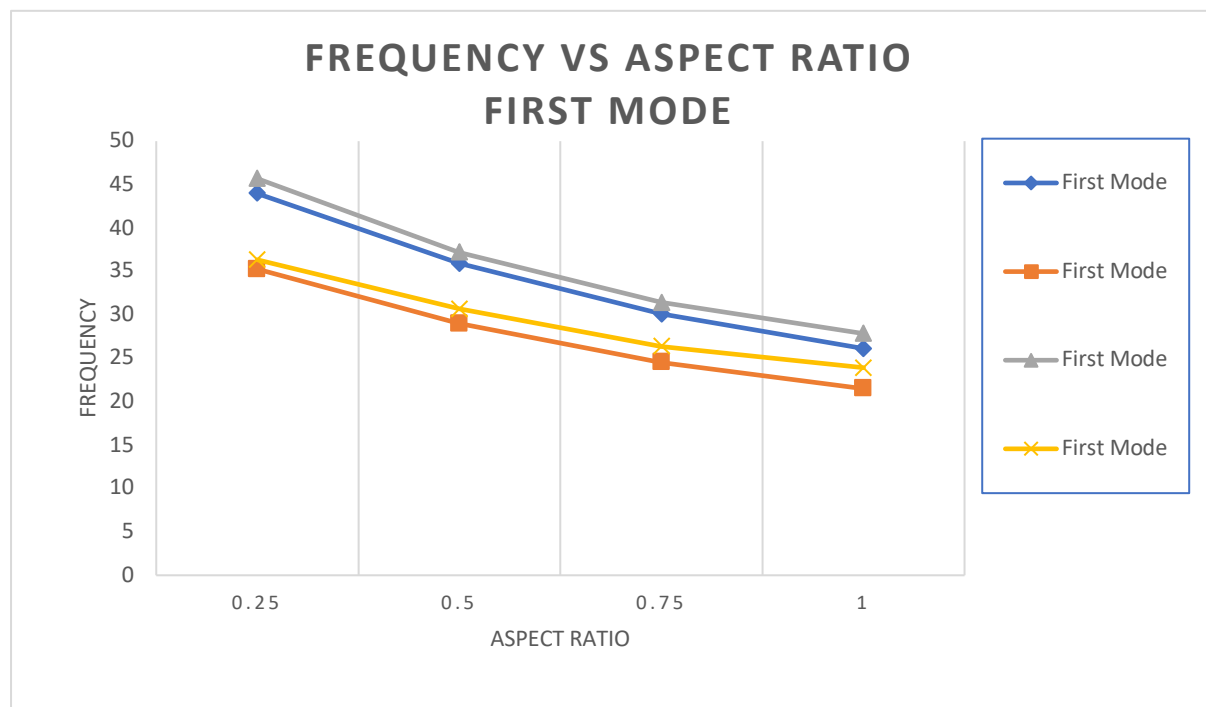


Figure 6. Frequency Vs Aspect Ratio First Mode

Figure 6. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratio $\frac{a}{b} = 1.0$, non-homogeneity constant $\beta=0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

When $a/b=1.0$

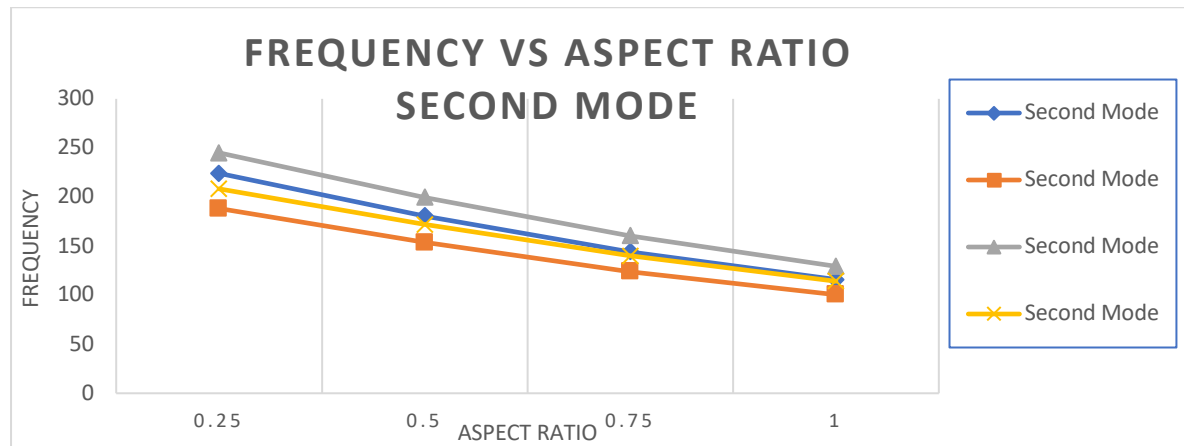


Figure 7. Frequency Vs Aspect Ratio Second Mode

Figure 7. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of thermal gradient δ and constant aspect ratio $\frac{a}{b} = 1.0$, non-homogeneity constant $\beta=0.0$, Thermal gradient $\delta = 0.0, 0.4$ and Taper Constant $\delta_1 = \delta_2 = 0.0, 0.6$.

Table 3.4. In the following table two different values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, $\delta = 0.0$ and 0.4 , $\delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ are calculated.

β	$\delta_1 = \delta_2 = 0.0$				$\delta_1 = 0.2, \delta_2 = 0.6$			
	$\delta = 0.0$		$\delta = 0.4$		$\delta = 0.0$		$\delta = 0.4$	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	35.9037	180.673	28.9851	153.54	36.4769	188.309	28.7572	155.577
0.2	33.6306	169.257	27.1491	143.843	34.1413	176.149	26.9092	145.521
0.4	31.7407	159.776	25.6229	135.789	32.2035	166.097	25.3769	137.21
0.6	30.1374	151.738	24.328	128.96	30.5621	157.602	24.0796	130.188
0.8	28.7547	144.807	23.2115	123.072	29.1484	150.297	22.9629	124.15
1.0	27.5464	138.75	22.2359	117.926	27.9143	143.927	22.7223	122.946

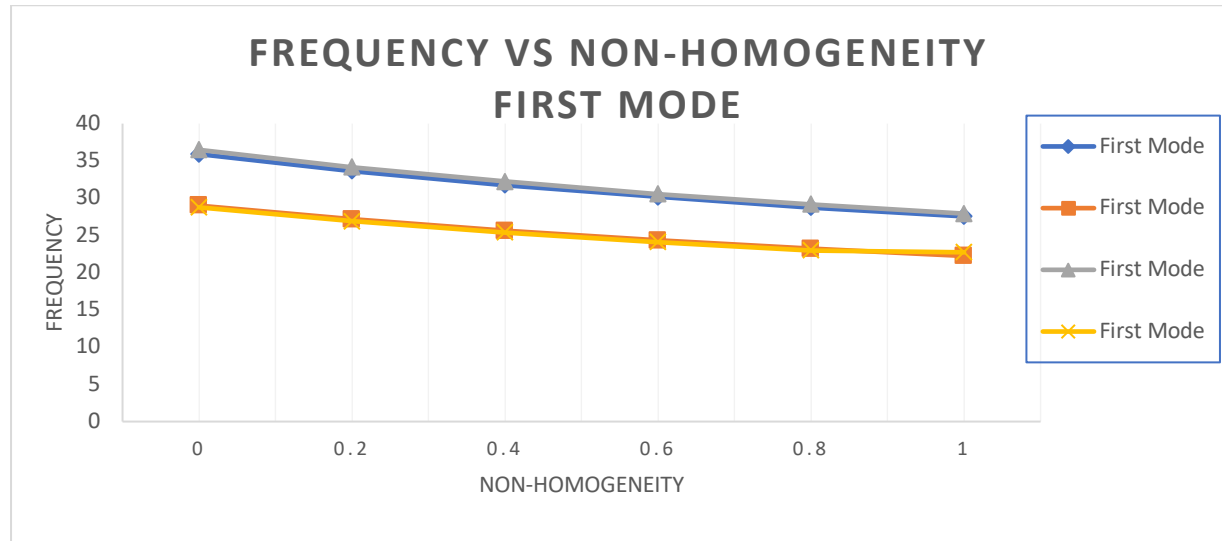


Figure 8. Frequency Vs Non-Homogeneity First Mode

Figure 8. Represents the first mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, $\delta = 0.0, 0.4$ $\delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ respectively.

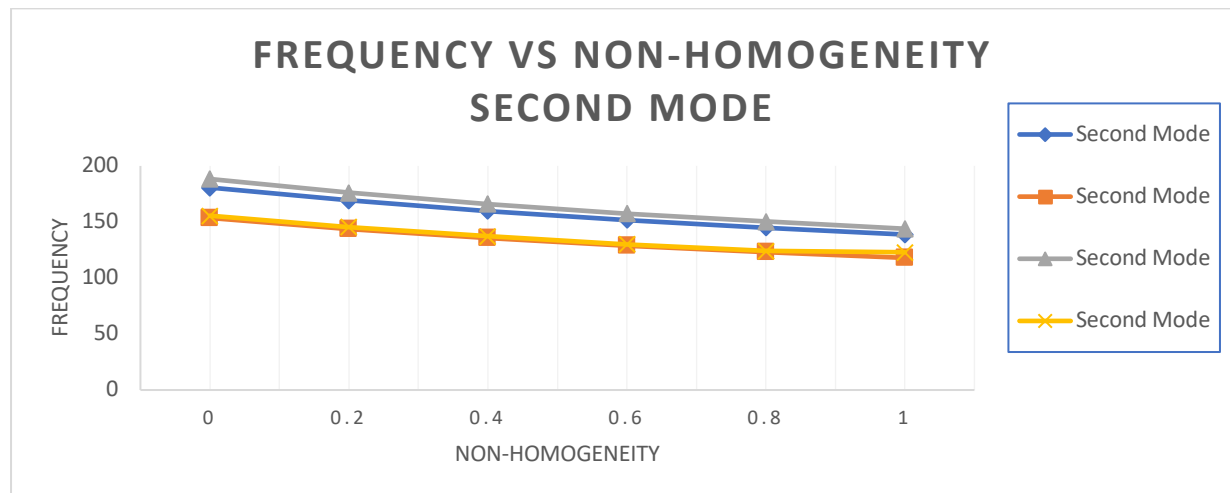


Figure 9. Frequency Vs Non-Homogeneity Second Mode

Figure9. Represents the second mode values of frequency parameter μ for a orthotropic trapezoidal plate for different values of non-homogeneity constant β and aspect ratios $\frac{a}{b} = 1.0, \frac{c}{b} = 0.5$, $\delta = 0.0$ and $0.4, \delta_1 = \delta_2 = 0.0$ and $\delta_1 = 0.2, \delta_2 = 0.6$ respectively.

CONCLUSION

1. It is concluded that with the increase of thermal gradient the value of frequency parameter is decreases. The results are verified with reference [2,4] and are close fit to previous results.
2. It is concluded that with the increase of aspect ratio the value of frequency parameter is decreases. The results are verified with reference [2,4] and are close fit to previous results.

3. It is concluded that with the increase non-homogeneity constant the value of frequency parameter is also decreases. The results are verified with reference [2,4] and are close fit to previous results.

These types of mathematical models are very useful in engineering structures. The main aim to develop such type of mathematical model is to attract the mind of mechanical engineers in that direction. By using such type of models we increase the efficiency and minimizing the cost of manufacturing. Also developing new models to grow the technology towards excellence.

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