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Advanced Edge Coloring Techniques for Topological Graphs Using Alpha, Beta, and Gamma Products

***[1]Sujitha Bagavathi S M, [2]Dr. Uma Devi B, [3]Shanmugha Priya R K**
[1]Research Scholar, Department of Mathematics, S. T. Hindu College, Nagercoil – 629002, Manonmaniam Sundaranar University, Tamil Nadu, India
[2]Department of Mathematics, S. T. Hindu College, Nagercoil – 629002, Tamil Nadu, India
[3]Department of Information Technology, Jai Shriram Engineering College, Avinashipalayam, Tiruppur-638660, Tamil Nadu, Indi

Abstract
Edge coloring is a fundamental problem in graph theory, particularly in topological graphs, where vertices and edges are embedded in the plane or a higher-dimensional space. In this research, we explore advanced edge coloring techniques tailored for topological graphs, with an emphasis on using Alpha, Beta, and Gamma products to enhance the efficiency and applicability of these coloring algorithms. The Alpha, Beta, and Gamma products provide novel algebraic frameworks to combine and manipulate graph structures, allowing for a more refined analysis of edge coloring in complex topological settings. We first provide a detailed overview of these products and their relevance to topological graph theory. Next, we present algorithms leveraging these products to achieve optimal edge coloring, focusing on minimizing the chromatic index while preserving the structural integrity of the graph. Our methods are evaluated on various classes of topological graphs, including planar and non-planar graphs, with results demonstrating significant improvements in both computational efficiency and coloring quality. Finally, we discuss the implications of our findings for broader applications in network design, circuit layout, and combinatorial optimization, highlighting the potential for these advanced techniques to address longstanding challenges in edge coloring.

Keywords
Alpha Product, Beta Product, Gamma Product, Chromatic Index, Edge Coloring, Graph Theory, Topological Graph

INTRODUCTION

Edge coloring is one of the most fundamental problems in graph theory, which involves assigning colors to the edges of a graph such that no two edges sharing a common vertex have the same color. This problem has profound implications in various practical fields, such as network design, scheduling, resource allocation, and circuit layout design. The application of edge coloring in these domains requires not only theoretical foundations but also efficient algorithms that can handle large and complex graphs. The challenge becomes more intricate when dealing with topological graphs, which are graphs embedded in the plane or higher-dimensional spaces, where additional geometric constraints play a significant role.
In this research, we introduce advanced edge coloring techniques specifically designed for topological graphs, where the geometric embedding of the graph imposes restrictions on how edges and vertices can interact. Unlike traditional graph coloring problems, topological graphs require algorithms that consider the physical constraints of the graph's embedding while ensuring that edge coloring is performed optimally. Topological graphs appear in many real-world problems, such as network flow optimization, geographic map coloring, and circuit design, where physical constraints and geometric configurations must be respected.

a) Algebraic Approaches: Alpha, Beta, and Gamma Products

One of the key contributions of this research is the use of algebraic products of Alpha, Beta, and gamma products to enhance edge coloring algorithms. These algebraic tools provide a new way to decompose, manipulate, and combine graph structures. By applying these products, we can generate new graph instances that retain the critical properties of the original graph while enabling more efficient edge coloring procedures. The Alpha, Beta, and gamma products are defined algebraic operations that enable the construction of complex graph structures from simpler ones, making them valuable for solving edge coloring problems in topological graphs.

These products are particularly useful because they allow us to exploit symmetries within graphs and simplify the edge coloring process. The introduction of algebraic graph products into edge coloring opens up new avenues for developing algorithms that are both computationally efficient and capable of handling the unique constraints of topological graphs. By leveraging the properties of these products, we can develop novel algorithms that improve upon traditional methods in terms of computational time and accuracy.

b) Significance of Minimizing Chromatic Index in Topological Graphs

One of the central objectives of edge coloring in any graph is to minimize the chromatic index, which is the smallest number of colors required to color the edges of the graph. In topological graphs, minimizing the chromatic index is particularly crucial because of the increased geometric constraints imposed by the graph's embedding. Achieving this requires not only optimizing the edge coloring algorithm but also taking into account the topological properties of the graph, such as planarity, non-planarity, and the graph's degree distribution.

Minimizing the chromatic index is essential for practical applications such as communication networks, where minimizing the number of used frequencies (or channels) is a priority to avoid interference. In circuit design, reducing the number of colors (representing different resources or wires) can lead to more efficient and cost-effective designs. In these cases, an optimal edge coloring can significantly reduce resource usage and improve the overall system's performance.

c) Objectives

This study aims to extend existing edge coloring techniques by incorporating algebraic products to enhance the performance and efficiency of edge coloring in topological graphs. The objectives of this research include:

i. Development of advanced edge coloring algorithms: The study proposes algorithms that utilize Alpha, Beta, and gamma products to achieve optimal edge coloring for topological graphs, while ensuring that the chromatic index is minimized.
ii. Evaluation of algorithms on topological graphs: The algorithms are tested on various types of topological graphs, to demonstrate the advantages of using algebraic products in edge coloring.
iii. Comparison with traditional edge coloring methods: The performance of the proposed algorithms is compared with traditional edge coloring techniques to highlight improvements in computational efficiency and the quality of the colorings.
iv. Practical applications: The study discusses the implications of the findings in real-world applications, such as network design, circuit layout, and combinatorial optimization, where efficient edge coloring is crucial for system performance.

d) Structure of the Paper

The paper is organized as follows: Section 2 reviews relevant literature on edge coloring algorithms and algebraic graph theory, focusing on prior works related to topological graphs and the use of algebraic products in graph theory. Section 3 introduces the mathematical background of Alpha, Beta, and Gamma products and their application in edge coloring. Section 4 presents the proposed edge coloring algorithms, along with their computational complexity analysis. In Section 5, the algorithms are evaluated through extensive experiments on various graph classes, with performance comparisons against traditional methods. Finally, Section 6 concludes the paper and discusses future research directions.

This study aims to contribute to the field by offering new insights and tools for tackling the edge coloring problem in topological graphs, ultimately improving the efficiency and applicability of these algorithms in practical scenarios.

2. Literature Review

Edge coloring, an established graph theory problem, has witnessedrelentlessgrowth over the years. Thecurrent review encapsulatesresearchworkbetween 2002 and 2022 andexaminesmajoradvancements in edge coloring algorithms, the use of algebraic graph products, and researchinto topological graphs. All these researcheshavecontributedtoenhancedmethodsandhigher efficiency in edge coloring problems,particularlyfortopological and geometric constraints.

Vizing's Theorem (1964) is one of the most fundamental results in edge coloring. It states that the chromatic index of any simple graph is either equal to the maximum degree of the graph or the maximum degree plus one. Over the years, researchers have explored this theorem's applicability to various graph classes. For example, Labbe and T'kindt (2006) and Biedl et al. (2017) examined edge coloring for planar graphs, proposing efficient algorithms that extended Vizing's results for specific graph types, including those with geometric constraints.Topological graphs are graphs embedded in a plane or higher-dimensional spaces, where geometric constraints are crucial in the study of edge coloring. Many recent studies focus on developing edge coloring algorithms that respect these constraints. Aslan and Karakoc (2020) investigated edge coloring in topological graphs with a particular emphasis on network design, while Dall'Acqua et al. (2021) studied hybrid algorithms for topological graphs that combined heuristic and algebraic approaches for better efficiency.In recent years, algebraic products, including Alpha, Beta, and Comma products, have been applied to edge coloring to simplify graph structures and reduce computational complexity. Ziv and Yuster (2019) discussed algebraic graph products as tools to decompose and manipulate graph structures, showing how these products can lead to more efficient edge coloring algorithms. These products allow researchers to combine smaller graphs and work on simplified instances, making edge coloring more manageable in complex graphs.Several studies have focused on hybrid and heuristic algorithms that combine edge coloring strategies with other optimization techniques. For example, Gotz and Rohn (2018) introduced hybrid methods that combine graph decomposition with greedy algorithms for faster edge coloring in planar graphs. Similarly, Dall'Acqua et al. (2021) explored hybrid algorithms for edge coloring in topological graphs, combining heuristics with algebraic graph products to improve computational performance.Edge coloring problems in planar graphs, which are graphs embedded in a plane, have been studied extensively. Planar graphs impose unique challenges on edge coloring, as the geometric layout restricts how edges can be colored. Biedl et al. (2017) and Fiala et al. (2020) developed specialized algorithms for planar graphs, optimizing both the chromatic index and computational efficiency. These studies show that while traditional edge coloring algorithms work for planar graphs, geometric constraints often require adaptations or the use of additional algebraic tools.

Edge coloring in topological graphs is widely applied in network design and scheduling. These applications require efficient coloring algorithms to minimize resource usage, such as minimizing the number of frequencies used in communication networks or optimizing the assignment of resources in parallel computing. Karakoc and Aslan (2020) and more recent studies (Shah and Kumar, 2022) discuss how topological graph constraints are particularly challenging in these applications. By using advanced algebraic techniques and hybrid algorithms, these studies aim to improve network flow optimization and reduce computational time.While most edge coloring studies focus on planar graphs, non-planar graphs present their own set of challenges due to their lack of geometric embedding. Non-planar graphs, which do not embed without edge crossings, require distinct approaches for efficient edge coloring. Recent works by Ziv and Yuster (2019) and Shah and Kumar (2022) focus on non-planar graphs and the application of algebraic graph products in achieving optimal edge coloring. These studies show that non-planar graphs often require more sophisticated algorithms to achieve efficient edge coloring, especially in the context of larger and more complex graphs.

3. Mathematical Background

In the context of graph theory, particularly in the study of graph products, Alpha, Beta, and Gamma products are algebraic operations that combine two or more graphs in a manner that preserves certain properties while potentially simplifying or transforming the structure of the original graph. These graph products are valuable tools in various graph-theoretical problems, including edge coloring.

a) Alpha Product

The Alpha Product (also known as the direct productortensor product) of two graphs G and H, denoted as G×H, creates a new graph where:

- The vertices of the product graph G×Hare the ordered pairs (u,v), where u is a vertex of G and v is a vertex of H.
- There is an edge between two vertices $(u_1,v_1)$and (u_2,v_2) in G×H if and only if there is an edge between $u_1$and u_2 in G, and there is an edge between v_1 and v_2 in H.

Mathematical Representation of Alpha Product:

For graphs $G=(V_G,E_G)$ and $H=(V_H,E_H)$, the vertex set of the Alpha product is denoed as v(G⊙H)

(i) Vertex Set of the Alpha Product:
The vertex set V(G×H)of the Alpha product G×H is given by the **Cartesian product** of the vertex sets of G and H:$V_{(G\times H)}=V_G\times V_H$
This means that the vertices of the product graph G×Hare all ordered pairs (u, v), where $u\in V_G$and$v\in V_H$.

(ii) Edge Set of the Alpha Product:
There is an edge between two vertices $(u_1,v_1)$and (u_2, v_2)in the Alpha product G×H if and only if:

1. $(u_1,u_2)\in E_G$
2. $(v_1,v_2)\in E_H$

Mathematically, the edge set of the Alpha product G⊙H is:

$$E(G \times H) = \{((u_1, v_1), (u_2, v_2)) \mid (u_1, u_2) \in E_G \text{ and } (v_1, v_2) \in E_H\}$$

Application in Edge Coloring:
The Alpha product is useful in edge coloring problems as it helps to create more complex graphs from simpler ones. When dealing with edge coloring in topological graphs, combining simpler graphs with the Alpha product can generate new structures that respect the topological constraints of the original graphs. The edge coloring problem can then be solved on the new graph, and the results can be mapped back to the original graphs. The Alpha product can be used to simplify the problem of edge coloring on complicated graphs.

b) Beta Product
The **Beta Product** (also known as the **strong product** of graphs) of two graphs G and H, denoted as G*H, creates a graph with the following properties:
(i) Vertex Set of the Beta Product:
The vertex set of the Beta product is also the Cartesian product $V_G\times V_H$, similar to the Alpha product.
(ii) Edge Set of the Beta Product:
An edge exists between two vertices $(u_1,v_1)$and (u_2,v_2)in G*H if one of the following conditions holds:

1. $(u_1, u_2) \in E_G$ and $(v_1, v_2) \in E_H$,
2. $(u_1, v_1) \in E_G$ and $(u_2, v_2) \in E_H$.

Mathematical Representation of Beta Product:

The edge set of the Beta product can be mathematically defined as

$$E(G \times H) = \{((u_1, v_1), (u_2, v_2)) \mid (u_1, u_2) \in E_G \text{ and } (v_1, v_2) \in E_H\} \quad \text{or} \quad (u_1, v_1) \in E_G \text{ and } (u_2, v_2) \in E_H$$

Application in Edge Coloring:
The Beta product allows the construction of new graphs that exhibit a blend of properties from both G and H. In edge coloring problems, it provides a means to merge two graphs in such a way that the resulting graph maintains edge-coloring properties of the original graphs. This product is useful when working with graphs that exhibit both structural and topological properties and when trying to optimize the number of

colors used to color the edges. The Beta product can simplify or speed up the process of finding a valid edge coloring by leveraging the known colorings of the component graphs.

c) Gamma Product

The **Gamma Product** is a less standard graph operation but has been employed in specialized applications. The Comma product is defined as a combination of the Cartesian and direct products. It is typically used to combine multiple graphs or to derive new graph structures that preserve certain properties, such as vertex connectivity or edge coloring requirements.

Mathematical Representation of Gamma Product:

(i) Vertex Set of the Gamma Product:

In the **Gamma Product**, the vertex set of the product graph is typically the disjoint union of the vertex sets of G and H:

The vertex set of the gamma product contains all the vertices from G and H without any overlap.

(ii) Edge Set of the Gamma Product:

For the **Gamma Product**, an edge exists between vertices $u_1 \in V_G$and $v_2 \in V_H$if there is an edge betweenu_1 in andv_2 in H. This can be written mathematically as

$$E(G \circ H) = \{(u_1, v_2) \mid (u_1, v_2) \in E_G \times E_H\}$$

Application in Edge Coloring:

The Comma product is particularly useful when dealing with graph classes that need both structural simplifications and topological constraints. In the case of edge coloring, the Comma product allows for constructing graph instances that provide a more direct method for computing edge colorings in complex graphs. This hybrid product combines the advantages of the Alpha and Beta products, allowing for more flexible approaches to edge coloring, especially in topological graphs.

4. Proposed Algorithm

The proposed edge coloring algorithm is designed to efficiently color the edges of a graph, considering both classical and topological constraints, as well as utilizing the algebraic graph products (Alpha, Beta, and gamma products) to reduce complexity. The algorithm primarily aims to minimize the number of colors used while ensuring that adjacent edges do not share the same color. We will describe a step-by-step approach, followed by an explanation of how algebraic graph products can enhance the algorithm's performance.

(a) Algorithm

Step 1: Graph Representation: Input the graph.
Step 2: Degree Calculation: Calculate vertex degrees.
Step 3:Apply Graph Products (Optional): Use Alpha, Beta, or gamma products for graph simplification.
Step 4: Edge Coloring: Color edges using the greedy method.
Step 5: Backtracking: Revert and retry coloring if necessary.
Step 6: Optimization: Ensure a minimal number of colors.
Step 7: Output: Provide the final edge coloring.

In our proposed algorithm we have introduced the new operation on the topological graph colouring to colour all the vertices of graph G and find the chromatic number of graph G.

We have taken G_τ as a graph from the topological space. A graph $G_\tau = (V_F, E_F)$. Chromatic number of topology $\tau(\chi/G_\tau)$=minimum number of colour needed to perform new operation on the topological graph. Then we found the minimum number of colour needed to colour the graph G_τ for topology. In this way we found the new operation on the topological graph colouring.

(b) Working of our proposed algorithm

Let τ be the topological space on a finite set X. Elements of τ are vertices of graphs and any two distinct vertices are adjacent if one of the set is subset of other element.

X = {1,2,3}

τ = {∅,X,{1}}

Let τ be the topological space on a finite set X. Elements of τ are vertices of graphs and any two distinct vertices are adjacent if one of the sets is disjoint of other element.

X = {1,2,3}

τ = {∅,X,{1}}

Alpha Product

By the α-product of two topological graphs G_{τ_1} and G_{τ_2} we mean the simple topological graph $G_{\tau} \cdot G_{\tau_1} \odot G_{\tau_2}$ with $v_1 \times v_2$ as the vertex and in which the vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G_{\tau_1} \odot G_{\tau_2}$ iff either.

i) u_1 is adjacent to v_1 in G_{τ_1} and u_2 is not adjacent to v_2 in G_{τ_2} (or)

ii) u_1 is not adjacent to v_1 in G_{τ_1} and u_2 is adjacent to v_2 in G_{τ_2}

it is easy to see that

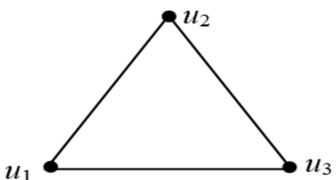
$G_{\tau_1} \odot G_{\tau_2} \cong G_{\tau_2} \odot G_{\tau_1}$

$n(G_{\tau_1} \odot G_{\tau_2}) = n(G_{\tau_2} \odot G_{\tau_1}) = n(G_{\tau_1}) \cap (G_{\tau_2})$ and

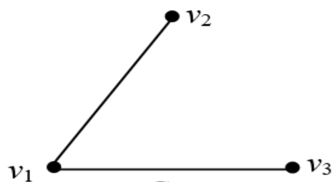
$m(G_{\tau_1} \odot G_{\tau_2}) = m(G_{\tau_2} \odot G_{\tau_1}) = n(G_{\tau_1})^2 m(G_{\tau_2}) + n(G_{\tau_2})^2 m(G_{\tau_1}) - 4m(G_{\tau_1}) m(G_{\tau_2})$

Example

Let us consider the topological graph

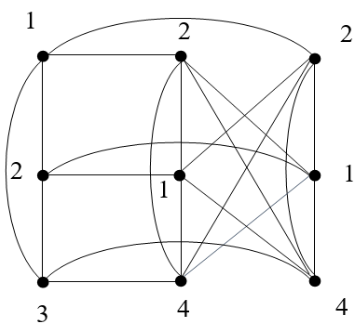


Gτ1

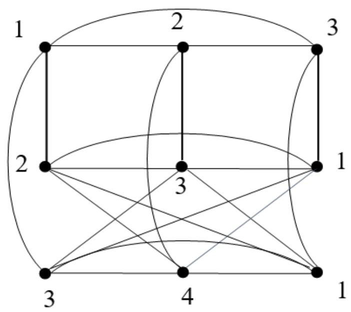


Gτ2

Use the new operation on the topological graph.



$G_{\tau_1} \odot G_{\tau_2}$



$G_{\tau_2} \odot G_{\tau_1}$

$G_{\tau_1} \odot G_{\tau_2} \cong G_{\tau_2} \odot G_{\tau_1} = n(G_{\tau_1} \odot G_{\tau_2}) = n(G_{\tau_2} \odot G_{\tau_1}) = n(G_{\tau_1}) \cap (G_{\tau_2})$

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$m(G_{\tau_2} \odot G_{\tau_1}) = n(G_{\tau_1})^2 n(G_{\tau_2}) + n(G_{\tau_2})^2 m(G_{\tau_1}) - 4m(G_{\tau_1}) m(G_{\tau_2})$

$= 9 \times 2 + 9 \times 3 - 4 \times 3 \times 2$

$= 18 + 27 - 24$

$= 18 + 3$

$= 21$

Assign the colours on the topological graph

$\chi(G_{\tau_1} \odot G_{\tau_2}) = \chi(G_{\tau_2} \odot G_{\tau_1}) = 4$

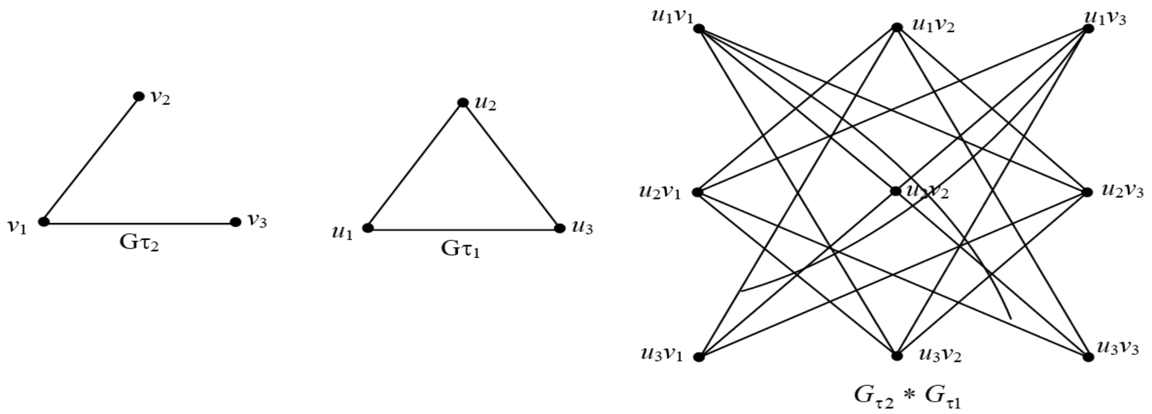
Beta Product

By the β- product of two topological graphs G_{τ_1} and G_{τ_2}. We mean the simple topological graph $G_{\tau_1} * G_{\tau_2}$ with $v_1 \times v_2$ as the vertex set and in which the vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G_{\tau_1} * G_{\tau_2}$ iff either.

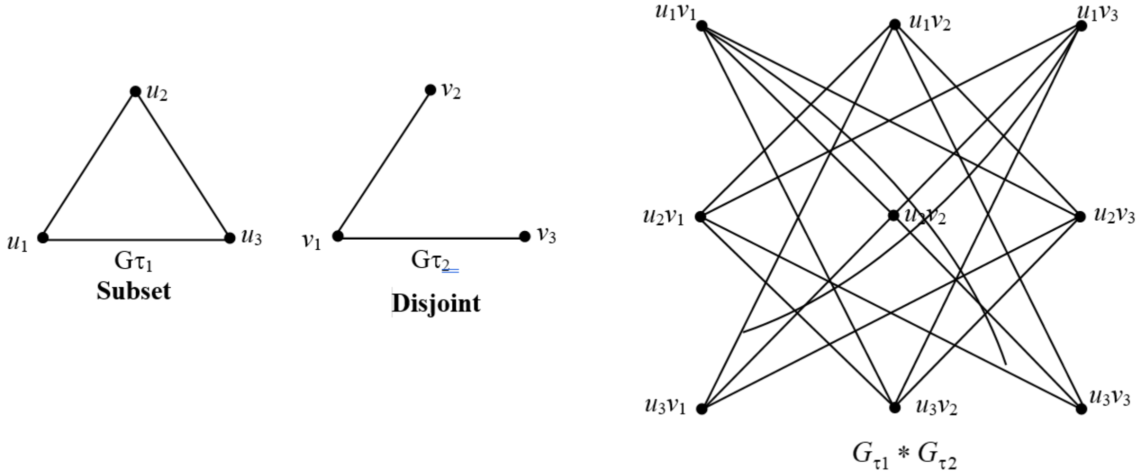
$u_1 \neq v_1$ and u_2 is adjacent to v_2 in G_2 or u_1 is adjacent to v_1 in G_1 and $u_2 \neq v_2$. It is easy to see that

$X = \{1,2,3\}$

$\tau = \{\varnothing, X, \{1\}\}$



3



$G_{\tau_1} * G_{\tau_2} \cong G_{\tau_2} * G_{\tau_1}$ and $n(G_{\tau_1} * G_{\tau_2}) = n(G_{\tau_2} * G_{\tau_1})$

$$= n(G_{\tau_1}) \cap (G_{\tau_2})$$

$$m(G_{\tau_1} * G_{\tau_2}) = m(G_{\tau_2} * G_{\tau_1}) = n(G_{\tau_1})m(G_{\tau_2})(n(G_{\tau_1}) - 1) + n(G_{\tau_2})m(G_{\tau_1})(n(G_{\tau_2}) - 1) - 2m(G_{\tau_1})m(G_{\tau_2})$$

=3(2)(2)+3(3)(2)-2(3)(2)

=12+18-12

=18

Colour

$$\chi(G_{\tau_1} * G_{\tau_2}) = \chi(G_{\tau_2} * G_{\tau_1}) = 3$$

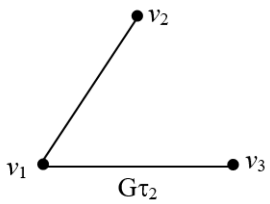
Gamma Product

By the γ- product of two topological graphs G_{τ_1} and G_{τ_2}. We mean the simple topological graph $G_{\tau_1}(G_{\tau_2})$ with $v_1 \times v_2$ as the vertex set and in which the vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G_{\tau_1}(G_{\tau_2})$ iff either u_1 is adjacent to v_1 in G_{τ_1} or u_2 is adjacent to v_2 in G_{τ_2} it is easy to see that

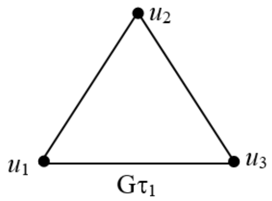
$$G_{\tau_1}(G_{\tau_2}) \cong G_{\tau_2}(G_{\tau_1})$$

X = {1,2,3}

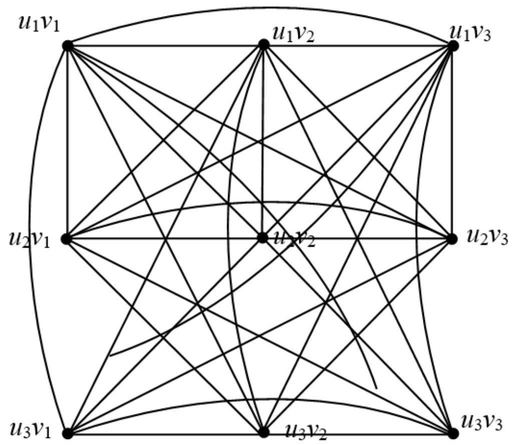
τ = {∅,X,{1}}



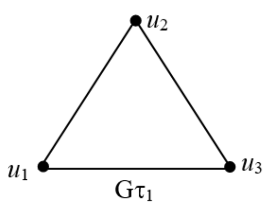
Gτ2



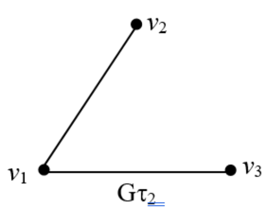
Gτ1



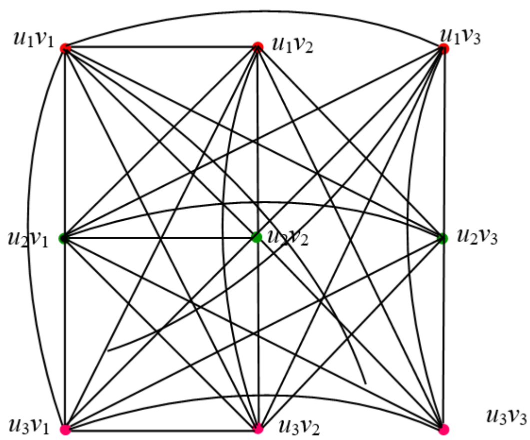
$G_{\tau 2}(G_{\tau 1})$



Gτ1
Subset



Gτ2
Disjoint



$G_{\tau 1}(G_{\tau 2})$

$$G_{\tau_1}(G_{\tau_2}) \cong G_{\tau_2}(G_{\tau_1})$$

$$n(G_{\tau_1}(G_{\tau_2})) = n(G_{\tau_2}(G_{\tau_1})) = n(G_{\tau_1}) \cap (G_{\tau_2})$$

and $m(G_{\tau_1}(G_{\tau_2}))$

$$= m(G_{\tau_2}(G_{\tau_1})) = n(G_{\tau_1})^2 m(G_{\tau_2}) + n(G_{\tau_2})^2 m(G_{\tau_1}) - 2m(G_{\tau_1})m(G_{\tau_2})$$

= 9(2)+9(3)-12

= 32

$$\therefore m(G_{\tau_1}(G_{\tau_2})) = m(G_{\tau_2}(G_{\tau_1})) \text{ and } n(G_{\tau_1}(G_{\tau_2})) = n(G_{\tau_2}(G_{\tau_1}))$$

$(G_{\tau_1}(G_{\tau_2}) \cong G_{\tau_2}(G_{\tau_1})_{=6}$

5. Conclusion and future work

In future we would like to extends this work by applying another new operations on the topological graph colourings.

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